SYLLABUS

MA 8452 - STATISTICS AND NUMERICAL METHODS

UNIT - I

TESTING OF HYPOTHESIS

Sampling distributions - Estimation of parameters - Statistical hypothesis - Large sample tests based on Normal distribution for single mean and difference of means -Tests based on t, Chi-square and F distributions for mean, variance and proportion - Contingency table (test for independent) - Goodness of fit.

UNIT - II

DESIGN OF EXPERIMENTS

One way and two way classifications - Completely randomized design – Randomized block design – Latin square design - 2^2 factorial design.

UNIT - III

SOLUTION OF EQUATIONS AND EIGENVALUE PROBLEMS

Solution of algebraic and transcendental equations - Fixed point iteration method – Newton Raphson method - Solution of linear system of equations - Gauss elimination method – Pivoting - Gauss Jordan method – Iterative methods of Gauss Jacobi and Gauss Seidel - Eigenvalues of a matrix by Power method and Jacobi's method for symmetric matrices.

UNIT - IV

INTERPOLATION, NUMERICAL DIFFERENTIATION AND NUMERICAL INTEGRATION

Lagrange's and Newton's divided difference interpolations – Newton's forward and backward difference interpolation – Approximation of derivates using interpolation polynomials – Numerical single and double integrations using Trapezoidal and Simpson's 1/3 rules.

UNIT - V

NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATIONS

Single step methods : Taylor's series method - Euler's method - Modified Euler's method - Fourth order Runge-Kutta method for solving first order equations - Multi step methods : Milne's and Adams - Bash forth predictor corrector methods for solving first order equations.

MA6452 STATISTICS AND NUMERICAL METHODS

QUESTION BANK

UNIT I

TESTING OF HYPOTHESIS

PART A

1. What you mean by test of hypothesis?

Solution:

On the basis of sample statistics, the study of difference between the observed sample statistics and the hypothetical population parameter value is significant or not is defined as test of significance.

2. State the difference between parameter and statistic. $[AU2010]$

Solution:

The statistical constants like mean μ , variance σ^2 computed from population are called parameters where as statistical constants like mean $\bar{x}$, variance s^2 etc., computed from a sample are called sample statistics.

3. Define power of a statistical test?

Solution:

The probability of type - II error is denoted by β and $(1-\beta)$ is defined as power of the statistical test.

4. Define level of significance.

Solution:

The probability of Type I error is called the level of significance of the test and is denoted by α.

5. Define Type I error $[2012, 2014]$

Solution:
The rejection of H_0 when it is true.

6. Define Type II error $[2012, 2014]$

Solution:
The acceptance of H_0 when it is false.

7. Define critical region.

Solution:
For a test statistic, the area under the probability curve, which is normal, is divided into two regions namely the region of acceptance of H_0 and the region of rejection of H_0. The region in which H_0 is rejected is called the critical region. The area of the critical region is α, the level of significance.

8. Define statistical hypothesis

Solution:
In making statistical decisions, we make assumptions or guesses about populations involved. Such assumptions, which may be true or false are called statistical hypothesis.

9. Define null hypothesis $[AU2012]$

Solution:
For applying the test of significance, we first set up a hypothesis which is a statement about the population parameter. This statement is usually a hypothesis of no difference and so it is called null hypothesis and is denoted by H_0. The purpose of null hypothesis is for possible rejection under the assumption that it is true.

10. Define alternate hypothesis $[AU2012]$

Solution:
Suppose the null hypothesis is false, then something else must be true. This is called an alternate hypothesis and is denoted by H_1.

11. Define One tailed and Two-tailed test.

Solution:

One tailed test: In a test of any statistical hypothesis, if the alternate hypothesis is one-sided then it is called one tailed test. It may be right tailed or left tailed.

Two tailed test: In a test of statistical hypothesis, if the alternate hypothesis is two sided, then it is called a two tailed test.

12. Define Large and small samples

Solution:

The number of elements in a sample is greater than or equal to 30, then the sample is called a large sample and if it is less than 30, then the sample is called a small sample.

13. State the application of t test

Solution:

Testing the significance of the difference between
(i) The mean of a sample and the mean of the population.
(ii) The means of two samples.

14. State the application of F test

Solution:

(i) Testing the significance of the difference between the variances of two populations from which two samples are drawn.
(ii) analysis of variance.

15. State the application of Chi-square test [$AU2010$]

Solution:

1. It is used to test the goodness of fit.
2. It is used to test the independence of attributes.
3. To test the homogeneity of a given data.

16. Write conditions for the application of Chi-square test. $[AU2014]$

Solution:

1. The experimental data (or sample deviations) must be independent of each other.
2. The sample size should be reasonably large, $\geq$ 50.
3. The theoretical cell frequency should be atleast 5. If it is less than 5, it is combined with adjacent frequencies so that the pooled frequency is $>$ 5.
4. The constraints on the cell frequencies should be linear.
   eg., $\sum O_i = \sum E_i = N \geq 50$.

17. Define attribute

Solution:

An attribute is a characteristic or a quality which may be present amongst the members of a population.
For example, tall, short, healthy, black etc.

18. Define 2×2 contigency table.

Solution:

Let A and B two attributes. Dividing A into A_1 and A_2 and B into B_1, B_2, we get the following 2×2 table, called the 2×2 table.

B \ A	A_1	A_2	Total
B_1	$(A_1 B_1)$	$(A_2 B_1)$	(B_1)
B_2	$(A_1 B_2)$	$(A_2 B_2)$	(B_2)
Total	(A_1)	(A_2)	N

19. Give the formula for the χ^2-test of independence for

a	b
c	d

[AU2012]

Solution:

A \ B	***A***	***α***	**Total**
B	a	b	$a+b$
β	c	d	$c+d$
Total	$a+c$	$b+d$	$N=a+b+c+d$

$\therefore$ the value of $\chi^2 = \dfrac{N(ad-bc)^2}{(a+b)(c+d)(a+c)(b+d)}$

20. A random sample of 200 tins of coconut oil gave an average weight of 4.95 kgs. With a standard deviation of 0.21 kg. Do we accept the net weight is 5 kgs per tin at 5% level? [AU2013]

Solution:

Given $n = 200$, $\bar{x} = 4.95$, $s^2 = 0.21$ and $\mu = 5$

Let $H_0 = \mu = 5$

$H_1 = \mu \neq 5$

$\therefore \quad Z = \dfrac{\bar{x}-\mu}{s/\sqrt{n}} = \dfrac{4.95-5}{0.21/\sqrt{200}} = \dfrac{-0.05}{0.21} 10\sqrt{2} = -3.37$

$\therefore |z| = 3.37 > 1.96 \therefore \quad H_0$ is rejected.

21. Mention the various steps involved in testing of hypothesis. [$AU2010$]

Solution:

Step 1. State the null hypothesis H_0.

Step 2. Decide the alternate hypothesis H_1.

Step 3. Choose the level of significance $\alpha(\alpha = 5\%$ or $\alpha = 1\%)$

Step 4. Compute the test statistic $Z = \dfrac{t - E(t)}{S.Eof(t)}$

Step 5. Compare the computed value of $|Z|$ with the table value of Z and decide the acceptance or the rejection of H_0.

Step 6. Inference.

PART B

1. A sample of 100 people during the past year showed an average life span of 71.8 years. If the standard deviation of the population is 8.9 years, test whether the mean life span today is greater than 70 years. $[AU2010]$

2. A sample of 900 items has mean 3.4 cms and standard deviation 2.61 cms. Can the sample be regarded as drawn from a population with mean 3.25 cms at 5% level of significance.$[AU2008, 2010]$

3. A sample of heights of 6400 Englishmen has a mean of 170 cms and a standard deviation of 6.4 cms, while a sample of heights of 1600 Australians has a mean of 172 cm and standard deviation of 6.3 cm. Do the data indicate that the Australians are on the average taller than the Englishmen. $[AU2007]$

4. A machine produces 16 imperfect articles in a sample of 500. After it was overhauled, it puts ou 3 imperfect articles in a sample of 100. Has the machine improved in its performance? $[AU2012]$

5. A mathematics test was given to 50 girls and 75 boys. The girls made an average grade of 76 with a SD of 6, while boys made an average grade of 82 with a SD of 2. Test whether there is any significant difference between the performance of boys and girls. $[AU2012]$

6. Test whether there is any significant difference between the variances of the populations from which the following samples are taken: $[AU2012]$

Sample I:	20	16	26	27	23	22	
Sample II:	27	33	42	35	32	34	38

7. A sample of 10 boys had the I.Q's:70, 120, 110, 101, 88, 83, 95, 98, 100 and 107. Test whether the population mean I.Q may be 100. $[AU2012]$

8. Two independent sample of sizes 9 and 7 from a normal population had the following values of the variables. $[AU2014]$

Sample I:	18	13	12	15	12	14	16	14	15
Sample II:	16	19	13	16	18	13	15		

9. Out of 8000 graduates in a town 800 are females, out of 1600 graduate employees 120 are females. Use χ^2 to determine if any distinction is made in appointment on the basis of sex. Value of χ^2 at 5% level for one degree of freedom is 3.84. $[AU2010]$

10. An automobile company gives you the following information about age groups and the liking for particular model of car which it plans to introduce. On the basis of this data can it be concluded that the model appeal is independent of the age group. $(\chi^2_{0.05}(3) = 7.815)$ $[AU2010]$

Age Group

Persons who :	Below 20	20 − 39	40 − 59	60 and above
Liked the car :	140	80	40	20
disliked the car :	60	50	30	80

UNIT II

DESIGN OF EXPERIMENTS

PART A

1. State the basic principles of design of Experiments.[2014]

Solution:

The three basic principles of experimental design are 1. Randomisation, 2. Replication and 3. Local control.

2. What is the aim of the design of experiments? [AU 2013]

Solution:

The prime objective of design of experiments is to control the extraneous variables so that the results could be attributed only to the experimental variables.

3. What do you understand by "Design of Experiment"? [2013]

Solution:

It is defined as the logical construction of the experiment in which the degree of uncertainty with which the inference is drawn, may be well defined.

4. State the assumptions involved in ANOVA. [2012]

Solution:

1. The samples are drawn from normal populations.
2. The samples are drawn independently from these populations.
3. All the populations have the same variance.

ANOVA should not be used if we cannot make these assumptions.

5. When do you apply ANOVA. [AU 2005]

Solution:

When we have to test the differences between means of more than two samples we use analysis of variance.

6. Write down the ANOVA table for one way classification. [2013]

Solution:

One way classification ANOVA table

Source of Variation	**Sum of squares (SS)**	**Degrees of freedom (df)**	**Mean square (Ms)**	**Variance ratio**
Between samples	SSB	$r-1$	MSB= $\dfrac{SSB}{r-1}$	F= $\dfrac{MSB}{MSW}$
Within samples (or error)	SSW	$N-r$	MSW= $\dfrac{SSW}{N-r}$	(or) $\dfrac{MSW}{MSB}$
Total	SST	$N-1$		

7. Compare one-way classification model with two-way classification model. [AU 2010]

Solution:

1. CRD analysis results in one-way classification, whereas RBD analysis results in two-way classification.
2. Experimental errors are large in CRD compared to RBD and RBD is popular.

8. State the null and alternate hypothesis for a CRD. [AU 2009]

Solution:

$H_0 : \mu_1 = \mu_2 = \cdots = \mu_k = \mu$ i.e., the treatment means are equal.
H_1 : not all μ_i's are equal.

9. State any two advantages of a CRD [AU 2014]

Solution:

1. It has a simple layout.
2. There is complete flexibility as the number of replication is not fixed.

10. Discuss the advantages and disadvantages of Randomized block design. [2010]

Solution:

Advantages:

1. It has a simple layout but it is more efficient than CRD because of reduction of experimental error.
2. It is flexible and so any number of treatments and any number of replication may be used.

Disadvantages:

1. If the number of treatments is large, then the size of the blocks will increase this may cause heterogeneity within blocks.
2. The shape of the experimental material should be rectangular.

11. Define RBD. [2012]

Solution:

Suppose we want to test the effect of r fertilizers on the yield of paddy. We divide the plots into h blocks, each block is relatively homogeneous and each block contains r plots. Within each block the plots are selected at random and the r treatments (i.e., fertilizers) are given. Thus in each block only one plot receives one fertiliser. This repeated for all the h blocks. This design is called randomized block design.

12. Write any two differences between RBD and CRD. [2011]

Solution:

1. CRD analysis results in one-way classification, whereas RBD analysis results in two-way classification.
2. Experimental errors are large in CRD compared to RBD and RBD is popular.

13. What is meant by Latin square? [AU 2010]

Solution:

Latin square design controls variation in two directions of the experimental material as rows and columns resulting in the reduction of experimental error.

14. what are the advantages of LSD? [2012]

Solution:

- The analysis of the design results in a three-way classification of analysis of variance.
- The analysis of remains relatively simple even with missing data.

15. Explain the situations in which randomized block design is considered an improvement over a completely randomized design. [AU 2014]

Solution:

- RBD has a simple layout but it is more efficient than CRD because of reduction of experimental error.
- The analysis of the design is simple as it results in a two-way classification anaylsis of variance.

16. What is the purpose of blocking in a randomized block design.[AU 2009]

Solution:
If the variation due to heterogeneity in experimental units is so large then the sensitivity of detecting treatment differences is reduced because of large value of s^2. A better idea would be to "block off" variation due to these units and thus reduce the extraneous variation by smaller homogeneous blocks.

17. State the advantages of a factorial Experiment over a simple experiment. [2010,2014]

Solution:
Factorial design is one of the fruitful advancement in the endevour to improve the logical foundations of experimental designs. In experiments based on factorial design, the experiment can evaluate the combined effect of two or more factors when used simultaneously.

18. Define 2^2 factorial design. [2013]

Solution:
When there are two factors A, B and two levels 'high' and 'low' for each factor we have a 2^2 factorial design. Inspite of its simplicity, the 2^2 design is a powerful tool to improve products and processes.

19. Define mean square. [AU 2007]

Solution:

Mean square= $\dfrac{\text{corresponding } SS}{df}$

20. Why a 2×2 Latin square is not - possible? Explain.[AU 2006]

Solution:

Consider a $n \times n$ Latin square design, then the degrees of freedom for SSE is

$$
\begin{aligned}
&= (n^2 - 1) - (n - 1) - (n - 1) - (n - 1) \\
&= n^2 - 1 - 3n + 3 \\
&= n^2 - 3n + 2 \\
&= (n - 1)(n - 2)
\end{aligned}
$$

For $n = 2$, d.f of SSE$= 0$ and hence MSE is not defined.

$\therefore$ comparisons are not possible. Hence a 2×2 Latin Square Design is not possible.

PART B

1. A set of data involving four "four tropical feed stuffs A, B, C, D" tried on 20 chicks is given below. All the twenty chicks are treated alike in all respects except the feeding treatments and each feeding treatment is given to 5 chicks. Analyze the data [AU 2010]
Weight gain baby chicks fed on different feeding materials composed of tropical feed stuffs:

						TOTAL T_i
A	55	49	42	21	52	219
B	61	112	30	89	63	355
C	42	97	81	95	92	407
D	169	137	169	85	154	714
Total						1695

2. An experiment was planned to study the effect of sulphate of potash and super phosphate on the yield of potatoes. All the combinations of 2 levels of super phosphate and 2 levels of sulphate of potash were studied in a randomized block design with 4 replications for each. The yields (per plot) obtained are given below. [AU 2010]

Block	Yields(lbs per plot)			
I	(1)	k	p	kp
	23	25	22	38
II	p	(1)	k	kp
	40	26	36	38
III	(1)	k	kp	p
	29	20	30	20
IV	kp	k	p	(1)
	34	31	24	28

Analyze the data and comment on your findings. $(F_{0.05}(3, 9) = 3.86, F_{0.05}(1, 9) = 5.12)$.

3. Carry out ANOVA(Analysis of Variance) for the following. [AU 2010]

		A	B	C	D
Workers	1	44	38	47	36
	2	46	40	52	43
	3	34	36	44	32
	4	43	38	46	33
	5	38	42	49	39

4. Perform Latin Square Experiment for the following.[AU 2010]

Roam	I	II	III	→	Three equally spaced concentrations of poison as extracted from the scorpion fish.
Arabic	1	2	3	→	Three equally spaced body weights for the animals tested.
Latin	A	B	C	→	Three equally spaced times of storage of the poison before it is administered to the animals.

	I	II	III
1	0.194	0.73	1.187
	A	B	C
2	0.758	0.311	0.589
	C	A	B
3	0.369	0.558	0.311
	B	C	A

5. The following are the number of mistakes made in 5 successive days by 4 technicians working for a photographic laboratory test at a level of significance $\alpha = 0.01$. Test whether the difference among the four sample means can be attributed to chance.[2011]

Technician			
I	II	III	IV
6	14	10	9
14	9	12	12
10	12	7	8
8	10	15	10

6. The following data represent the number of units of production per day turned out by different workers using 4 different types of machines[AU 2011, 2013]

		Machine type			
		A	B	C	D
Workers	1	44	38	47	36
	2	46	40	52	43
	3	34	36	44	32
	4	43	38	46	33
	5	38	42	49	39

   1. Test whether the five men differ with respect to mean productivity and
   2. Test whether the mean productivity is the same for the four different machine types.

7. (a) What are the basic assumptions involved in ANOVA?[2011] [4]

   (b) In a Latin square experiment given below are the yields in quintals per acre on the paddy crop carried out for testing the effect of five fertilizers A, B, C, D, E. Analyze the data for variations.[AU 2011] [12]

8. The sales of 4 salesmen in 3 seasons are tabulated here. Carry out an analysis of variance.[AU 2012]

	Salesmen			
Seasons	A	B	C	D
Summer	36	36	21	35
Winter	28	29	31	32
Monsoon	26	28	29	29

9. A farmer wishes to test the effect of 4 fertilizers A, B, C, D on the yield of wheat. The fertilizers are used in a LSD and the result are tabulated here. Perform an analysis of variance.[AU 2012]

A 18	C 21	D 25	B 11
D 22	B 12	A 15	C 19
B 15	A 20	C 23	D 24
C 22	D 21	B 10	A 17

10. Four varities A, B, C, D of a fertilizer are tested in RBD with 4 replications. The plot yields in pounds are as follows:[AU 2012,2013,2014]

A 12	D 20	C 16	B 10
D 18	A 14	B 11	C 14
B 12	C 15	D 19	A 13
C 16	B 11	A 15	D 20

Analyse the experimental yield.

11. A variable trial was conducted on wheat with 4 varities in a Latin Square design. The plan of the experiment and per plot yield are given below:[AU 2012]

C 25	B 23	A 20	D 20
A 19	D 19	C 21	B 18
B 19	A 14	D 17	C 20
D 17	C 20	B 21	A 15

12. The following is a Latin square of a design when 4 varieties of seeds are being tested. Set up the analysis of variance table and state your conclusion. You may carry out suitable change of origin and scale.[AU 2013]

A 105	B 95	C 125	D 115
C 115	D 125	A 105	B 105
D 115	C 95	B 105	A 115
B 95	A 135	D 95	C 115

13. Compare and contrast the Latin square Design with the Randomized Block Design. [AU 2013]

14. Analyse the variance in the Latin square of yields in kgs of paddy where P, Q, R, S denote the different methods of cultivation. [AU 2014]

S 122	P 121	R 123	Q 122
Q 124	R 123	P 122	S 125
P 120	Q 119	S 120	R 121
R 122	S 123	Q 121	P 122

Examine whether different method of cultivation have significatly different yields.

15. A company wants to produce cars for its own use. It has to select the make of the car out of the four makes A, B, C and D available in the market. For this he tries four cars of each make by assigning the cars to four drivers to run on four different routes. The efficiency of cars is measured in terms of time in hours. The layout and time consumed is as given below.

	Drivers			
Routes	1	2	3	4
1	C 18	D 12	A 16	B 20
2	D 26	A 34	B 25	C 31
3	B 15	C 22	D 10	A 28
4	A 30	B 20	C 15	D 9

Analyze the experimental data and draw conclusions. ($F_{0.05}(3, 5) = 5.41$)

UNIT III

SOLUTION OF EQUATIONS AND EIGENVALUE PROBLEMS

PART A

1. What is the order of convergence of Newton-Raphson method? $[AU2011]$

Solution:
The order of convergence is 2.

2. Write down the condition for convergence of Newton-Raphson method for $f(x) = 0$. $[AU2007, 2010, 2011]$

Solution:
The condition is $|f(x).f'(x)| < (f'(x))^2$ in a neighbourhood of the root.

3. What are the merits of Newton's method of iteration?

Solution:

1. It can be used for finding root of both algebraic and transcendental equations.
2. The convergence of Newton's method is faster and so it is preferred compared to other methods.
3. It is simple and easy to deal with and it is used to improve the results obtained by other methods.

4. Using Gauss elimination method solve $x + y = 2$, $2x + 3y = 5$

Solution:
The augmented matrix is

$$[A, B] = \begin{bmatrix} 1 & 1 & : & 2 \\ 2 & 3 & : & 5 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & : & 2 \\ 0 & 1 & : & 1 \end{bmatrix} R_2 \to R_2 - 2R_1$$

$$\therefore \quad y = 1,\ x + y = 2 \Rightarrow\ x = 1$$

$$\therefore \quad x = 1, y = 1$$

5. Give two direct methods to solve a system of linear equations.

Solution:

Gauss-elimination method and Gauss-Jordan method are the direct methods.

6. What is the condition for convergence of Gauss-seidel method?

Solution:

The process of iteration will converge if in each equation one coefficient is much larger than the other two and the largest coefficient must be attached to different coefficients in different equations.

7. Why Gauss-seidel method is better than Jacobi's-iterative method?

Solution:

In Jacobi's method at each iteration the values of the variables in the previous iteration are used, where as in Gauss-Seidel method in each iteration the latest available values of the variables are used. Hence the convergence of Gauss-Seidel method is twice faster than Jacobi's method.

8. Find the inverse of $A = \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$ by Gauss-Jordan method.

Solution:

The augmented matrix is

$$[A, B] = \begin{bmatrix} 1 & 3 & : & 1 & 0 \\ 2 & 7 & : & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & : & 1 & 0 \\ 0 & 1 & : & -2 & 1 \end{bmatrix} R_2 \to R_2 + (-2)R_1$$

$$\sim \begin{bmatrix} 1 & 0 & : & 7 & -3 \\ 0 & 1 & : & -2 & 1 \end{bmatrix} R_1 \to R_1 + (-3)R_2$$

$$A^{-1} = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix} R_1 \to R_1 + (-3)R_2$$

9. Write down all possible types of initial vectors to determine the largest eigen value and the corresponding eigen vector of a matrix size 2×2

Solution:

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

10. Compare Gauss Jacobi with Gauss Jordan.[AU 2010]

Solution:

Sl.	COMPARISON	
No.	**Gauss Jordan**	**Gauss Jacobi**
1	Direct method	Itterative method
2	This method produce exact solution after a finite number of steps	This method give a sequence of approximate solutions, which ultimately approach the actual solution.
3	Applicable if the coefficient matrix is non-singular	Applicable if the coefficient matrix is diagonally dominant.

11. Compare Gauss elimination with Gauss Jacobi methods.[AU 2012]

Solution:

Sl.	COMPARISON	
No.	**Gauss elimination**	**Gauss Jacobi**
1	Direct method	Itterative method
2	This method produce exact solution after a finite number of steps	This method give a sequence of approximate solutions, which ultimately approach the actual solution.
3	Applicable if the coefficient matrix is non-singular	Applicable if the coefficient matrix is diagonally dominant.

12. What is the order of convergence and also state the error term for Newton Raphson method? [AU 2011]

Solution:

Order of convergence is 2 and the error term is $\epsilon_{k+1} \approx \dfrac{f''(x)}{2f'(x)}\epsilon_k^2$

13. Find the dominant eigen value of the matrix $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ by power method. [AU 2011]

Solution:

Given $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$. Take $X_0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ as the initial vector (II row contains the largest element and largest sum)

Then $AX_0 = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix} = 4 \begin{bmatrix} 0.5 \\ 1 \end{bmatrix} = 4X_1$

$AX_1 = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 0.5 \\ 1 \end{bmatrix} = \begin{bmatrix} 2.5 \\ 5.5 \end{bmatrix} = 5.5 \begin{bmatrix} 0.4545 \\ 1 \end{bmatrix} = 5.5X_2$

$$AX_2 = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 0.4545 \\ 1 \end{bmatrix} = \begin{bmatrix} 2.4545 \\ 5.3635 \end{bmatrix} = 5.3635 \begin{bmatrix} 0.4576 \\ 1 \end{bmatrix} = 5.3635X_3$$

$$AX_3 = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 0.4576 \\ 1 \end{bmatrix} = \begin{bmatrix} 2.4576 \\ 5.3728 \end{bmatrix} = 5.3728 \begin{bmatrix} 0.4574 \\ 1 \end{bmatrix} = 5.3728X_4$$

$$AX_4 = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 0.4574 \\ 1 \end{bmatrix} = \begin{bmatrix} 2.4574 \\ 5.3722 \end{bmatrix} = 5.3722 \begin{bmatrix} 0.4574 \\ 1 \end{bmatrix} = 5.3722X_5$$

Since $X_4 = X_5$, we stop the iteration and the largest eigen value is 5.3722

14. Using Newton-Raphson method, find the iteration formula to compute $\sqrt{N}$.[AU 2010]

Solution:

Let $x = \sqrt{N}$, then $x^2 = N \Rightarrow x^2 - N = 0$

Let $f(x) = x^2 - N$, then $f'(x) = 2x$

Newton's iteration formula is $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)},\ n = 0, 1, 2, \cdots$

$$\begin{aligned} \therefore \qquad x_{n+1} &= x_n - \frac{x_n^2 - N}{2x_n} \\ &= \frac{2x_n^2 - x_n^2 + N}{2x_n} \\ &= \frac{x_n^2 + N}{2x_n} \\ &= \frac{1}{2}\left(x_n + \frac{N}{x_n}\right) \end{aligned}$$

15. Explain the power method to determine the eigenvalue of a matrix.[AU 2010]

Solution:

Workingrule

1. Let x_0 be the initial which is usually chosen as a vector with all components equal to 1. (i.e., normalised)
2. Form the product AX_0 and express it in the form $AX_0 = \lambda_1 X_1$, where X_1 is normalized by taking out the largest component λ_1.
3. Form $AX_1 = \lambda_2 X_2$, where X_2 is normalized by taking out the largest component λ_2 and continue the process.
4. Thus we have a sequence of equations
   $AX_0 = \lambda_1 X_1,\ AX_1 = \lambda_2 X_2,\ AX_2 = \lambda_3 X_3, \cdots$

We stop at the stage where X_{r-1}, X_r are almost same.
Then λ_r is the largest eigen value and X_r is the corresponding eigen vector.

16. Arrive a formula to find the value of $\sqrt[3]{N}$, where $N \neq 0$, using Newton-Raphson method.[AU 2011]

Solution:
Let $x = \sqrt[3]{N}$, then $x^3 = N \Rightarrow x^3 - N = 0$
Let $f(x) = x^3 - N$, then $f'(x) = 3x^2$
Newton's iteration formula is $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)},\ n = 0, 1, 2, \cdots$

$$\begin{aligned}
\therefore \qquad x_{n+1} &= x_n - \frac{x_n^3 - N}{3x_n^2} \\
&= \frac{3x_n^3 - x_n^3 + N}{3x_n^2} \\
&= \frac{2x_n^3 + N}{3x_n^2} \\
&= \frac{1}{3}\left(2x_n + \frac{N}{x_n^2}\right)
\end{aligned}$$

17. Solve the following system of equations, using Gauss-Jordan elimination method $2x + y = 3,\ x - 2y = -1$. [AU 2012]

Solution:
Given $2x + y = 3,\ x - 2y = -1$.
Augmented matrix

$$\begin{aligned}
[A, B] &= \left[\begin{array}{rr|r} 2 & 1 & 3 \\ 1 & -2 & -1 \end{array}\right] \\
&\sim \left[\begin{array}{rr|r} 1 & 1/2 & 3/2 \\ 1 & -2 & -1 \end{array}\right] R_1 \longrightarrow \frac{R_1}{2} \\
&\sim \left[\begin{array}{rr|r} 1 & 1/2 & 3/2 \\ 0 & -5/2 & -5/2 \end{array}\right] R_2 \longrightarrow R_2 - R_1 \\
&\sim \left[\begin{array}{rr|r} 1 & 1/2 & 3/2 \\ 0 & 1 & 1 \end{array}\right] R_1 \longrightarrow \frac{R_2}{-5/2} \\
&\sim \left[\begin{array}{rr|r} 1 & 0 & 1 \\ 0 & 1 & 1 \end{array}\right] R_1 \longrightarrow R_1 - (1/2)R_2
\end{aligned}$$

$\therefore$ the solution is $x = 1,\ y = 1$

18. Form the divided difference table for the following data:[AU 2012]

x:	5	15	22
y:	7	36	160

Solution:

The Divided difference table

x	y	Δy	$\Delta^2 y$
5	7		
		$\frac{36-7}{15-5} = 2.6$	
15	36		$\frac{17.71-2.6}{22-5} = 0.88$
		$\frac{160-36}{22-15} = 17.71$	
22	160		

19. Find the real positive root of $3x - \cos x - 1 = 0$ by Newton's method correct to 6 decimal places.[2013]

Solution:

Given eqn. is $3x - \cos x - 1 = 0$.
Let $f(x) = 3x - \cos x - 1$ then $f'(x) = 3 + \sin x$
$f(0) = -1 < 0$,
$f(1) = 3 - \cos 1 - 1 = 1.4597 > 0$
$\therefore$ a root lies between 0 and 1.
Now $f(0.5) = 3(0.5) - \cos 0.5 - 1 = -0.3776$, which is closer to 0.
So, the root is indeed between 0.5 and 1 and is nearer to 0.5.
Take $x_0 = 0.6$ Iteration formula is $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)},\ n = 0, 1, 2, \cdots$

I approximation:

$$\begin{aligned} x_1 &= x_0 - \frac{f(x_0)}{f'(x_0)} \\ &= 0.6 - \frac{f(0.6)}{f'(0.6)} \\ &= 0.6 - \frac{-0.0253356}{3.564642} \\ &= 0.6 + 0.0071 \\ &= 0.6071 \end{aligned}$$

II approximation:

$$\begin{aligned} x_2 &= x_1 - \frac{f(x_1)}{f'(x_1)} \\ &= 0.6071 - \frac{f(0.6071)}{f'(0.6071)} \\ &= 0.6 - \frac{-0.000006}{3.570488} \\ &= 0.6 + 0.00000168 \\ &= 0.60710168 \end{aligned}$$

$\because$ $x_1 = x_2$ upto 4 decimal places, the root is 0.6071

20. Solve the equations $A+B+C=6,\ 3A+3B+4C=20,\ 2A+B+3C=13$ using Gauss elimination method.[2013]

Solution:

Given $A+B+C=6$, $3A+3B+4C=20$, $2A+B+3C=13$

$$\begin{aligned}
[A,B] &= \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 3 & 3 & 4 & 20 \\ 2 & 1 & 3 & 13 \end{array}\right] \\
&\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 0 & 1 & 2 \\ 2 & 1 & 3 & 13 \end{array}\right] R_2 \longrightarrow R_2 - 3R_1 \\
&\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 0 & 1 & 2 \\ 0 & -1 & 1 & 1 \end{array}\right] R_3 \longrightarrow R_3 - 2R_1 \\
&\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right] R_3 \longleftrightarrow R_2 \\
&\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 2 \end{array}\right] R_2 \longrightarrow -R_2
\end{aligned}$$

$\therefore x+y+z=6$

$y-z=-1$

and $z=2$,

By back substitution method, we have $y=-1+z=-1+2=1$ and

$x=6-y-z=6-1-2=3$

$\therefore$ the solution is $x=3,\ y=1,\ z=2$.

PART B

Newton-Raphson method

1. Find a positive root of $x^3 - 5x + 3 = 0$. $[AU2007]$
2. Find by Newton Raphson method a positive root of the equation $3x - cosx - 1 = 0$. $[AU2006, 2009]$
3. Using Newton's method, find a real root of $xlog_{10}x = 1.2$ correct to 4 decimals.$[AU2004, 2007]$
4. Use Newton's Raphson method to find a root of the equation $cosx - xe^x = 0$ correct to 3 decimal places.$[AU2000, 2006]$
5. Find a solution of $3x + sinx - e^x = 0$ correct to four decimal places by Newton's method. $[AU2006]$
6. Find an iterative formula to find $\sqrt{N}$, where N is a positive integer, using Newton's method and hence find $\sqrt{11}$ $[AU2006]$
7. Find an iterative formula to find $\sqrt{N}$, where N is a positive integer, using Newton's method and hence find $\sqrt{142}$ $[AU2006, 2005, 2009]$
8. Find an iterative formula to find $\sqrt{N}$, where N is a positive integer, using Newton's method and hence find $\sqrt{5}$ $[AU2000, 2006]$
9. Find the iterative formula for finding the value of $\frac{1}{N}$ where N is a real number, using Newton-Raphson method. Hence evaluate $\frac{1}{26}$ correct to 4 decimal places. $[AU2006]$
10. Find the Newton's iterative formula for the reciprocal of N and hence find the value of $\frac{1}{23}$, correct to 5 decimal places. $[AU2006,\ 2012]$
11. Find an iterative formula for $\sqrt[3]{N}$, where N is a positive integer, using Newton's method and hence find $\sqrt[3]{24}$, $\sqrt[3]{41}$
12. Find the iterative formula by Newton's formula for (i) $\frac{1}{N}$ (ii) $\frac{1}{\sqrt{N}}$, where N is a positive integer. Hence find $\frac{1}{31}$ and $\frac{1}{\sqrt{15}}$
13. Solve for a positive root of the equation $x^4 - x - 10 = 0$ using Newton's Raphson's method.$[AU2010]$
14. Find a root of $3x^3 - 9x^2 + 8 = 0$ using Newton Raphson method. $[AU2003]$
15. Find a root of $x^3 - 6x + 4 = 0$ between 0 and 1 to 5 places of decimals. $[AU2000, 2008]$
16. Find a root of $2x^3 - 3x - 6 = 0$ between 1 and 2 to five places of decimals. $[AU2008]$
17. Using Newton's method, solve $x \log_{10} x = 12.34$ taking the initial value x_0 as 10 . $[AU2004, 2012, 2014]$

Gauss Elimination Method

1. Solve by Gauss elimination method the equations
   $x + y + z = 9,\ 2x - 3y + 4z = 13;\ 3x + 4y + 5z = 40.$[AU 2012]
2. Solve by Gauss elimination method the equations
   $2x + y + z = 10,\ 3x + 2y + 3z = 10;\ x + 4y + 9z = 16.$[AU 2010]

3. Solve by Gauss elimination method the equations
$2x - y + 8z = 13$, $3x + 4y + 5z = 18$; $5x - 2y + 7z = 20$.[AU 2013]
4. Solve by Gauss elimination method the equations
$x + 2y + z = 4$, $3x - y + 2z = -3$; $x + 2y + 4z = 7$.
5. Solve by Gauss elimination method the equations
$10x - 2y + 3z = 23$, $2x + 10y - 5z = -33$; $3x - 4y + 10z = 41$.

Gauss Jordan Method

1. Solve by Gauss-Jordan method the equations
$x + 2y + z = 3$, $2x + 3y + 3z = 10$; $3x - y + 2z = 13$. $[AU2004]$
2. Solve by Gauss-Jordan method the equations
$x + 3y + 3z = 16$, $x + 4y + 3z = 18$; $x + 3y + 4z = 19$. $[AU2005]$
3. Solve by Gauss-Jordan method the equations
$10x + y + z = 12$, $2x + 10y + z = 13$; $x + y + 5z = 7$. $[AU2004, 2008, 2010]$
4. Solve by Gauss-Jordan method the equations
$2x - y + 3z = 8$, $-x + 2y + z = 4$; $3x + y - 4z = 0$. $[AU2007]$
5. Solve by Gauss-Jordan method the equations
$2x + y + z = 10$, $3x + 2y + 3z = 18$; $x + 4y + 9z = 16$. $[AU2006]$
6. Solve by Gauss-Jordan method the equations
$x + y + z = 9$, $2x - 3y + 4z = 13$; $3x + 4y + 5z = 40$. $[AU2011]$

Gauss Jacobi method

1. Solve by Gauss Jacobi method the equations
$x + 17y - 2z = 48$, $30x - 2y + 3z = 75$; $2x + 2y + 18z = 30$.
2. Solve by Gauss Jacobi method the equations
$8x - y + z = 18$, $x + y - 3z = -6$; $2x + 5y - 2z = 3$.
3. Solve by Gauss Jacobi method the equations
$5x + 2y + z = 12$, $x + 4y + 2z = 15$; $x + 2y + 5z = 20$.
4. Solve by Gauss Jacobi method the equations
$4x_1 + x_2 + x_3 = 6$, $x_1 + 4x_2 + x_3 = 6$; $x_1 + x_2 + 4x_3 = 6$. $[AU2007]$

Gauss - Seidel method

1. Solve by Gauss - Seidel method the equations
$10x + 2y + z = 9$,
$x + 10y - z = -22$,
$-2x + y - 10z = -22$ $[AU2012]$
2. Solve by Gauss - Seidel method the equations
$27x + 6y - z = 85$, $x + y + 54z = 110$; $6x + 15y + 2z = 72$. $[AU2006, 2011, 2012]$
3. Solve by Gauss - Seidel method the equations
$20x + y - 2z = 17$, $3x + 20y - z = -18$; $2x - 3y + 20z = 25$. $[AU2008, 2009]$

4. Solve by Gauss - Seidel method the equations
$28x + 4y - z = 32$, $x + 3y + 10z = 24$; $2x + 17y + 4z = 35$. $[AU2007]$

5. Solve by Gauss - Seidel method the equations starting with (0,0,0,0) as solution. Do 5 iterations only.
$4x_1 - x_2 - x_3 = 2$
$-x_1 + 4x_2 - x_4 = 2$
$-x_1 + 4x_3 - x_4 = 2$
$-x_2 - x_3 + 4x_4 = 1$ $[AU2008]$

6. Use Gauss - Seidel method to obtain the solution of the equations
$9x - y + 2z = 9$, $x + 10y - 2z = 15$; $2x - 2y - 3z = -17$. $[AU2010]$

7. Use Gauss - Seidel method to obtain the solution of the equations
$4x + 2y + z = 14$, $x + 5y - z = 10$; $x + y + 8z = 20$. $[AU2005]$

8. Use Gauss - Seidel method to obtain the solution of the equations
$x + 3y + 52z = 173.61$, $x - 27y + 2z = 71.31$; $41x - 2y + 3z = 65.46$ starting with $x = 1,\ y = -2,\ z = 3$ $[AU2004]$

9. Solve by Gauss - Seidel method the equations
$10x - 5y - 2z = 3$,
$4x - 10y - 3z = -3$,
$x + 6y + 10z = -3$ $[AU2006, 2008]$

10. Solve by Gauss - Seidel method the equations
$6x - 3y + z = 11$,
$x - 7y + z = 10$,
$2x + y - 8z = -15$ $[AU2009]$

11. Solve by Gauss - Seidel method the equations
$6x + 3y + 12z = 35$,
$8x - 3y + 12z = 20$,
$4x + 11y - z = 33$ $[AU2007, 2010]$

12. Solve by Gauss - Seidel method the equations
$5x - y + z = 10$,
$2x + 4y = 12$,
$x + y + 5z = -1$ $[AU2010]$

Matrix inversion by Gauss Jordan Method

1. Using Gauss Jordan method, find the inverse of the matrix A= $\begin{bmatrix} 1 & 2 & 6 \\ 2 & 5 & 15 \\ 6 & 15 & 46 \end{bmatrix}$ $[AU2012]$

2. Using Gauss Jordan method, find the inverse of the matrix A= $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$
$[AU2001, 2004, 2006, 2010, 2012]$

3. Using Gauss Jordan method, find the inverse of the matrix A= $\begin{bmatrix} 4 & 1 & 2 \\ 2 & 3 & -1 \\ 1 & -2 & 2 \end{bmatrix}$ $[AU2007, 2011]$

4. Using Gauss Jordan method, find the inverse of the matrix A= $\begin{bmatrix} 2 & 6 & 6 \\ 2 & 8 & 6 \\ 2 & 6 & 8 \end{bmatrix}$ $[AU2006]$

5. Using Gauss Jordan method, find the inverse of the matrix A= $\begin{bmatrix} 2 & 2 & 3 \\ 2 & 1 & 1 \\ 1 & 3 & 5 \end{bmatrix}$ $[AU2004]$

6. Using Gauss Jordan method, find the inverse of the matrix A= $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ $[AU2010]$

7. Using Gauss Jordan method, find the inverse of the matrix A= $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 2 & 0 \\ 3 & -1 & -4 \end{bmatrix}$ $[AU2010]$

8. Using Gauss Jordan method, find the inverse of the matrix A= $\begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & 1 \\ 3 & 5 & 3 \end{bmatrix}$ $[AU2011]$

Eigen value of a square matrix using Power method

1. Find the numerically largest eigen value of A= $\begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix}$ and the corresponding eigen vector. $[AU2006, 2008, 2011]$

2. Find the dominant eigen value and the corresponding eigen vector of $\begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ $[AU2007, 2008, 2010]$

3. Using power method, find all the eigen values of $\begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix}$ $[AU2006, 2009, 2013, 2014]$

4. Find the numerically largest eigen value and the corresponding eigen vector using power method, given A= $\begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix}$. $[AU2012, 2014]$

UNIT IV
INTERPOLATION, NUMERICAL DIFFERENTIATION AND NUMERICAL INTEGRATION

PART A

1. Write down the Lagrange's interpolating formula. [AU 2010, 2014]

Solution:

$$y = f(x) = \frac{(x-x_1)(x-x_2)\cdots(x-x_n)}{(x_0-x_1)(x_0-x_2)\cdots(x_0-x_n)}y_0 + \frac{(x-x_0)(x-x_2)\cdots(x-x_n)}{(x_1-x_0)(x_1-x_2)\cdots(x_1-x_n)}y_1 + \frac{(x-x_0)(x-x_1)\cdots(x-x_n)}{(x_2-x_0)(x_2-x_1)\cdots(x_2-x_n)}y_2 + \ldots + \frac{(x-x_0)(x-x_2)\cdots(x-x_{n-1})}{(x_n-x_0)(x_n-x_2)\cdots(x_n-x_{n-1})}y_n$$

2. Use Lagrange's formula to fit a polynomial to the data and find y at $x = 1$. [2013]

X	-1	0	2	3
Y	-8	3	1	12

Solution:

Given $x_0 = -1$ $x_1 = 0$ $x_2 = 2$ $x_3 = 3$
and $y_0 = -8$ $y_1 = 3$ $y_2 = 1$ $y_3 = 12$

$$y = f(x) = \frac{(1-0)(1-2)(1-3)}{(-1-0)(-1-2)(-1-3)}(-8) + \frac{(1+1)(1-2)(1-3)}{(0+1)(0-2)(0-3)}(3) + \frac{(1+1)(1-0)(1-3)}{(2+1)(2-0)(2-3)}(1) + \frac{(1+1)(1-0)(1-2)}{(3+1)(3-0)(3-2)}(12)$$

$$= \frac{(1)(-1)(-2)}{(-1)(-3)(-4)}(-8)^2 + \frac{(2)(-3)(-2)}{(1)(-2)(-3)}(3) + \frac{(2)(1)(-2)}{(3)(2)(-1)}(1) + \frac{(2)(1)(-1)}{(4)(3)(1)}(12)$$

$$= \frac{2}{3}2 + \frac{4}{2}3 + \frac{4}{6} - 2 = \frac{8+36+4-12}{6} = \frac{36}{6} = 6$$

3. Write down the Simpson's 1/3 Rule in numerical integration.[AU 2010]

Solution:

$$\int_{x_0}^{x_n} f(x)dx = \frac{h}{3}\left[(y_0 + y_n) + 4(y_1 + y_3 + y_5 + \ldots + y_{n-1}) + 2(y_2 + y_4 + \ldots + y_{n-2})\right]$$

where $h = \frac{x_n - x_0}{n}$ and $n = even$ no. of sub intervals.

4. Using Trapezoidal rule, evaluate $\int\limits_0^1 \frac{dx}{1+x^2}$ with $h = 0.2$. Hence obtain an approximate value of π. [AU 2011]

Solution:

Given h=0.2

x	0	0.2	0.4	0.6	0.8	1
$y = \frac{1}{1+x^2}$	1	0.96	0.86	0.74	0.61	0.50
	y_0	y_1	y_2	y_3	y_4	y_5

$$\therefore \int\limits_0^1 y dx = \frac{h}{2}\left[(y_0 + y_5) + 2(y_1 + y_2 + y_3 + y_4)\right]$$
$$= \frac{0.2}{2}\left[(1 + 0.5) + 2(0.96 + 0.86 + 0.74 + 0.61 + 0.50)\right]$$
$$= 0.1\left[(1.5) + 2(3.67)\right]$$
$$= 0.1\left[(1.5) + 7.34\right]$$
$$= 0.884$$

5. State the formula to find the second order derivative using the forward differences. [AU 2011]

Solution:

1. $y'(x) = \frac{1}{h}\left[\Delta y_0 + \left(\frac{2u-1}{2}\right)\Delta^2 y_0 + \left(\frac{3u^2-6u+2}{6}\right)\Delta^3 y_0 + \left(\frac{4u^3-18u^2+22u-6}{24}\right)\Delta^4 y_0 + \ldots\right]$
2. $y''(x) = \frac{1}{h^2}\left[\Delta^2 y_0 + (u-1)\Delta^3 y_0 + \left(\frac{6u^2-18u+11}{12}\right)\Delta^4 y_0 + \ldots\right]$

6. Evaluate $\int\limits_{0.5}^1 \frac{dx}{x}$ by Trapezoidal rule, dividing the range into equal 4 parts.[AU 2012]

Solution:

Here n=4 $\therefore h = \frac{1-0.5}{4} = 0.125$

x	0.5	0.625	0.75	0.875	1
$y = \frac{1}{x}$	2	1.6	1.33	1.14	1
	y_0	y_1	y_2	y_3	y_4

Trapezoidal rule is

$$\int\limits_{0.5}^1 f(x)dx = \frac{h}{2}\left[(y_0 + y_4) + 2(y_1 + y_2 + y_3)\right]$$
$$= \frac{0.125}{2}\left[(2 + 1) + 2(1.6 + 1.33 + 1.14)\right]$$
$$= \frac{0.125}{2}\left[(3) + 2(4.08)\right]$$
$$= 0.697$$

7. Find the area under the curve passing through the points (0,0),(1,2),(2,2.5),(3,2.3),(4,2),(5,1.7) and (6,1.5).

Solution:

Given

x	0	1	2	3	4	5	6
y	0	2	2.5	2.3	2	1.7	1.5
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

Here n=6 $\therefore h = \dfrac{6-0}{6} = 1$

Trapezoidal rule is

$$\begin{aligned}\int_0^6 y\,dx &= \tfrac{h}{2}\left[(y_0+y_6)+2(y_1+y_2+y_3+y_4+y_5)\right]\\ &= \tfrac{1}{2}\left[(0+1.5)+2(2+2.5+2.3+2+1.7+1.5)\right]\\ &= \tfrac{1}{2}\left[(1.5)+2(12)\right]\\ &= 12.75\end{aligned}$$

8. State any two properties of divided differences.[2014]

Solution:

1. The divided differences are symmetrical in all their arguments, that is, the value of any difference is independent of the order of the arguments.
2. The divided difference of the product of a constant and a functions is equal to the product of the constant and the divided difference of the function.

9. Evaluate $\int_1^2 \frac{dx}{1+x^2}$ by Trapezoidal rule, taking $h = 0.5$.[AU 2013]

Solution:

Given $h = 0.5$

x	1	1.5	2
$y = \dfrac{1}{1+x^2}$	0.5	0.31	0.2
	y_0	y_1	y_2

$$\therefore \int_1^2 y\,dx = \tfrac{h}{2}\left[(y_0+y_2)+2(y_1)\right] = \tfrac{0.5}{2}\left[(0.5+0.2)+2(0.31)\right] = \tfrac{0.5}{2}\left[(0.7)+0.62\right] = 0.33$$

10. What is inverse interpolation?[AU 2014]

Solution:

It is the process of finding a value of x for the corresponding value of y and we use Lagrange's interpolation formula by taking the independent variable as y and the dependent variable as x. It is the inverse process of direct interpolation in which we find the values of y corresponding to a value of x, not present in the table.

11. What is the need of Newton's and Lagrange's interpolation formulae?

Solution:

The process of computing intermediate values of a function from a given set of tabular values of the function.
Newton's forward and backward interpolation formula can be used only when the values of the independent variable x are equally spaced and also be when the differences of the dependent variable y become smaller finally. But Lagrange's interpolation formula can be used whether the values of x, the independent variable, are equally spaced or not,whether the differences of y becomes smaller or not.

12. Find the parabola of the form $y = ax^2 + bx + c$ passing through the points $(0, 0), (1, 1), (2, 20)$. [AU 2011]

Solution:

Given

	x_0	x_1	x_2
x	0	1	2
y	0	1	20
	y_0	y_1	y_2

$$\therefore y = f(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}y_0$$

$$+\frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}y_1$$

$$+\frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}y_2$$

$$= \frac{(x-1)(x-2)}{(0-1)(0-2)}0 + \frac{(x-0)(x-2)}{(1-0)(1-2)}1 + \frac{(x-0)(x-1)}{(2-0)(2-1)}20$$

$$= 0 + \frac{x^2-2x}{(1)(-1)} + \frac{x^2-x}{\cancel{(2)}(1)}\cancel{20}^{10} = -x^2 + 2x + 10x^2 - 10x = 9x^2 - 7x$$

13. Write down trapezoidal rule to evaluate $\int_6^1 f(x)dx$ with $h = 0.5$, function $f(x)$ is unknown.[AU 2005]

Solution:

Here h=0.5 $\therefore n = \frac{6-1}{0.5} = 10$ $x_0 = 1$

x	1	1.15	2	2.5	3	3.5	4	4.5	5	5.5	6
$f(x)$	$f(1)$	$f(1.5)$	$f(2)$	$f(2.5)$	$f(3)$	$f(3.5)$	$f(4)$	$f(4.5)$	$f(5)$	$f(5.5)$	$f(6)$
	y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}

Trapezoidal rule is

$$\int_1^6 f(x)dx = \frac{h}{2}[(y_0 + y_{10}) + 2(y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7 + y_8 + y_9)]$$

$$= \frac{0.5}{2}[(f(1) + f(6)) + 2(f(1.5) + f(2) + f(2.5) + f(3) + f(3.5) + f(4) + f(4.5) + f(5) + f(5.5))]$$

14. What are the errors in trapezoidal and simpson's rule of numerical integration.[AU 2005]

Solution:

When evaluating $\int_a^b f(x)dx$, the error in the trapezoidal rule is $< \frac{(b-a)^2}{12}h^2M$, where $h = \frac{b-a}{n}$, n is the number of subintervals of (a, b), and
$M = max\{|y_0''|, |y_1''|, \cdots, |y_{n-1}''|\},\ y_r'' = f''(x_r)$
The error in Simpson's rule is $< \frac{(b-a)}{180}h^4M$, where $h = \frac{b-a}{2n}$, $2n$ is the number of subintervals of (a, b), and
$M = max\{|y_0^{(1)}|, |y_1^{(4)}|, \cdots, |y_{n-1}^{(4)}|\},\ y_r^{(4)} = f^{(4)}(x_r)$

15. What is the order of error in Simpson's $\frac{1}{3}$ rule?

Solution:

Total error in Simpson's forumula is of order h^4 where h is the length of the subintervals.

16. State the local error term in Trapezoidal rule.

Solution:

The local error term in the interval (x_0, x_1) is $\frac{-h^2}{12}y_0''$

17. When does Simpson's rule give exact result?[AU 2006]

Solution:

If the integrand f(x) in $\int_a^b f(x)dx$ is a second degree polynomial, then Simpson's formula will give exact result.

18. State Newton's backward formula for interpolation [AU 2007]

Solution:

$$y = f(x_n + hv)$$

$$= y_0 + v\nabla y_n + \frac{v(v+1)}{2!}\nabla^2 y_n + \frac{v(v+1)(v+2)}{3!}\nabla^3 y_n + \cdots + \frac{v(v+1)(v+2)...(v+(n-1))}{n!}\nabla^n y_n$$

where $v = \frac{x-x_n}{h}$

19. State Newton's forward formula for interpolation [AU 2008]

Solution:

$$y = f(x_0 + hu)$$

$$= y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \cdots + \frac{u(u-1)(u-2)...(u-(n-1))}{n!}\Delta^n y_0$$

where $u = \frac{x-x_0}{h}$

20. In order to evaluate $\int_a^b f(x)dx$ by Trapezoidal and simpson's rule. What is the restriction on the number of intervals?

Solution:

In trapezoidal rule number of intervals can be any positive integer, where as in Simpson's rule the number of intervals is even.

PART B

Newton's forward and backward difference method

1. Using Newton's forward interpolation formula find the cubic polynomial which takes the following values:

x	0	1	2	3
y	1	2	1	10

[AU2000, 2009]

2. A third degree polynomial passes through the points $(0,-1)$, $(1,1)$, $(2,1)$ and $(3,-2)$. Using Newton's forward formula, find the polynomial. Hence find the value at 15. [AU2000].

3. Construct Newton's forward interpolating polynomial for the following data

x	4	6	8	10
y	1	3	8	16

Hence find y when $x = 5$. [AU2006, 2012].

4. Find a polynomial of degree two for the data by Newton's forward difference method.

x	0	1	2	3	4	5	6	7
y	1	2	4	7	11	16	22	29

[AU2007].

5. The population of a city in a Census taken once in 10 years is given below. Estimate the population in the year 1955.

Year	1951	1961	1971	1981
Population in thousands	35	42	58	84

[AU2008].

6. Find $y(1976)$ from the following [AU2010].

x	1941	1951	1961	1971	1981	1991
y	20	24	29	36	46	51

7. From the data given below find the number of students whose weight is between 60 to 70.

Weight in lbs:	$0-40$	$40-60$	$60-80$	$80-100$	$100-120$
No. of students	250	120	100	70	50

[AU2003].

8. The following are data from the steam table.

Temperature °C	140	150	160	170	180
No. of students	3.685	4.854	6.302	8.076	10.225

Find t he pressure at temperature 142° and 175° [AU2008].

9. From the following table of half yearly premium for policies maturing at different ages estimate the premium for policies maturing at age 46 and 63. [AU2006, 2008, 2011].

Age x	45	50	55	60	65
Premium y	114.84	96.16	83.32	74.48	68.48

Lagrange's interpolation formula

1. Find $f(x)$ as a polynomial in x from the given data and find f(8).

x	3	7	9	10
f(x)	168	120	72	63

[$AU2008$].

2. Find the polynomial $f(x)$ by using Lagrange's formula and hence $f(3)$ for the following values of x and y.

x	0	1	2	5
y	2	3	12	147

and hence find f(3). [$AU2005, 2012$].

3. Using Lagrange's interpolation formula find $y(10)$ from the following table:

x	5	6	9	11
y	12	13	14	16

[$AU2008, 2011$].

4. Using Lagrange's interpolation formula to fit a polynomial to the following data hence find $y(1)$.

x	-1	0	2	3
y	-8	3	1	12

[$AU2010$].

5. Given the values

x	5	7	11	13	17
y	150	392	1452	2366	5202

Evaluate $f(9)$ using Lagrange's formula. [$AU2009, 2011$].

6. Use Lagrange's method to find $log_{10}656$, given that $log_{10}654 = 2.8156, log_{10}658 = 2.8182$, $log_{10}659 = 2.8189$ and $log_{10}661 = 2.8202$ [$AU2012$].

Newton's Divided Difference Method

1. By using Newton's divided difference formula find $f(8)$, given

x	4	5	7	10	11	13
f(x)	48	100	294	900	1210	2028

Also find $f(6)$, $f(9)$, $f(15)$ [$AU2005, 2006, 2011, 2013$].

2. Use Newton's divided difference formula to calculate $f(8)$, $f'(3)$ $f''(3)$ from the following table [$AU2010$].

x	0	1	2	4	5	6
f(x)	1	14	15	5	6	19

3. If $f(0) = 0$, $f(1) = 0$, $f(2) = -12$, $f(4) = 0$, $f(5) = 600$, $f(7) = 7308$, find a polynomial that satisfies this data using Newton's divided difference formula. Hence find $f(6)$. $[AU2007]$.

4. From the following table $f(x)$ and hence $f(6)$ using Newton's divided difference formula.

x	1	2	7	8
y	1	5	5	4

$[AU2007]$.

5. Determine $f(x)$ as a polynomial in x for the following data:

x	-4	-1	0	2	5
y	1245	33	5	9	1335

$[AU2004]$.

6. Using Newton's divided difference formula, find $f(x)$ from the following table and hence $f(4)$.

x	0	1	2	5
f(x)	2	3	12	147

$[AU2012]$.

Derivative using Newton's forward & backward difference interpolating formula

1. Find sec 31°, from the following data:

$\theta°$	31	32	33	34
$tan\theta$	0.6008	0.6249	0.6494	0.6745

$[AU2004, 2011]$.

2. Find the value of cos 1.74. Using the values given in the table below:

x	1.70	1.74	1.78	1.82	1.86
$sinx$	0.9916	0.9857	0.9781	0.9691	0.9584

$[AU2000]$.

3. Compute $f'(0)$ and $f'(4)$ from the following data.

x	0	1	2	3	4
y	1	2.718	7.381	20.086	54.598

$[AU2012]$.

4. Find the first and second derivatives of y w.r.to x at x=10 from the data given below.

x	3	5	7	9	11
y	31	43	57	41	27

$[AU2008]$.

5. A jet fighter's position on an aircraft carrier's runway was timed during landing.

t, sec	1.0	1.1	1.2	1.3	1.4	1.5	1.6
y, m	7.989	8.403	8.781	9.129	9.451	9.750	10.031

where y is the distance from the end of the carrier. Estimate velocity $\frac{dy}{dt}$ and acceleration $\frac{d^y}{dt^2}$ at (i) t=1.1, (ii) t=1.6 using numerical differentiation. $[AU2007, 2008, 2010]$.

Derivative using Newton's divided difference formula

1. Find f'(10) from the following data:

x	3	5	11	27	34
f(x)	−13	23	899	17315	35606

$[AU2011]$.

2. Using the following data, $f'(1)$, $f'(5)$ and $f'(6)$ $[AU2007, 2009, 2010, 2012]$.

x	0	2	3	4	7	9
f(x)	4	26	58	112	466	922

Trapezoidal rule, Simpson's rule

1. Dividing the range into 10 equal parts, find the approximate value of $\int_0^{\pi} sinxdx$ by (i) Trapezoidal rule, (ii) Simpson's rule. $[AU2006, 2010, 2012]$

2. Evaluate $\int_0^2 \frac{dx}{x^2+4}$ using trapezoidal rule taking $h = 0.25$ $[AU2007]$.

3. Evaluate $\int_0^2 \frac{dx}{x^2+x+1}$ to three decimal places, dividing the range of integration into 8 equal parts using Simpson's rule $[AU2012]$.

4. A rocket is launched from the ground. Its acceleration is registered during the first 80 seconds and it is in the table below. Using trapezoidal rule and Simpson's $\frac{1}{3}$ rule, find the velocity of the rocket at $t = 80secs$. $[AU2010]$.

t secs	0	10	20	30	40	50	60	70	80
f: cm/sec	30	31.63	33.34	35.47	37.75	40.33	43.25	46.69	40.67

5. Evaluate the length of the curve $3y = x^3$ from $(0, 0)$ to $(0, \frac{1}{3})$ rule and using 8 sub intervals $[AU2010]$.

6. Evaluate $\int_0^6 \frac{1}{1+x^2}dx$ using Trapezoidal rule. Verify the answer with direct integration.$[AU2010]$.

7. The velocity v of a particle at a distance s from a point on its path is given by the table

s ft:	0	10	20	30	40	50	60
v: ft/sec:	47	58	64	65	61	52	38

Estimate the time taken to travel 60ft by using Simpson's $\frac{1}{3}$ rule. Compare the result with Simpson's $\frac{3}{8}$ rule.$[AU2003]$.

8. The following table gives the velocity v of a particle at time t:

$t\ sec:$	0	2	4	6	8	10	12
$v : m/sec:$	4	6	16	34	60	94	136

Find the distance moved by the particle in 12 sec and also the acceleration at $t = 2$ sec.$[AU2003, 2011]$.

Trapezoidal rule, Simpson's rule for double integration

1. Evaluate $\int_0^2\int_0^2 f(x,y)dxdy$ by Trapezoidal rule for the following data:

y \ x	0	0.5	1	1.5	2
0	2	3	4	5	5
1	3	4	6	9	11
2	4	6	8	11	14

$[AU2005]$.

2. Use trapezoidal rule to evaluate $\int_1^2\int_1^2 \frac{dxdy}{x+y}$ taking 4 subintervals $[AU2004, 2007]$.

3. Evaluate $\int_0^1\int_0^1 \frac{1}{1+x+y}dxdy$ by Trapezoidal rule $[AU2009, 2011]$.

4. Use trapezoidal rule to evaluate $\int_1^{1.2}\int_1^{1.4} \frac{1}{x+y}dxdy$ by taking $h = k = 0.1$ $[AU2010]$.

5. Evaluate $\int_0^1\int_1^2 \frac{2xy}{(1+x^2)(1+y^2)}dxdy$ by Trapezoidal rule with $h = k = 0.25$ $[AU2004, 2009]$.

6. Evaluate $\int_1^2\int_1^2 \frac{dxdy}{x+y}$ by using Trapezoidal rule with h=k=0.5 and h=k=0.25. Improve the estimate by Romberg's formula$[AU2007]$.

7. Using Simpson's $\frac{1}{3}$ rule, evaluate $\int_0^1\int_0^1 \frac{1}{1+x+y}dxdy$ by taking $h = k = 0.5$. $[AU2004, 2009, 2013]$.

8. Evaluate $\int_2^{2.4}\int_4^{4.4} xydxdy$ using Simpson rule, given $h = k = 0.1$ $[AU2008, 2010]$.

UNIT V
NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATIONS

PART A

1. Using Euler's method, solve the following differential equation $y' = -y$ subject to $y(0) = 1$. [AU 2010]

Solution:

Given $y' = f(x, y) = -y$
$x_0 = 0, y_0 = 1$
By Euler's method,

$$\begin{aligned} y_1 &= y_0 + hf(x_0, y_0) \\ &= y_0 + h(-y_0) \\ &= 1 - h(1) \\ &= 1 - h \end{aligned}$$

2. Write down the Milne's predictor-corrector formula for solving initial value problem in first order differential equation. [AU 2010,2011]

Solution:

Milne's Predictor Formula is

$$y_{n+1} = y_{n-3} + \frac{4h}{3}\left[2y'_{n-2} - y'_{n-1} + 2y'_n\right]$$

Milne's Corrector Formula is

$$y_{n+1} = y_{n-1} + \frac{h}{3}\left[y'_{n-1} + 4y'_n + y'_{n+1}\right]$$

3. Given $y' = -y$ and $y(0) = 1$, determine the value of $y(0.01)$ by Euler's method.[AU 2011]

Solution:

Given $y' = f(x, y) = -y$
$x_0 = 0,\ y_0 = 1\ h = 0.01$
By Euler's method,

$$\begin{aligned} y_1 &= y_0 + hf(x_0, y_0) \\ &= 1 + 0.01(-1) \\ &= 1 - 0.01 \\ &= 0.99 \\ \therefore \quad y(0.01) &= 0.99 \end{aligned}$$

4. State Runge-kutta 4th order formula.

Solution:

$k_1 = hf(x_n, y_n)$

$k_2 = hf(x_n + \frac{h}{2}, y_n + \frac{k_1}{2})$

$k_3 = hf(x_n + \frac{h}{2}, y_n + \frac{k_2}{2})$

$k_4 = hf(x_n + h, y_n + k_3)$

$\Delta y = \frac{[k_1+2k_2+2k_3+k_4]}{6}$

$y_{n+1} = y_n + \Delta y \quad n = 0,\ 1,\ 2,\ 3, \ldots$

5. Using Runge-Kutta method of fourth order to find the value of y when x=1, given that $\dfrac{dy}{dx} = \dfrac{y-x}{y+x}$ with $y(0) = 1$ [AU 2014]

Solution:

Given $\dfrac{dy}{dx} = f(x, y) = \dfrac{y-x}{y+x},\ y(0) = 1$. Take $h = 1$
Fourth order Runge-Kutta formula are
$k_1 = hf(x_n, y_n)$

$k_2 = hf(x_n + \frac{h}{2}, y_n + \frac{k_1}{2})$

$k_3 = hf(x_n + \frac{h}{2}, y_n + \frac{k_2}{2})$

$k_4 = hf(x_n + h, y_n + k_3)$

$\Delta y = \frac{[k_1+2k_2+2k_3+k_4]}{6}$

$y_{n+1} = y_n + \Delta y \quad n = 0,\ 1,\ 2,\ 3, \ldots$
Put $n = 0$ to find $y_1 = y(1)$
$k_1 = hf(x_0, y_0) = 1\left(\dfrac{1-0}{1+0}\right) = 1$

$k_2 = hf(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}) = 1f(0 + \frac{1}{2}, 1 + \frac{1}{2}) = f(\frac{1}{2}, \frac{3}{2}) = \left(\dfrac{\frac{3}{2} - \frac{1}{2}}{\frac{3}{2} + \frac{1}{2}}\right) = \frac{1}{2}$

$k_3 = hf(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}) = 1f(0 + \frac{1}{2}, 1 + \frac{1}{4}) = f(\frac{1}{2}, \frac{5}{4}) = \left(\dfrac{\frac{5}{4} - \frac{1}{2}}{\frac{5}{4} + \frac{1}{2}}\right) = \frac{3}{7}$

$k_4 = hf(x_0 + h, y_0 + k_3) = 1f(0 + 1, 1 + \frac{3}{7}) = f(1, \frac{10}{7}) = \left(\dfrac{\frac{10}{7} - 1}{\frac{10}{7} + 1}\right) = \dfrac{\frac{3}{7}}{\frac{17}{7}} = \frac{3}{17}$

$\Delta y = \frac{[k_1+2k_2+2k_3+k_4]}{6} = \frac{\left[1+2\frac{1}{2}+2\frac{3}{7}+\frac{3}{17}\right]}{6} = \frac{\left[2+\frac{6}{7}+\frac{3}{17}\right]}{6} = 0.4342$

$y_1 = y_0 + \Delta y = 1 + 0.43 = 1.4342$

$\therefore$ $y(1) = 1.4342$

6. Given that $y' = y + x^2$, $y(0) = 1$ find $y(0.02)$ using modified Euler's method. [AU 2014]

Solution:

Given $h = 0.02$ $x_0 = 0$ $y_0 = 1$
We have, **MODIFIED EULER'S METHOD:**

$$y_{n+1} = y_n + hf\left[x_n + \tfrac{h}{2}, y_n + \tfrac{h}{2}f(x_n, y_n)\right], \quad n = 0,\ 1,\ 2,\ 3, \ldots$$

To find $y(0.02)$ put $n = 0$, we get

$$\begin{aligned} y_1 &= y_0 + hf\left[x_0 + \tfrac{h}{2}, y_0 + \tfrac{h}{2}f(x_0, y_0)\right] \\ &= 1 + (0.02)f\left[0 + \tfrac{0.02}{2}, 1 + \tfrac{0.02}{2}f(0,1)\right] \\ &= 1 + (0.02)f\left[0.01, 1.01)\right] \\ &= 1 + (0.02)f\left(1.01 + 0.01^2\right) \\ &= 1.0202 \end{aligned}$$

7. State the merits of RK-method over Taylor series method. [AU 2012]

Solution:

Runge-kutta method is a single step method. This method does not require prior calculation of higher derivatives likes Taylor's series method. To find the value of y at $x = x_{r+1}$ we need the value of y at x_r only.

8. Write the central difference approximations for $\dfrac{dy}{dx}$, $\dfrac{d^2y}{dx^2}$.[AU 2012]

Solution:

- $y'_i = \frac{y_{i+1}-y_i}{h}$
- $y''_i = \frac{y_{i+1}-2y_i+y_{i-1}}{h^2}$

9. Bring out the merits and demerits of Taylor series method. [AU 2012]

Solution:

- Taylor series method is a powerful single step method if we are able to get the successive derivatives easily. This method is useful to give some initial values for powerful numerical methods like Runge-kutta method.
- The disadvantage is that it became tedious if the higher derivatives are complicated.

10. Find $y(0.1)$ by Euler's method, if $\dfrac{dy}{dx} = x^2 + y^2,\ y(0) = 1$. [AU 2012]

Solution:

Given $y' = f(x, y) = x^2 + y^2,\ y(0) = 1$. Take $h = 0.1,\ x_0 = 0,\ x_1 = 1$
By Euler's method,

$$\begin{aligned} y_1 &= y_0 + hf(x_0, y_0) \\ &= 1 + 0.1(x_0^2 + y_0^2) \\ &= 1 + 0.1(0 + 1) \\ &= 1.1 \\ \therefore \quad y(0.1) &= 1.1 \end{aligned}$$

11. Using Taylor's series method, find y at $x = 0.1$ given $\dfrac{dy}{dx} = x^2 - y, y(0) = 1$. [AU 2000,2006,2013]

Solution:

Given $y' = x^2 - y$, $y(0) = 1$
i.e., $x_0 = 0$ and $y_0 = 1$
Taylor's series is

$$y_{r+1} = y_r + \tfrac{h}{1!}y_r' + \tfrac{h^2}{2!}y_r'' + \tfrac{h^3}{3!}y_r''' + \ldots \quad r = 0,\ 1,\ 2,\ 3, \ldots - - - - - - - - (1)$$

At $(x_0, y_0) = (0, 1)$

$\because\ y' = x^2 - y \qquad y' = 0^2 - 1 = 0 - 1 = -1$
$\therefore\ y'' = 2x - y' \qquad y'' = 2(0) - (-1) = 1$
$y''' = 2 - y'' \qquad y''' = 2 - 1 = 1$

Putting $r = 0$ in (1), we get

$$y_1 = y_0 + \tfrac{h}{1!}y_0' + \tfrac{h^2}{2!}y_0'' + \tfrac{h^3}{3!}y_0''' + \ldots$$

$$\therefore \quad y_1 = y(0.1) = 1 + \tfrac{0.1}{1!}(-1) + \tfrac{(0.1)^2}{2!}(1) + \tfrac{(0.1)^3}{3!}(1)$$

$$= 1 - 0.1 + 0.005 + 0.0001666 = 0.905$$

$\therefore \quad y(0.1) = 0.905$

12. Compute y at $x = 0.25$ by Modified Euler method given $y' = 2xy$, $y(0) = 1$. [AU 2013]

Solution:

Given $h = 0.25$ $x_0 = 0$ $y_0 = 1$

$$y_{n+1} = y_n + hf\left[x_n + \frac{h}{2}, y_n + \frac{h}{2}f(x_n, y_n)\right], \quad n = 0,\ 1,\ 2,\ 3, \ldots$$

To find $y(0.25)$ put $n = 0$, we get

$$\begin{aligned} y_1 &= y_0 + hf\left[x_0 + \tfrac{h}{2}, y_0 + \tfrac{h}{2}f(x_0, y_0)\right] \\ &= 1 + (0.25)f\left[0 + \tfrac{0.25}{2}, 1 + \tfrac{0.25}{2}f(0, 1)\right] \\ &= 1 + (0.25)f\left[0.125, 1\right] \\ &= 1 + (0.25)(2\,(0.125)\,(1)) \\ &= 1.0625 \end{aligned}$$

13. Using Taylor's method, find y at x=1.1 given $\frac{dy}{dx} = x^3 + y$, $Y(1) = 1$. [AU 2013]

Solution:

Given $y' = x^3 + y$, $y(1) = 1$ $h = 0.1$
i.e., $x_0 = 1$ and $y_0 = 1$

Taylor's series is

$$y_{r+1} = y_r + \frac{h}{1!}y'_r + \frac{h^2}{2!}y''_r + \frac{h^3}{3!}y'''_r + \ldots \quad r = 0,\ 1,\ 2,\ 3, \ldots -------- (1)$$

At $(x_0, y_0) = (1, 1)$

$\because \quad y' = x^3 + y \qquad y' = 1^2 + 1 = 2$

$\therefore \quad y'' = 3x^2 + y' \qquad y'' = 3(1) + (1) = 4$

$y''' = 6x + y'' \qquad y''' = 6 + 4 = 10$

Putting $r = 0$ in (1), we get

$$y_1 = y_0 + \frac{h}{1!}y'_0 + \frac{h^2}{2!}y''_0 + \frac{h^3}{3!}y'''_0 + \ldots$$

$$\therefore \quad y_1 = y(1.1) = 1 + \frac{0.1}{1!}(2) + \frac{(0.1)^2}{2!}(4) + \frac{(0.1)^3}{3!}(10)$$

$$= 1 + 0.2 + 0.02 + 0.001666 = 1.2217$$

$\therefore \quad y(1.1) = 1.2217$

14. Obtain the finite difference scheme for differential equation $2\frac{d^2y}{dx^2} + y = 5$[AU 2013]

Solution:

Given $2y'' + y = 5 \Rightarrow y'' = \frac{1}{2}(5 - y)$
$\Rightarrow \frac{y_{i+1}-2y_i+y_{i-1}}{h^2} = \frac{1}{2}(5 - y_i)$
$\Rightarrow y_{i+1} - 2y_i + y_{i-1} = \frac{h^2}{2}(5 - y_i)$
$\Rightarrow y_{i+1} + \left(\frac{h^2}{2} - 2\right) y_i + y_{i-1} = 5\frac{h^2}{2}$

15. Convert the differential equation $y''(x) + y'(x) + y = 0$ into finite difference equivalent form.

Solution:

Given $y''(x) + y'(x) + y = 0$

$\Rightarrow \; y'' + y' + y = 0$

$\Rightarrow \; \frac{y_{i+1}-2y_i+y_{i-1}}{h^2} + \frac{y_{i+1}-y_i}{h} + y_i = 0$

16. How many pairs of prior values are required to predict the next value in Milne's method?

Solution:
Four pairs of prior values are required to apply Milne's Predictor formula.

17. State the Taylor's series formula to find $y(x_1)$ for solving $y' = f(x, y),\; y(x_0) = y_0$[AU 2007]

Solution:
$y_1 = y(x_1) = y_0 + \frac{h}{1!}y_0' + \frac{h^2}{2!}y_0'' + \frac{h^3}{3!}y_0''' + \dots$

18. Write down the modified Euler's method for ordinary differential equations. [AU 2007]

Solution:
$y_{n+1} = y_n + hf\left[x_n + \frac{h}{2}, y_n + \frac{h}{2}f(x_n, y_n)\right], \quad n = 0,\ 1,\ 2,\ 3, \dots$

where $x_n = x_0 + nh,\; y(x_n) = y_n$

19. State the difference between single step and multi step methods in solving ODE numerically?

Solution:

Euler and RK methods are single step methods, because they use only information from the last step computed. But multistep methods use more than one past values to compute the next value of y i.e., two or more step values are required for computation of the next.

20. Write down the Euler algorithm to solve differential equation $\frac{dy}{dx} = f(x, y)$

Solution:

If x_0 is the starting value,

$$
\begin{aligned}
y_1 &= y_0 + hf(x_0, y_0) \\
y_2 &= y_1 + hf(x_1, y_1) \\
&\vdots \\
y_{n+1} &= y_n + hf(x_n, y_n)
\end{aligned}
$$

PART B

Taylor series method

1. Given $\frac{dy}{dx} = 1+y^2$, $y(0) = 0$ by Taylor series method. Find the values of y at $x = 0.2$, $x = 0.4$ and $x = 0.6$ $[AU2010]$
2. Solve $y' = x + y$, $y(0) = 1$ by Taylor series method. Find the values of y at $x = 0.1$ and $x = 0.2$ $[AU2005, 2006, 2010]$
3. Solve $\frac{dy}{dx} = x + y$, $y(0) = 1$ by Taylor series method. Find $y(1.1)$ and $y(1.2)$ by Taylor series method $[AU2010]$
4. Using Taylor series method, find y at $x = 0.1$ given $\frac{dy}{dx} = x^2 - y$, $y(0) = 1$ correct to 4 decimal places.$[AU2000, 2006]$
5. Using Taylor method, find $y(0.2)$ and $y(0.4)$ given $\frac{dy}{dx} = 1 - 2xy$, and $y(0) = 1$ by taking $h = 0.2$ $[AU2011]$
6. Using Taylor series method solve $\frac{dy}{dx} = x^2 - y$, $y(0) = 1$ at $x = 0.1, 0.2, 0.3$ Also compare the values with exact solution. correct to 4 decimal places.$[AU2012]$
7. Using Taylor series method find y at $x = 0$, if $\frac{dy}{dx} = x^2y - 1$, $y(0) = 1$. $[AU2006, 2008]$
8. Using Taylor series method find $y(1.1)$ and $y(1.2)$ correct to four places given $y' = xy^{1/3}$ $[AU2006]$
9. Given $y' = xy + y^2$, $y(0) = 1$, use Taylor series method to get the values of $y(0.1)$, $y(0.2)$ and $y(0.3)$. $[AU2009]$
10. Use Taylor series method to find $y(0.1)$, $y(0.2)$, given that $\frac{dy}{dx} = 3e^x + 2y$, $y(0) = 0$ correct to 4 decimal accuracy. $[AU2008, 2010]$

Euler's method and Modified Euler method

1. Using Euler's method, find $y(0.2)$ and $y(0.4)$ from $\frac{dy}{dx} = x + y$, $y(0) = 1$ with $h = 0.2$ $[AU2001, 2010, 2012]$
2. Apply modified Euler's method to find $y(0.2)$ and $y(0.4)$ given $y' = x^2 + y^2$, $y(0) = 1$ by taking $h = 0.2$ $[AU2007, 2010]$
3. Using modified Euler method find $y(0.1)$ and $y(0.2)$ given $\frac{dy}{dx} = x^2 + y^2$, $y(0) = 1$ by taking $h = 0.2$ $[AU2010, 2011]$
4. Solve $\frac{dy}{dx} = log_{10}(x + y), y(0) = 2$ by Euler's modified method and find the values of $y(0.2)$, $y(0.4)$, and $y(0.6)$ by taking $h = 0.2$ $[AU2007]$
5. Using modified Euler's method solve given that $y' = y - x^2 + 1$, $y(0) = 0.5$ find $y(0.2)$ $[AU2003, 2006, 2012]$
6. Given that $\frac{dy}{dx} = 2 + \sqrt{xy}$ and $y = 1$ when $x = 1$. Find approximate value of y at $x = 2$ in steps of 0.2, using Euler's modified method. $[AU2004]$

7. Using Eulers modified method compute $y(0.1)$ with $h = 0.1$ from $y' = y - \frac{2x}{y}$, $y(0) = 1$. $[AU2011]$

8. Using modified Euler method solve, given that $y' = 1 - y$, $y(0) = 0$ find $y(0.1)$, $y(0.2)$ and $y(0.3)$. $[AU2005]$

9. Using Modified Euler's method, find $y(4.1)$ and $y(4.2)$ if $5x\frac{dy}{dx} + y^2 - 2 = 0$; $y(4) = 1$. $[AU2012]$

R-K method

1. Given $y'' + xy' + y = 0$, $y(0) = 1$, $y'(0) = 0$ find the value of $y(0.1)$ by R.K method of fourth order.$[AU2007, 2013]$

2. Consider the second order initial value problem $y'' - 2y' + 2y = e^{2t} sin\ t$ with $y(0) = -0.4$ and $y'(0) = -0.6$. Using fourth order Runge-Kutta method, find $y(0.2)$ $[AU2003]$

3. Compute $y(0.1)$ and $y(0.2)$ by Runge-Kutta method of 4^{th} order for the differential equation $\frac{dy}{dx} = xy + y^2$, $y(0) = 1$ $[AU2006, 2011]$

4. Solve for $y(0.1)$ and $z(0.1)$ from the simultaneous differential equations $\frac{dy}{dx} = 2y + z$; $\frac{dz}{dx} = y - 3z$; $y(0) = 0$, $z(0) = 0.5$ using Runge-Kutta method of fourth order. $[AU2012]$

5. Solve $y'' = xy' - y$ given $y(0) = 3$, $y'(0) = 0$ to find the value of $y(0.1)$, using RK-method of order 4. $[AU2012]$

6. Using Runge-Kutta method of fourth order solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ with $y(0) = 1$ at $x = 0.2,\ 0.4$ $[AU2004, 2005, 2006, 2007, 2010, 2011, 2012, 2013]$

7. Apply Runge-Kutta method to find approximate value of y for $x = 0.2$ in steps of 0.1 if $\frac{dy}{dx} = x + y^2$ given that $y = 1$ when $x = 0$. $[AU2008, 2010]$

8. Find $y(0.8)$ given that $y' = y - x^2$, $y(0.6) = 1.739$ by using Runge-Kutta method of fourth order. Take $h = 0.1$ $[AU2000, 2012]$

9. Given that $\frac{dy}{dx} = x^3 + y$, $y(0) = 2$. Compute $y(0.2)$, $y(0.4)$, $y(0.6)$ by Runge-Kutta method of fourth order. $[AU2004]$

10. Using Runge-Kutta method of fourth order solve $\frac{dy}{dx} = \frac{y^2 - 2x}{y^2 + x}$ with $y(0) = 1$ find y at $x = 0.2$ taking $h = 0.2$ $[AU2009]$

11. Apply 4^{th} order Runge-Kutta method to determine $y(0.2)$ with $h = 0.1$ from $\frac{dy}{dx} = x^2 + y^2$, $y(0) = 1$. $[AU2000]$

Milne's Predictor−Corrector Method

1. Solve $y' = \frac{1}{2}(1 + x^2)y^2$, $y(0) = 1$, $y(0.1) = 1.06$, $y(0.2) = 1.12$, $y(0.3) = 1.21$. Compute $y(0.4)$, using Milne's Predictor Corrector formula. $[AU2006, 2010]$

2. Given $5xy' + y^2 = 2$, $y(4) = 1$, $y(4.1) = 1.0049$, $y(4.2) = 1.0097$, $y(4.3) = 1.0143$ Compute $y(4.4)$ using Miline's method. $[AU2004, 2007, 2010, 2011]$

3. Solve $y' = x - y^2$, $0 \leq x \leq 1$, $y(0) = 0$, $y(0.2) = 0.02$, $y(0.4) = 0.0795$, $y(0.6) = 0.1762$ by Milne's method to find $y(0.8)$ and $y(1)$. $[AU1990, 2005, 2010]$

4. Given $y' = xy+y^2$, $y(0) = 1$, find $y(0.1)$ by Taylor's method, $y(0.2)$ by Euler's method, $y(0.3)$ by Runge-Kutta method and $y(0.4)$ by Milne's method. $[AU2003, 2006, 2008]$

5. Given $\frac{dy}{dx} = x^3 + y$, $y(0) = 2$. $y(0.2) = 2.073$, $y(0.4) = 2.452$, $y(0.6) = 3.023$. Compute $y(0.8)$ by Milne's predictor-corrector method taking $h = 0.2$ $[AU2004, 2006]$

6. Given $\frac{dy}{dx} = x^2(1 + y)$ and $y(1) = 1$, $y(1.1) = 1.233$, $y(1.2) = 1.548$, $y(1.3) = 1.979$, evaluate $y(1.4)$ by Milne's predictor corrector method. $[AU2009]$

7. Using Milne's predictor-corrector method, find $y(0.2)$ given $y' = xy(1 + y)$ given $y(-0.2) = 1.0412$, $y(0) = 1.0108$ and $y(-0.1) = 1.0108$ $[AU2009]$

8. Given the initial value problem $\frac{dy}{dx} = \frac{1}{2}(x + y)$, $y(0) = 2$, $y(0.5) = 2.636$, $y(1) = 3.595$, $y(1.5) = 4.968$ find $y(2)$ by Milne method. $[AU2000, 2009]$

9. Given $y' = x(x^2 + y^2)e^{-x}$, $y(0) = 1$, find y at $x = 0.1$, 0.2 and 0.3 by Taylors series method and compute $y(0.4)$ by Milne's method. $[AU2007]$

Finite differences

1. Using finite differences solve the boundary value problem $y'' + 3y' - 2y = 2x + 3$, $y(0) = 2,\ y(1) = 1$ with $h = 2[2010]$

2. Solve $u_{n+2} - 4u_{n+1} + 4u_n = 2^n.[2010]$

3. Solve the BVP $\dfrac{d^2y}{dx^2} - y = 0$ with $y(0) = 0,\ y(1) = 1$, using finite difference method with $h = 0.2[2012]$

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