



# **KCG**

**COLLEGE OF TECHNOLOGY**

**B.E - Mechanical Engineering**

**(Anna University Regulation - 2017)**

**THIRD SEMESTER**

**QUESTION BANK**



**DEPARTMENT OF MECHANICAL  
ENGINEERING**

**KCG COLLEGE OF TECHNOLOGY,**

**CHENNAI – 600097**

**SUBJECTS FOR III SEMESTER MECHANICAL ENGINEERING**

S. No.	Course Code	Course Title	Cat	Contact Periods	L	T	P	C
<b>THEORY</b>								
1.	MA8353	Transforms and Partial Differential Equations	BS	4	4	0	0	4
2.	ME8391	Engineering Thermodynamics	PC	5	3	2	0	4
3.	CE8394	Fluid Mechanics and Machinery	ES	4	4	0	0	4
4.	ME8351	Manufacturing Technology - I	PC	3	3	0	0	3
5.	EE8353	Electrical Drives and Controls	ES	3	3	0	0	3
<b>PRACTICAL</b>								
6.	ME8361	Manufacturing Technology Laboratory - I	PC	4	0	0	4	2
7.	ME8381	Computer Aided Machine Drawing	PC	4	0	0	4	2
8.	EE8361	Electrical Engineering Laboratory	ES	4	0	0	4	2
9.	HS8381	Interpersonal Skills/Listening & Speaking	EEC	2	0	0	2	1
			<b>TOTAL</b>	<b>33</b>	<b>17</b>	<b>2</b>	<b>14</b>	<b>25</b>

### **VISION OF THE COLLEGE**

KCG College of Technology aspires to become a globally recognized centre of excellence for science, technology & engineering education, committed to quality teaching, learning, and research while ensuring for every student a unique educational experience which will promote leadership, job creation, social commitment and service to nation building.

### **MISSION OF THE COLLEGE**

- Disseminate knowledge in a rigorous and intellectually stimulating environment
- Facilitate socially responsive research, innovation and entrepreneurship
- Foster holistic development and professional competency
- Nurture the virtue of service and an ethical value system in the young minds

### **VISION OF THE DEPARTMENT**

The department aspires to become a globally recognized centre of excellence by producing competent professionals in Mechanical Engineering to serve as a valuable resource for industry and society.

### **MISSION OF THE DEPARTMENT**

- ❖ Impart intellectually rigorous and holistic education to the students in the field of Mechanical Engineering.
- ❖ Establish state of-the-art facilities for research and consultancy work.
- ❖ Enhance the knowledge and skills of the faculty with the latest advancements in the mechanical engineering domain.
- ❖ Mentor the students to develop research and entrepreneurial capabilities.
- ❖ Inculcate a high degree of professionalism and contribute to the needs of industry and Society.

## **PROGRAMME EDUCATIONAL OBJECTIVES:**

**On completion of the program, the students will achieve the following:**

<b>PEO 1</b>	Excel as competent professional or entrepreneur or researcher in related fields of Mechanical Engineering.
<b>PEO 2</b>	Analyze, design/develop innovative solutions for real world engineering problems using appropriate modern tools.
<b>PEO 3</b>	Exhibit professionalism, ethical attitude and adapt to the changes in the industry and society supporting sustainable development.
<b>PEO 4</b>	Lead and manage teams for effective execution of projects.

## **PROGRAMME OUTCOMES**

**After successful completion of B.E./B.Tech./ M.E/ M.Tech. (Branch or Specialization) programme, the students will be able to:**

<b>PO No.</b>	<b>Description of the PO</b>
<b>1</b>	Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization to the solution of complex engineering problems.
<b>2</b>	Identify, formulate, review research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences
<b>3</b>	Design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal, and environmental considerations
<b>4</b>	Use research based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.
<b>5</b>	Create, select, and apply appropriate techniques, resources, and modern engineering and IT tools including prediction and modelling to complex engineering activities with an understanding of the limitations

<b>6</b>	Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal and cultural issues and the consequent responsibilities relevant to the professional engineering practice.
<b>7</b>	Understand the impact of the professional engineering solutions in societal and environmental contexts, and demonstrate the knowledge of, and need for sustainable development.
<b>8</b>	Apply ethical principles and commit to professional ethics and responsibilities and norms of the engineering practice.
<b>9</b>	Function effectively as an individual, and as a member or leader in diverse teams, and in multidisciplinary settings.
<b>10</b>	Communicate effectively on complex engineering activities with the engineering community and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions.
<b>11</b>	Demonstrate knowledge and understanding of the engineering and management principles and apply these to one's own work, as a member and leader in a team, to manage projects and in multidisciplinary environments.
<b>12</b>	Recognize the need for, and have the preparation and ability to engage in independent and lifelong learning in the broadest context of technological change.

### **PROGRAMME SPECIFIC OUTCOMES**

<b>PSO No.</b>	<b>Description of PSO</b>
<b>1</b>	Model, analyze, design and realize physical systems, components or process by applying principles of three core streams of Mechanical Engineering, i.e.Design, Manufacturing, Thermal and Fluid Engineering.
<b>2</b>	Apply the knowledge of AutoCAD, SolidWorks, ANSYS,CNC, Simulation softwares, MATLAB, Machine tool practices, Material & Machine testing, Fluid & Thermal machinery to solve real time Mechanical Engineering problems
<b>3</b>	Engage in lifelong learning and follow professional ethics, codes and standards of Professional practices.

**MA 8353**

**TRANSFORMS &  
PARTIAL  
DIFFERENTIAL  
EQUATIONS**

## **MA8353 TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS**

### **OBJECTIVES:**

- To introduce the basic concepts of PDE for solving standard partial differential equations.
- To introduce Fourier series analysis which is central to many applications in engineering apart from its use in solving boundary value problems.
- To acquaint the student with Fourier series techniques in solving heat flow problems used in various situations.
- To acquaint the student with Fourier transform techniques used in wide variety of situations.
- To introduce the effective mathematical tools for the solutions of partial differential equations that model several physical processes and to develop Z transform techniques for discrete time systems.

### **UNIT I PARTIAL DIFFERENTIAL EQUATIONS**

**12**

Formation of partial differential equations – Singular integrals - Solutions of standard types of first order partial differential equations - Lagrange's linear equation - Linear partial differential equations of second and higher order with constant coefficients of both homogeneous and non-homogeneous types.

### **UNIT II FOURIER SERIES**

**12**

Dirichlet's conditions – General Fourier series – Odd and even functions – Half range sine series – Half range cosine series – Complex form of Fourier series – Parseval's identity – Harmonic analysis.

### **UNIT III APPLICATIONS OF PARTIAL DIFFERENTIAL EQUATIONS 12**

Classification of PDE – Method of separation of variables - Fourier Series Solutions of one dimensional wave equation – One dimensional equation of heat conduction – Steady state solution of two dimensional equation of heat conduction.

### **UNIT IV FOURIER TRANSFORMS**

**12**

Statement of Fourier integral theorem – Fourier transform pair – Fourier sine and cosine transforms – Properties – Transforms of simple functions – Convolution theorem – Parseval's identity.

## UNIT V Z - TRANSFORMS AND DIFFERENCE EQUATIONS 12

Z-transforms - Elementary properties – Inverse Z-transform (using partial fraction and residues) – Initial and final value theorems - Convolution theorem - Formation of difference equations – Solution of difference equations using Z - transform.

**TOTAL : 60 PERIODS**

### COURSE OUTCOMES :

**Upon successful completion of the course, students should be able to:**

<b>CO 1</b>	Understand how to solve the given standard partial differential equations.
<b>CO 2</b>	Solve differential equations using Fourier series analysis which plays a vital role in engineering applications.
<b>CO 3</b>	Appreciate the physical significance of Fourier series techniques in solving one and two dimensional heat flow problems and one dimensional wave equations.
<b>CO 4</b>	Understand the mathematical principles on transforms and partial differential equations would provide them the ability to formulate and solve some of the physical problems of engineering.
<b>CO 5</b>	Use the effective mathematical tools for the solutions of partial differential equations by using Z transform techniques for discrete time systems.

### TEXT BOOKS :

1. Grewal B.S., "Higher Engineering Mathematics", 43rd Edition, Khanna Publishers, New Delhi, 2014.
2. Narayanan S., Manicavachagom Pillay.T.K and Ramanaiah.G "Advanced Mathematics for Engineering Students", Vol. II & III, S.Viswanathan Publishers Pvt. Ltd, Chennai, 1998.

### REFERENCES :

1. B.V Ramana.., "Higher Engineering Mathematics", McGraw Hill Education Pvt. Ltd, New Delhi, 2016.
2. Erwin Kreyszig, "Advanced Engineering Mathematics ", 10th Edition, John Wiley, India, 2016.
3. G. James, "Advanced Modern Engineering Mathematics", 3rd Edition, Pearson Education, 2007.
4. L.C Andrews, L.C and Shivamoggi, B, "Integral Transforms for Engineers" SPIE Press, 1999.
5. N.P. Bali. and Manish Goyal, "A Textbook of Engineering Mathematics", 9th Edition, Laxmi Publications Pvt. Ltd, 2014.
6. R.C. Wylie, and Barrett, L.C., "Advanced Engineering Mathematics "Tata McGraw Hill Education Pvt. Ltd, 6th Edition, New Delhi, 2012.



## UNIT-I PARTIAL DIFFERENTIAL EQUATIONS

### PART-A

1. **Form the partial differential equation of the family of spheres having their centres on the line  $x = y = z$ .**

Solution: Let the centre of the sphere having their centre of the line  $x = y = z$  is  $(a, a, a)$  and its radius be  $r$ .

$$\text{Hence its equation is } (x-a)^2 + (y-a)^2 + (z-a)^2 = r^2 \text{ ----}$$

----- (1)

Differentiating (1) partially with respect to  $x$ , we have

$$2(x-a) + 2(z-a) \frac{\partial z}{\partial x} = 0$$

$$2(x-a) + 2(z-a)p = 0$$

$$2[(x-a) + (z-a)p] = 0$$

$$(x-a) + (z-a)p = 0$$

$$x-a + zp - ap = 0$$

$$a + ap = x + zp$$

$$(1+p)a = x + zp$$

$$a = \frac{x+zp}{1+p} \text{ ..... (2)}$$

Differentiating (1) partially with respect to  $y$ , we have

$$2(y-a) + 2(z-a) \frac{\partial z}{\partial y} = 0$$

$$2(y-a) + 2(z-a)q = 0$$

$$2[(y-a) + (z-a)q] = 0$$

$$(y-a) + (z-a)q = 0$$

$$y-a + zq - aq = 0$$

$$a + aq = y + zq$$

$$(1+q)a = y + zq$$

$$a = \frac{y+zq}{1+q} \dots\dots\dots(3)$$

From (2) and (3), we have

$$\frac{x+z p}{1+p} = \frac{y+z q}{1+q}$$

$$(1+q)(x+z p) = (1+p)(y+z q)$$

$$x+z p+q x+z p q = y+z q+p y+z p q$$

$$x+z p+q x = y+z q+p y$$

$$y p-z p+z q-x q = x-y$$

$$(y-z) p+(z-x) q = x-y, \text{ which}$$

is the required partial differential equation.

2. **Form the partial differential equation by eliminating a and b from  $z = a(x+y) + b$ .**

Solution: Given  $z = a(x+y) + b \dots\dots\dots(1)$

Differentiating (1) partially with respect to  $x$ , we have

$$\frac{\partial z}{\partial x} = a \text{ implies } p = a \dots\dots\dots(2)$$

Differentiating (1) partially with respect to  $y$ , we have

$$\frac{\partial z}{\partial y} = a \text{ implies } q = a \dots\dots\dots(3)$$

From (2) and (3), we have

$$p = q$$

$p - q = 0$ , which is the required partial differential equation.

3. **Form a partial differential equation by eliminating the arbitrary constants from  $z = ax^2 + by^2$ . [N/D13]**

Solution:

$$\text{Given } z = ax^2 + by^2 \dots\dots\dots(1)$$

Differentiating (1) partially with respect to  $x$ , we have

$$\frac{\partial z}{\partial x} = 2ax$$

$$p = 2ax$$

$$a = \frac{p}{2x} \text{ ----- (2)}$$

Differentiating (1) partially with respect to  $y$ , we have

$$\frac{\partial z}{\partial y} = 2by$$

$$q = 2by$$

$$b = \frac{q}{2y} \text{ ----- (3)}$$

using (2) and (3) in (1), we have

$$z = \left( \frac{p}{2x} \right) x^2 + \left( \frac{q}{2y} \right) y^2$$

$$z = \frac{1}{2} [px + qy]$$

$2z = px + qy$ , which is the required partial differential equation.

4. **Form a partial differential equation by eliminating the arbitrary constants from  $z = ax^3 + by^3$ . [M/J14]**

Solution:

$$\text{Given } z = ax^3 + by^3 \text{ ----- (1)}$$

Differentiating (1) partially with respect to  $x$ , we have

$$\frac{\partial z}{\partial x} = 3ax^2$$

$$p = 3ax^2$$

$$a = \frac{p}{3x^2} \text{ ----- (2)}$$

Differentiating (1) partially with respect to  $y$ , we have

$$\frac{\partial z}{\partial y} = 3by^2$$

$$q = 3by^2$$

$$b = \frac{q}{3y^2} \text{ ----- (3)}$$

using (2) and (3) in (1), we have

$$z = \left( \frac{p}{3x^2} \right) x^3 + \left( \frac{q}{3y^2} \right) y^3$$

$$z = \frac{1}{3} [p x + q y]$$

$3z = p x + q y$ , which is the required partial differential equation.

5. **Form the partial differential equation by eliminating a and b from  $z = (x^2 + a^2)(y^2 + b^2)$ . [A/M10].**

Solution: Given  $z = (x^2 + a^2)(y^2 + b^2)$  ----- (1)

Differentiating (1) partially with respect to  $x$ , we have

$$\frac{\partial z}{\partial x} = 2x(y^2 + b^2)$$

$$p = 2x(y^2 + b^2)$$

$$y^2 + b^2 = \frac{p}{2x} \text{ ----- (2)}$$

Differentiating (1) partially with respect to  $y$ , we have

$$\frac{\partial z}{\partial y} = (x^2 + a^2)(2y)$$

$$q = 2y(x^2 + a^2)$$

$$x^2 + a^2 = \frac{q}{2y} \text{ ----- (3)}$$

using (2) and (3) in (1), we have

$$z = \left( \frac{q}{2y} \right) \left( \frac{p}{2x} \right)$$

$4xyz = pq$ , which is the required partial differential equation.

6. Find the partial differential equation of all planes passing through the origin.

Solution: Let the equation of the plane be

$$a x + b y + c z + d = 0 \quad \text{----- (1)} \quad \text{where } a, b, c$$

and  $d$  are constants.

Since plane (1) passes through the origin, we have

$$a(0) + b(0) + c(0) + d = 0$$

$$d = 0$$

substituting  $d = 0$  in (1), we have

$$a x + b y + c z = 0 \quad \text{----- (2)}$$

Differentiating (2) partially with respect to  $x$ , we have

$$a + c \frac{\partial z}{\partial x} = 0$$

$$a + c p = 0$$

$$a = -c p \quad \text{----- (3)}$$

Differentiating (2) partially with respect to  $y$ , we have

$$b + c \frac{\partial z}{\partial y} = 0$$

$$b + c q = 0$$

$$b = -c q \quad \text{----- (4)}$$

using (3) and (4) in (2), we have

$$(-c p)x + (-c q)y + c z = 0$$

$$-c p x - c q y + c z = 0$$

$$-c [p x + q y - z] = 0$$

$$x p + y q - z = 0$$

$x p + y q = z$ , which is the required partial differential equation.

7. Find the partial differential equation of all planes having equal intercepts on the  $x$  and  $y$  axis. [N/D09].

Solution: Let  $a$ ,  $c$  be the intercepts on  $x$  and  $z$  axes respectively.

Hence, the equation of the plane be

$$\frac{x}{a} + \frac{y}{a} + \frac{z}{c} = 1 \quad \text{----- (1)}$$

Differentiating (1) partially with respect to  $x$ , we have

$$\frac{1}{a} + \frac{1}{c} \frac{\partial z}{\partial x} = 0$$

$$\frac{1}{a} + \frac{1}{c} p = 0$$

$$\frac{1}{a} = -\frac{p}{c} \text{ ----- (2)}$$

Differentiating (1) partially with respect to  $y$ , we have

$$\frac{1}{a} + \frac{1}{c} \frac{\partial z}{\partial y} = 0$$

$$\frac{1}{a} + \frac{1}{c} q = 0$$

$$\frac{1}{a} = -\frac{q}{c} \text{ ----- (3)}$$

From (2) and (3), we have

$-\frac{p}{c} = -\frac{q}{c} \Rightarrow p = q \Rightarrow p - q = 0$  which is the required partial differential equation.

8. **Eliminate the arbitrary function  $f$  from  $z = f\left(\frac{xy}{z}\right)$  and form the partial differential equation.**

Solution: Given  $z = f\left(\frac{xy}{z}\right)$  ----- (1)

Differentiating (1) partially with respect to  $x$ , we have

$$\frac{\partial z}{\partial x} = f'\left(\frac{xy}{z}\right) \left[ \frac{z(y) - xy\left(\frac{\partial z}{\partial x}\right)}{z^2} \right]$$

$$p = f'\left(\frac{xy}{z}\right) \left[ \frac{yz - xy p}{z^2} \right] \text{ ----- (2)}$$

Differentiating (1) partially with respect to  $y$ , we have

$$\frac{\partial z}{\partial y} = f'\left(\frac{xy}{z}\right) \left[ \frac{z(x) - xy \left( \frac{\partial z}{\partial y} \right)}{z^2} \right]$$

$$q = f'\left(\frac{xy}{z}\right) \left[ \frac{xz - xyq}{z^2} \right] \text{----- (3)}$$

Equations  $\frac{(2)}{(3)}$  implies

$$\frac{p}{q} = \frac{f'\left(\frac{xy}{z}\right) \left[ \frac{yz - xyp}{z^2} \right]}{f'\left(\frac{xy}{z}\right) \left[ \frac{xz - xyq}{z^2} \right]}$$

$$\frac{p}{q} = \frac{yz - xyp}{xz - xyq}$$

$$xzp - xypq = yzq - xypq$$

$$xzp = yzq$$

$$xp = yq$$

$$xp - yq = 0, \text{ which is the required partial differential equation.}$$

9. **Form partial differential equation by eliminating the arbitrary function from  $z = f(xy)$ .**

Solution: Given  $z = f(xy)$  ----- (1)

Differentiating (1) partially with respect to  $x$ , we have

$$\frac{\partial z}{\partial x} = f'(xy)y$$

$$p = yf'(xy) \text{----- (2)}$$

Differentiating (1) partially with respect to  $y$ , we have

$$\frac{\partial z}{\partial y} = f'(xy)x$$

$$q = xf'(xy) \text{----- (3)}$$

Equations  $\frac{(2)}{(3)}$  implies

$$\frac{p}{q} = \frac{y f'(xy)}{x f'(xy)}$$

$$\frac{p}{q} = \frac{y}{x}$$

$$x p = y q$$

$x p - y q = 0$ , which is the required partial differential equation.

10. **Form the partial differential equation by eliminating  $f$  from the relation  $z = f(x^2 + y^2) + x + y$ .**

Solution: Given  $z = f(x^2 + y^2) + x + y$  -----(1)

Differentiating (1) partially with respect to  $x$ , we have

$$\frac{\partial z}{\partial x} = f'(x^2 + y^2)(2x) + 1$$

$$p = 2x f'(x^2 + y^2) + 1$$

$$p - 1 = 2x f'(x^2 + y^2) \text{ -----(2)}$$

Differentiating (1) partially with respect to  $y$ , we have

$$\frac{\partial z}{\partial y} = f'(x^2 + y^2)(2y) + 1$$

$$q = 2y f'(x^2 + y^2) + 1$$

$$q - 1 = 2y f'(x^2 + y^2) \text{ -----(3)}$$

Equations  $\frac{(2)}{(3)}$  implies

$$\frac{p-1}{q-1} = \frac{2x f'(x^2 + y^2)}{2y f'(x^2 + y^2)}$$

$$\frac{p-1}{q-1} = \frac{x}{y}$$

$$y p - y = x q - x$$

$y p - x q = y - x$ , which is the required partial differential equation.



11. Obtain the partial differential equation by eliminating the arbitrary functions  $f$  and  $g$  from

$$z = f(x + it) + g(x - it).$$

Solution: Given  $z = f(x + it) + g(x - it)$  ----- (1)

Differentiating (1) partially with respect to  $x$ , we have

$$\frac{\partial z}{\partial x} = f'(x + it) + g'(x - it) \text{ ----- (2)}$$

Differentiating (1) partially with respect to  $t$ , we have

$$\frac{\partial z}{\partial t} = f'(x + it)(i) + g'(x - it)(-i)$$

$$\frac{\partial z}{\partial t} = i f'(x + it) - i g'(x - it) \text{ Error! Reference source}$$

**not found.** ----- (3)

Differentiating (2) partially with respect to  $x$ , we have

$$\frac{\partial^2 z}{\partial x^2} = f''(x + it) + g''(x - it) \text{ ----- (4)}$$

Differentiating (3) partially with respect to  $t$ , we have

$$\frac{\partial^2 z}{\partial t^2} = i f''(x + it)(i) - i g''(x - it)(-i)$$

$$\frac{\partial^2 z}{\partial t^2} = i^2 f''(x + it) + i^2 g''(x - it)$$

$$\frac{\partial^2 z}{\partial t^2} = - f''(x + it) - g''(x - it)$$

$$\frac{\partial^2 z}{\partial t^2} = - [f''(x + it) + g''(x - it)] \text{ ----- (5)}$$

From (4) and (5), we have

$$\frac{\partial^2 z}{\partial x^2} = - \frac{\partial^2 z}{\partial t^2} \Rightarrow \frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial t^2} = 0 \text{ which is the required}$$

partial differential equation.

12. **Solve**  $\frac{\partial^2 z}{\partial x^2} = \sin y$ .

Solution: Given  $\frac{\partial^2 z}{\partial x^2} = \sin y$  ----- (1)

Integrating (1) with respect to  $x$ , we have

$$\frac{\partial z}{\partial x} = x \sin y + f(y) \text{ ----- (2)}$$

Again integrating (2) with respect to  $x$ , we have

$$z = \frac{x^2}{2} \sin y + x f(y) + \phi(y).$$

13. **Find the complete integral of**  $p + q = 1$ .

Solution: Given  $p + q = 1$

This is of the form  $F(p, q) = 0$

Hence, the complete integral is

$$z = ax + by + c \text{ where } a + b = 1 \text{ (ie) } b = 1 - a$$

Therefore, the complete solution is  $z = ax + (1 - a)y + c$ .

14. **Find the complete integral of**  $p + q = pq$  .[M/J13]

Solution: Given  $p + q = pq$

This is of the form  $F(p, q) = 0$ .

Hence the complete integral is

$$z = ax + by + c \text{ where } a + b = ab$$

$$b - ab = -a$$

$$(1 - a)b = -a$$

$$b = \frac{-a}{1 - a}$$

$$b = \frac{a}{a - 1}$$

Therefore, the complete solution is  $z = ax + \frac{a}{a - 1}y + c$ .

15. Find the complete solution of the partial differential equation  $p^2 + q^2 - 4pq = 0$ .

Solution: Given  $p^2 + q^2 - 4pq = 0$ .

This is of the form  $F(p, q) = 0$ .

Hence the complete integral is

$$z = ax + by + c \quad \text{where} \quad a^2 + b^2 - 4ab = 0$$

$$b^2 - 4ab + a^2 = 0$$

$$b = \frac{4a \pm \sqrt{16a^2 - 4a^2}}{2}$$

$$b = \frac{4a \pm \sqrt{12a^2}}{2}$$

$$b = \frac{4a \pm 2\sqrt{3}a}{2}$$

$$b = 2a \pm \sqrt{3}a = (2 \pm \sqrt{3})a$$

Therefore, the complete solution is

$$z = ax + (2 \pm \sqrt{3})ay + c.$$

16. Write down the complete solution of

$$z = px + qy + c\sqrt{1 + p^2 + q^2}.$$

Solution: Given  $z = px + qy + c\sqrt{1 + p^2 + q^2}$

This is of the form  $z = px + qy + f(p, q)$

Therefore, the complete solution is

$z = ax + by + c\sqrt{1 + a^2 + b^2}$  where a and b are arbitrary constants.

17. Find the complete integral of the partial differential equation  $(1-x)p + (2-y)q = 3-z$ .

Solution: Given  $(1-x)p + (2-y)q = 3-z$

$$p - px + 2q - qy = 3 - z$$

$$z = px + qy - p - 2q + 3$$

This is of the form  $z = px + qy + f(p, q)$

Hence, the complete solution is  $z = ax + by - a - 2b + 3$ .

18. **Find the complete integral of**  $\frac{z}{pq} = \frac{x}{q} + \frac{y}{p} + \sqrt{pq}$  .

Solution: Given  $\frac{z}{pq} = \frac{x}{q} + \frac{y}{p} + \sqrt{pq}$  -----(1)

(1)  $\times pq \Rightarrow z = px + qy + pq\sqrt{pq}$

This is of the form  $z = px + qy + f(p, q)$

Hence, the complete solution is  $z = ax + by + ab\sqrt{ab}$

19. **Solve the partial differential equation**  $pq = x$  .

**[A/M10].**

Solution: Given  $pq = x$  -----(1)

This of the form  $F(x, p, q) = 0$

Assume  $q = a$

Substituting  $q = a$  in (1), we have  $pa = x$  implies  $p = \frac{x}{a}$

But  $dz = pdx + qdy$

$$dz = \frac{x}{a} dx + a dy$$

Integrating on both sides, we have

$$z = \frac{x^2}{2a} + ay + c \text{ -----(2) which is the complete integral}$$

Differentiating (1) partially with respect to  $c$  , we have

$$0 = 1 \text{ which is absurd}$$

Hence there is no singular solution.

Substituting  $c = f(a)$  in (2) , we have

$$z = \frac{x^2}{2a} + ay + f(a) \text{ -----(3)}$$

Differentiating (3) partially with respect to  $a$  , we have

$$0 = -\frac{x^2}{2a^2} + y + f'(a) \text{-----(4)}$$

Eliminating  $a$  between the equations (3) and (4) we get the general solution.

20. **Solve**  $px^2 + qy^2 = z^2$ . [N/D14]

Solution: Given  $px^2 + qy^2 = z^2$

This is of Lagrange's type.

Here  $P = x^2$   $Q = y^2$   $R = z^2$

The subsidiary equations are  $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$

$$\frac{dx}{x^2} = \frac{dy}{y^2} = \frac{dz}{z^2}$$

$$\frac{dx}{x^2} = \frac{dy}{y^2} \quad \frac{dy}{y^2} = \frac{dz}{z^2}$$

$$x^{-2} dx = y^{-2} dy \quad y^{-2} dy = z^{-2} dz$$

Integrating, we have

$$\frac{x^{-1}}{-1} = \frac{y^{-1}}{-1} + a \frac{y^{-1}}{-1} = \frac{z^{-1}}{-1} + b$$

$$\frac{1}{x} - \frac{1}{y} = a \frac{1}{y} - \frac{1}{z} = b$$

Hence the solution is  $\phi\left(\frac{1}{x} - \frac{1}{y}, \frac{1}{y} - \frac{1}{z}\right) = 0$ .

21. **Find the general solution of**  
 $(4D^2 - 12DD' + 9D'^2)z = 0$ .

Solution: Auxiliary equation is  $4m^2 - 12m + 9 = 0$

$$(2m-3)(2m-3) = 0$$

$$m = \frac{3}{2}, \quad m = \frac{3}{2}$$

Hence the solution is  

$$z = \phi_1 \left( y + \frac{3}{2} x \right) + x \phi_2 \left( y + \frac{3}{2} x \right) .$$

22. **Solve**  $(D^3 + D^2 D' - D D'^2 - D'^3)z = 0 .$

Solution: Auxiliary equation is  $m^3 + m^2 - m - 1 = 0$

$$(m-1)(m^2 + 2m + 1) = 0$$

$$(m-1)(m+1)(m+1) = 0$$

$$m = 1 , m = -1 , m = -1$$

Hence the solution is  $z = \phi_1 (y+x) + \phi_2 (y-x) + x \phi_3 (y-x) .$

23. **Solve**  $(D^4 - D'^4)z = 0 .$  [M/J14]

Solution: The auxiliary equation is  $m^4 - 1 = 0$

$$(m^2 + 1)(m^2 - 1) = 0$$

$$m = 1 , -1 , i , -i$$

Hence the solution is  

$$z = \phi_1 (y+x) + \phi_2 (y-x) + \phi_3 (y+ix) + \phi_4 (y-ix) .$$

24. **Find the particular integral of**  
 $(D^2 - 2DD' + D'^2)z = e^{x-y} .$  [N/D10].

Solution:  $P.I = \frac{1}{D^2 - 2DD' + D'^2} e^{x-y}$

$$= \frac{1}{1-2(1)(1)+1} e^{x-y} = \frac{1}{0} e^{x-y} = \frac{1}{(D-D')^2} e^{x-y} = \frac{x^2}{2!} e^{x-y}$$

$$= \frac{x^2}{2} e^{x-y} .$$

25. **Solve**  $(D-D')(D+2D'+1)z = 0 .$

Solution: Given  $(D-D')(D+2D'+1)z = 0$

Here  $m_1 = 1$  ,  $c_1 = 0$  ,  $m_2 = -2$  ,  $c_2 = -1$

Hence the solution is  $z = \phi_1 (y+x) + e^{-x} \phi_2 (y-2x) .$

### **PART-B**

1. Form the partial differential equation of the family of planes that are at constant distance  $k$  from the origin. [A/M10].
2. Eliminate the arbitrary function  $f(x^2 + y^2 + z^2, x + y + z) = 0$ .
3. Form the partial differential equation by eliminating the arbitrary functions  $f$  and  $g$  in  $z = f(x^3 + 2y) + g(x^3 - 2y)$ .
4. Solve the equation  $pq + p + q = 0$ .
5. Find the singular integral of  $z = px + qy + pq$ .
6. Solve  $z = px + qy + p^2 q^2$  and find the complete and singular solutions. [N/D09].
7. Find the singular solution of  $z = px + qy + \sqrt{p^2 + q^2 + 16}$ .
8. Solve  $z = 1 + p^2 + q^2$ .
9. Solve  $p(1 - q^2) = q(1 - z)$ .
10. Solve  $p(1 + q) = qz$  [A/M10].
11. Solve  $p^2 + q^2 = x^2 + y^2$  [A/M10].
12. Solve  $x^2 p^2 + y^2 q^2 = z^2$ . [M/J14]
13. Solve  $z^2(p^2 + q^2) = x^2 + y^2$ .
14. Solve  $pqx y = z^2$ .
15. Solve  $(y + z)p + (z + x)q = x + y$ .

16. Solve  $(y-xz)p + (yz-x)q = (x+y)(x-y)$  [N/D09].

17. Solve  $x(y-z)p + y(z-x)q = z(x-y)$  [N/D11 , N/D14].

18. Solve  $(x^2-yz)p + (y^2-zx)q = z^2 - xy$  [A/M10].

19. Solve  $(2z-y)p + (x+z)q + 2x + y = 0$ .

20. Solve  $x(y^2-z^2)p + y(z^2-x^2)q = z(x^2-y^2)$  [M/J13 , N/D14].

21. Solve  $x(y^2+z)p + y(x^2+z)q = z(x^2-y^2)$ .

22. Solve the Lagrange's equation  $(x+2z)p + (2xz-y)q = x^2 + y$ . [M/J14]

23. Solve  $(2D^2 - 5DD' + 2D'^2)z = 5\sin(2x+y)$ .

24. Solve  $(D^3 + D^2D' - 4DD'^2 - 4D'^3)z = \cos(2x+y)$ . [N/D10, M/J12].

25. Solve  $(D^3 - 7DD'^2 - 6D'^3)z = e^{-2x+y} + \cos(x-2y)$ .

26. Solve  $(D^2 - D'^2)z = e^{x-2y} \sin(2x+y)$ .

27. Solve  $(D^2 - 3DD' + 2D'^2)z = \sin x \cos y$ .

28. Solve  $(D^2 + DD' - 6D'^2)z = y \cos x$  [M/J13 , M/J14].

29. Solve  $(D^2 - 2DD' + D'^2 - 3D + 3D' + 2)z = e^{2x-y}$  [N/D11].

30. Solve  $(D^2 - D'^2 - 3D + 3D')z = xy + 7$  [N/D09].





## UNIT-II FOURIER SERIES

### PART-A

1. State Dirichlet's conditions for a given function to expand in Fourier series. [N/D 09, A/M10, N/D11, M/J13, M/J14, N/D14, A/M15].

Solution:

A function  $f(x)$  defined in  $c \leq x \leq c + 2l$  can be expanded as an infinite trigonometric series of the form

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi}{l}\right)x + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi}{l}\right)x \text{ provided}$$

- (i)  $f(x)$  is single-valued and finite in  $(c, c + 2l)$
- (ii)  $f(x)$  is continuous or piecewise continuous with finite number of finite discontinuities in  $(c, c + 2l)$ .
- (iii)  $f(x)$  has no or finite number of maxima or minima in  $(c, c + 2l)$ .

2. State Euler's formula for Fourier co-efficients of a function defined in  $(c, c + 2l)$ .

Solution:

If a function  $f(x)$  defined in  $(c, c + 2l)$  can be expanded as the infinite trigonometric series

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi}{l}\right)x + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi}{l}\right)x, \text{ then}$$

$$a_0 = \frac{1}{l} \int_c^{c+2l} f(x) dx$$

$$a_n = \frac{1}{l} \int_c^{c+2l} f(x) \cos\left(\frac{n\pi}{l}\right)x dx$$

$$b_n = \frac{1}{l} \int_c^{c+2l} f(x) \sin\left(\frac{n\pi}{l}\right)x dx$$

Formulas given above for  $a_n$  and  $b_n$  are called Euler's formula for Fourier co-efficients.

**3. Does  $f(x) = \tan x$  possess a Fourier series expansion?**

Solution:

No,  $f(x) = \tan x$  does not possess a Fourier expansion. Because  $f(x) = \tan x$  has an infinite discontinuity. (ie) Dirichlet's condition is not satisfied.

**4. If  $f(x)$  is discontinuous at  $x = a$ , what does its Fourier series represent at that point?**

Solution:

If  $x = a$  is an interior point of discontinuity of  $f(x)$  in  $(c, c + 2l)$ , then the Fourier series of  $f(x)$  at  $x = a$  converges to  $\frac{1}{2} \lim_{h \rightarrow 0} [f(a-h) + f(a+h)]$

(ie) [ Sum of Fourier series of  $f(x)$  ] $_{x=a} = \frac{1}{2} \lim_{h \rightarrow 0} [f(a-h) + f(a+h)]$ .

**5. Determine  $b_n$  in the Fourier series expansion of**

$f(x) = \frac{1}{2} (\pi - x)$  in  $0 < x < 2\pi$  with period  $2\pi$ .

Solution:

Given  $f(x) = \frac{1}{2} (\pi - x)$

Here  $2l = 2\pi$  implies  $l = \pi$

$$b_n = \frac{1}{l} \int_0^{2l} f(x) \sin\left(\frac{n\pi}{l} x\right) dx$$

$$= \frac{1}{\pi} \int_0^{2\pi} \frac{1}{2} (\pi - x) \sin\left(\frac{n\pi}{\pi} x\right) dx = \frac{1}{2\pi} \int_0^{2\pi} (\pi - x) \sin nx dx$$

$$u = x$$

$$dv = \sin nx dx$$

$$u' = 1 \qquad v = -\frac{\cos nx}{n}$$

$$v_1 = -\frac{\sin nx}{n^2}$$

$$\text{Therefore, } b_n = \frac{1}{2\pi} \left[ -\frac{(\pi-x)}{n} \cos nx - \frac{\sin nx}{n^2} \right]_0^{2\pi}$$

$$\begin{aligned} b_n &= \frac{1}{2\pi} \left[ \left( -\frac{(\pi-2\pi)}{n} \cos 2n\pi - 0 \right) - \left( -\frac{(\pi-0)}{n} \cos 0 - 0 \right) \right] \\ &= \frac{1}{2\pi} \left[ \frac{\pi}{n} + \frac{\pi}{n} \right] = \frac{1}{2\pi} \left[ \frac{2\pi}{n} \right] = \frac{1}{n}. \end{aligned}$$

**6. Find the value of  $a_0$  in Fourier series expansion of**

$$f(x) = e^x \text{ in } (0, 2\pi) \text{ [N/D13]}$$

Solution: Here  $2l = 2\pi$  implies  $l = \pi$

$$a_0 = \frac{1}{l} \int_0^{2l} f(x) dx = \frac{1}{\pi} \int_0^{2\pi} e^x dx = \frac{1}{\pi} [e^x]_0^{2\pi} = \frac{1}{\pi} [e^{2\pi} - 1].$$

**7. If  $x^2 = \frac{\pi^2}{3} + 4 \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos nx$ , deduce that**

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots = \frac{\pi^2}{6}. \text{ [A/M10].}$$

Solution:

$$\text{Given } x^2 = \frac{\pi^2}{3} + 4 \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos nx$$

$$x^2 = \frac{\pi^2}{3} + 4 \left[ -\frac{1}{1^2} \cos x + \frac{1}{2^2} \cos 2x - \frac{1}{3^2} \cos 3x + \dots \dots \dots \right]$$

The point  $x = \pi$  is the point of continuity

$$\pi^2 = \frac{\pi^2}{3} + 4 \left[ -\frac{1}{1^2} \cos \pi + \frac{1}{2^2} \cos 2\pi - \frac{1}{3^2} \cos 3\pi + \dots \dots \dots \right]$$

$$\pi^2 - \frac{\pi^2}{3} = 4 \left[ -\frac{1}{1^2}(-1) + \frac{1}{2^2}(1) - \frac{1}{3^2}(-1) + \dots \dots \dots \right]$$

$$\frac{2\pi^2}{3} = 4 \left[ \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots \dots \dots \right]$$

$$\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots \dots \dots$$

$$\text{Therefore, } \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots \dots \dots = \frac{\pi^2}{6}.$$

**8. Give the expression for the Fourier series co-efficient  $b_n$  for the function  $f(x)$  defined in  $(-2, 2)$ . [A/M11].**

Solution:

The Fourier series co-efficient  $b_n$  for the function  $f(x)$  defined in  $(-2, 2)$  is given by

$$b_n = \frac{1}{2} \int_{-2}^2 f(x) \sin\left(\frac{n\pi}{2}\right) x dx.$$

**9. Find the constant term in the Fourier series corresponding to  $f(x) = \cos^2 x$  expressed in the interval  $(-\pi, \pi)$ . [N/D10, M/J12].**

Solution:

$$\text{Given } f(x) = \cos^2 x$$

$$f(x) = (\cos x)^2$$

$$f(x) = (\cos(-x))^2 = \cos^2 x = f(x)$$

Therefore,  $f(x)$  is an even function.

The constant term in the Fourier series is  $\frac{a_0}{2}$  where

$$\begin{aligned}
 a_0 &= \frac{2}{\pi} \int_0^{\pi} \cos^2 x \, dx \\
 &= \frac{2}{\pi} \int_0^{\pi} \frac{1 + \cos 2x}{2} \, dx \\
 &= \frac{1}{\pi} \left[ \int_0^{\pi} dx + \int_0^{\pi} \cos 2x \, dx \right] \\
 &= \frac{1}{\pi} \left[ (x)_0^{\pi} + \left( \frac{\sin 2x}{2} \right)_0^{\pi} \right] = \frac{1}{\pi} \left[ (\pi - 0) + \frac{1}{2} (0 - 0) \right] = \frac{1}{\pi} (\pi) = 1
 \end{aligned}$$

Therefore, the constant term  $\frac{a_0}{2}$  is  $\frac{1}{2}$ .

**10. What is the constant term  $a_0$  and the co-efficient of  $\cos nx$ ,  $a_n$  in the Fourier series expansion of  $f(x) = x - x^3$  in  $(-\pi, \pi)$ ?**

Solution:

$$\begin{aligned}
 \text{Given } f(x) &= x - x^3 \\
 f(-x) &= -x - (-x)^3 \\
 f(-x) &= -x - (-x^3) = -x + x^3 = -(x - x^3) = -f(x)
 \end{aligned}$$

Therefore,  $f(x)$  is an odd function.

Hence,  $a_0 = a_n = 0$ .

**11. Find  $b_n$  in the expansion of  $x^2$  as a Fourier series in  $(-\pi, \pi)$ .**

Solution:

$$\text{Given } f(x) = x^2$$

$$f(-x) = (-x)^2 = x^2 = f(x)$$

Therefore,  $f(x)$  is an even function.

Hence,  $b_n = 0$ .

**12. If  $f(x)$  is an odd function defined in  $(-l, l)$ , what are the values of  $a_0$  and  $a_n$ ?**

Solution:

Given  $f(x)$  is an odd function, the values of  $a_0 = a_n = 0$ .

**13. Obtain the first term of the Fourier series for the function  $f(x) = x^2$ ,  $-\pi < x < \pi$  [N/D09].**

Solution: Here  $2l = 2\pi$  implies  $l = \pi$

Given  $f(x) = x^2$

$$f(-x) = (-x)^2 = x^2 = f(x)$$

Therefore  $f(x)$  is an even function.

$$a_0 = \frac{2}{l} \int_0^l f(x) dx = \frac{2}{\pi} \int_0^{\pi} x^2 dx = \frac{2}{\pi} \left( \frac{x^3}{3} \right)_0^{\pi} = \frac{2}{3\pi} (\pi^3 - 0) = \frac{2}{3} \pi^2$$

Therefore the first term of Fourier series is

$$\frac{a_0}{2} = \frac{\frac{2}{3} \pi^2}{2} = \frac{\pi^2}{3}.$$

**14. Determine the Fourier series for the function  $f(x) = x$  in  $-\pi \leq x \leq \pi$ . [N/D14]**

Solution: Here  $2l = 2\pi$  implies  $l = \pi$

Given  $f(x) = x$

$$f(-x) = -x = -f(x)$$

Therefore  $f(x)$  is an odd function. Hence  $a_0 = a_n = 0$ .

Hence the Fourier series of  $f(x)$  is

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi}{l}\right)x$$

(i.e)  $f(x) = \sum_{n=1}^{\infty} b_n \sin nx$  where

$$b_n = \frac{2}{l} \int_0^l f(x) \sin\left(\frac{n\pi}{l}\right)x dx = \frac{2}{\pi} \int_0^{\pi} x \sin nx dx$$

$$u = x \quad dv = \sin nx dx$$

$$u' = 1 \quad v = -\frac{\cos nx}{n}$$

$$v_1 = -\frac{\sin nx}{n^2}$$

$$b_n = \frac{2}{\pi} \left[ -\frac{x \cos nx}{n} + \frac{\sin nx}{n^2} \right]_0^{\pi} = \frac{2}{\pi} \left[ \left( -\frac{\pi \cos n\pi}{n} + 0 \right) - (0 + 0) \right]$$

$$b_n = \frac{2}{\pi} \left[ -\frac{\pi(-1)^n}{n} \right] = -\frac{2(-1)^n}{n}$$

Therefore

$$f(x) = \sum_{n=1}^{\infty} -\frac{2(-1)^n}{n} \sin nx = -2 \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \sin nx.$$

**15. Find the Fourier constants  $b_n$  for  $x \sin x$  in  $(-\pi, \pi)$ .**

Solution:

Given  $f(x) = x \sin x$

$$f(-x) = -x \sin(-x) = -x(-\sin x) = x \sin x = f(x)$$

Therefore,  $f(x)$  is an even function.

Hence,  $b_n = 0$ .



16. If  $f(x) = x^2 + x$  is expressed as a Fourier series in the interval  $(-2, 2)$ , to which value this series converges at  $x = 2$ ?

Solution:

$$\begin{aligned} \text{Fourier series of } f(x) \text{ converges at } x = 2 \text{ is} \\ = \frac{f(-2) + f(2)}{2} \\ = \frac{[(-2)^2 - 2] + [2^2 + 2]}{2} = \frac{4 - 2 + 4 + 2}{2} = 4. \end{aligned}$$

17. Find the half range sine series expansion of  $f(x) = 1$  in  $(0, 2)$ . [N/D13]

Solution: Here  $l = 2$

Fourier sine series is given by

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi}{l}\right)x$$

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi}{2}\right)x$$

where

$$b_n = \frac{2}{l} \int_0^l f(x) \sin\left(\frac{n\pi}{l}\right)x dx = \frac{2}{2} \int_0^2 1 \sin\left(\frac{n\pi}{2}\right)x dx = \int_0^2 \sin\left(\frac{n\pi}{2}\right)x dx$$

$$= -\frac{2}{n\pi} \left[ \cos\left(\frac{n\pi}{2}\right)x \right]_0^2 = -\frac{2}{n\pi} [\cos n\pi - 1] = -\frac{2}{n\pi} [(-1)^n - 1] = \begin{cases} \frac{4}{n\pi}, & \text{if } n \text{ is odd} \\ 0, & \text{if } n \text{ is even} \end{cases}$$

Therefore

$$f(x) = \sum_{n=1,3,5}^{\infty} \frac{4}{n\pi} \sin\left(\frac{n\pi}{2}\right)x = \frac{4}{\pi} \sum_{n=1,3,5}^{\infty} \frac{1}{n} \sin\left(\frac{n\pi}{2}\right)x$$

**18. Find the half range cosine series of**

$$f(x) = \cos x, \quad 0 < x < \pi.$$

Solution:

$$\text{Here } l = \pi$$

Cosine series is given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi}{l}x\right)$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx \quad \dots\dots\dots(1)$$

where

$$a_0 = \frac{2}{l} \int_0^l f(x) dx = \frac{2}{\pi} \int_0^{\pi} f(x) dx = \frac{2}{\pi} \int_0^{\pi} \cos x dx = \frac{2}{\pi} [\sin x]_0^{\pi} = \frac{2}{\pi} [0 - 0] = 0$$

$$a_n = \frac{2}{l} \int_0^l f(x) \cos\left(\frac{n\pi}{l}x\right) dx = \frac{2}{\pi} \int_0^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_0^{\pi} \cos x \cos nx dx$$

$$= \frac{2}{\pi} \int_0^{\pi} \frac{\cos(nx+x) + \cos(nx-x)}{2} dx = \frac{1}{\pi} \left[ \int_0^{\pi} \cos(n+1)x dx + \int_0^{\pi} \cos(n-1)x dx \right]$$

$$= \frac{1}{\pi} \left[ \left( \frac{\sin(n+1)x}{n+1} \right)_0^{\pi} + \left( \frac{\sin(n-1)x}{n-1} \right)_0^{\pi} \right]$$

$$= \frac{1}{\pi} \left\{ \left[ \frac{1}{n+1} (0-0) \right] + \left[ \frac{1}{n-1} (0-0) \right] \right\} = 0 \quad \text{if } n \neq 1$$

if

$$n = 1, a_1 = \frac{2}{\pi} \int_0^{\pi} f(x) \cos x dx = \frac{2}{\pi} \int_0^{\pi} \cos x \cos x dx = \frac{2}{\pi} \int_0^{\pi} \cos^2 x dx$$

$$\begin{aligned}
 &= \frac{2}{\pi} \int_0^{\pi} \left( \frac{1 + \cos 2x}{2} \right) dx = \frac{1}{\pi} \left[ (x)_0^{\pi} + \left( \frac{\sin 2x}{2} \right)_0^{\pi} \right] \\
 &= \frac{1}{\pi} \left[ (\pi - 0) + \frac{1}{2} (0 - 0) \right] = 1
 \end{aligned}$$

Equation (1) can be written as

$$f(x) = \frac{a_0}{2} + a_1 \cos x + \sum_{n=2}^{\infty} a_n \cos nx$$

$$f(x) = \frac{0}{2} + (1) \cos x + \sum_{n=2}^{\infty} (0) \cos nx$$

$$f(x) = \cos x .$$

**19. To which value, the half range sine series corresponding to  $f(x) = x^2$  expressed in the interval  $(0, 2)$  converges at  $x = 2$  ?**

Solution:

We define  $F(x)$  as

$$F(x) = \begin{cases} -x^2 & ; -2 < x < 0 \\ x^2 & ; 0 < x < 2 \end{cases}$$

$$\begin{aligned}
 [\text{Sum of Fourier Series}]_{x=2} &= \frac{f(-2) + f(2)}{2} \\
 &= \frac{-4 + 4}{2} = \frac{0}{2} = 0
 \end{aligned}$$

Therefore, at  $x = 2$ , the series converges to zero.

**20. Define root mean square value of a function  $f(x)$  over the interval  $(a, b)$ . [M/J12, N/D12].**

Solution:

The root mean square value of a function  $f(x)$  over the interval  $(a, b)$  is given by

$$\bar{y} = \sqrt{\frac{1}{b-a} \int_a^b [f(x)]^2 dx}.$$

**21. Find the root mean square value of the function  $f(x) = x$  in the interval  $(0, l)$ . [N/D11].**

Solution:

The root mean square value of  $f(x) = x$  in  $(0, l)$  is given by

$$\bar{y} = \sqrt{\frac{1}{l-0} \int_0^l [x]^2 dx} = \sqrt{\frac{1}{l} \left[ \frac{x^3}{3} \right]_0^l} = \sqrt{\frac{1}{l} \left[ \frac{l^3}{3} - 0 \right]} = \sqrt{\frac{l^2}{3}}.$$

**22. Write down Parseval's formula on Fourier coefficients. [N/D14]**

Solution:

If  $y = f(x)$  can be expanded as Fourier series of the form

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi}{l}x\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi}{l}x\right) \text{ in } (0, 2l)$$

, then the root-mean square

value  $\bar{y}$  of  $y = f(x)$  in  $(0, 2l)$  is given by

$$\bar{y}^2 = \frac{1}{4} a_0^2 + \frac{1}{2} \sum_{n=1}^{\infty} a_n^2 + \frac{1}{2} \sum_{n=1}^{\infty} b_n^2 \quad \text{where}$$

$$\bar{y}^2 = \frac{1}{2l} \int_0^{2l} [f(x)]^2 dx.$$

**23. State Parseval's identity for the half range cosine expansion of  $f(x)$  in  $(0, 1)$ .**

Solution:

Parseval's identity for the half range cosine expansion of  $f(x)$  in  $(0, 1)$  is

$$\text{given by } \bar{y}^2 = \frac{1}{4} a_0^2 + \frac{1}{2} \sum_{n=1}^{\infty} a_n^2$$

$$\frac{1}{1-0} \int_0^1 [f(x)]^2 dx = \frac{1}{4} a_0^2 + \frac{1}{2} \sum_{n=1}^{\infty} a_n^2$$

$$\int_0^1 [f(x)]^2 dx = \frac{1}{4} a_0^2 + \frac{1}{2} \sum_{n=1}^{\infty} a_n^2 .$$

**24. If the Fourier series corresponding to  $f(x) = x$  in the interval  $(0, 2\pi)$  is  $\frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos nx + b_n \sin nx]$ ,**

**without finding the values of  $a_0$ ,  $a_n$ ,  $b_n$ , find the value of**

$$\frac{a_0^2}{4} + \frac{1}{2} \sum_{n=1}^{\infty} (a_n^2 + b_n^2).$$

Solution:

By Parseval's identity, we have

$$\begin{aligned} \frac{a_0^2}{4} + \frac{1}{2} \sum_{n=1}^{\infty} (a_n^2 + b_n^2) &= \bar{y}^2 = \frac{1}{2\pi} \int_0^{2\pi} [f(x)]^2 dx = \frac{1}{2\pi} \int_0^{2\pi} x^2 dx = \frac{1}{2\pi} \left[ \frac{x^3}{3} \right]_0^{2\pi} \\ &= \frac{1}{6\pi} [8\pi^3 - 0] = \frac{4}{3} \pi^2. \end{aligned}$$

**25. What do you mean by Harmonic Analysis?[M/J13]**

Solution:

When a function  $f(x)$  is given by its numerical values at  $q$  equally spaced

points, the co-efficients in the Fourier series representing  $f(x)$

can be obtained

by numerical integration.

## **PART-B**

- 1) Find the Fourier series expansion of  
 $f(x) = \begin{cases} x & \text{for } 0 \leq x \leq \pi \\ 2\pi - x & \text{for } \pi \leq x \leq 2\pi \end{cases}$ . Also deduce that

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots + \infty = \frac{\pi^2}{8}. \text{ [N/D10].}$$

- 2) Expand  $f(x) = \begin{cases} \sin x; & 0 \leq x \leq \pi \\ 0 & ; \pi \leq x \leq 2\pi \end{cases}$  as a Fourier series of

periodicity  $2\pi$  and hence evaluate  $\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \dots \infty$ .

- 3) Determine the Fourier series for the function  
 $f(x) = x \sin x$  in  $0 < x < 2\pi$ . [N/D14]

- 4) Obtain Fourier series for  $f(x)$  of period  $2l$  and defined as follows  $f(x) = \begin{cases} l - x; & 0 < x \leq l \\ 0 & ; l \leq x < 2l \end{cases}$ . Hence deduce that

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots = \frac{\pi}{4} \text{ and } \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}.$$

- 5) Find the Fourier series of periodicity 3 for  $f(x) = 2x - x^2$  in  $0 < x < 3$ . [N/D11, N/D14].

- 6) Find the Fourier series of  $f(x) = x^2$  in  $-\pi < x < \pi$ . Hence deduce the value of  $\sum_{n=1}^{\infty} \frac{1}{n^2}$ . [N/D14]

- 7) Obtain the Fourier series for  $f(x) = 1 + x + x^2$  in  $(-\pi, \pi)$ .

Deduce that  $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots = \frac{\pi^2}{6}$ .

- 8) Find the Fourier series for  $f(x) = |\cos x|$  in the interval  $(-\pi, \pi)$ .

- 9) Find the Fourier series of  $f(x) = e^{-x}$  in  $(-\pi, \pi)$ .

10) Find the Fourier series of  $f(x) = x + x^2$  in  $(-\pi, \pi)$  of periodicity  $2\pi$ . Hence deduce that  $\sum \frac{1}{n^2} = \frac{\pi^2}{6}$ . [N/D12].

11) Find the Fourier series of  $f(x) = \begin{cases} 1-x, & -\pi < x < 0 \\ 1+x, & 0 < x < \pi \end{cases}$ .

Hence deduce that  $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$ . [A/M11, N/D13].

12) Obtain the Fourier series of the periodic function defined by  $f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$ . Deduce that

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots + \infty = \frac{\pi^2}{8}. \text{ [N/D09].}$$

13) Find the Fourier series expansion of the periodic function  $f(x)$  of period  $2l$  defined by  $f(x) = \begin{cases} l+x; & -l \leq x \leq 0 \\ l-x; & 0 \leq x \leq l \end{cases}$ . Deduce that

$$\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}.$$

14) Expand  $f(x) = \begin{cases} 1 + \frac{2x}{\pi}, & -\pi < x < 0 \\ 1 - \frac{2x}{\pi}, & 0 < x < \pi \end{cases}$  as a full range

Fourier series in the interval  $(-\pi, \pi)$ . Hence deduce that

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}. \text{ [M/J14]}$$

15) Find the Fourier series of  $f(x) = x + x^2, -L < x < L$ .

16) Expand  $f(x) = x \sin x$  as a cosine series in  $0 < x < \pi$  and show that  $1 + \frac{2}{1.3} - \frac{2}{3.5} + \frac{2}{5.7} - \dots = \frac{\pi}{2}$ . [N/D11].

17) Obtain the half range cosine series for  $f(x) = x$  in  $(0, \pi)$ . [N/D10 ,12].

18) Find the half range sine series for  $f(x) = x(\pi - x)$  in the interval  $(0, \pi)$ .

19) Find the half range cosine series given

$$f(x) = \begin{cases} x; & 0 \leq x \leq 1 \\ 2-x; & 1 \leq x \leq 2 \end{cases}$$

20) Obtain the Fourier cosine series of  $(x-1)^2$ ,  $0 < x < 1$  and

hence show that  $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots = \frac{\pi^2}{6}$ . [M/J13]

21) Obtain the half range cosine series for  $f(x) = (x-2)^2$  in

the interval  $0 < x < 2$ . Deduce that  $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$ .

22) Expand  $f(x) = x - x^2$  as a Fourier series in  $-L < x < L$  and using this series find the root mean square value of  $f(x)$  in the interval. [N/D09].

23) Find the Fourier series of periodicity  $2\pi$  for  $f(x) = x^2$ ,

in  $-\pi < x < \pi$ . Hence show that  $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \dots = \frac{\pi^4}{90}$ .

[M/J13]

24) Find the Fourier series expansion of period  $l$  for the

function  $f(x) = \begin{cases} x; & (0, \frac{l}{2}) \\ l-x, & (\frac{l}{2}, l) \end{cases}$ . Hence deduce the sum of the

series  $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^4}$ .

25) By using cosine series for  $f(x) = x$  in  $0 < x < \pi$ , show that

$$\frac{\pi^4}{96} = 1 + \frac{1}{3^4} + \frac{1}{5^4} + \dots \quad [\text{N/D14}]$$



26) Find the half range Fourier cosine series of  $f(x) = (\pi - x)^2$  in the interval  $(0, \pi)$ . Hence find the sum of the series  $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \dots \infty$ . [M/J12].

27) Find the half range cosine series for the function  $f(x) = x(\pi - x)$  in  $0 < x < \pi$ . Deduce that  $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \dots = \frac{\pi^4}{90}$ . [A/M10].

28) Find the Fourier series of period  $2\pi$  as far as the first harmonic to represent the function  $y = f(x)$  defined by the following table:

$x^\circ$	0	30	60	90	120	150	180	210	240	270	300	330	360
$y$	2.34	3.01	3.69	4.15	3.69	2.20	0.83	0.51	0.88	1.09	1.19	1.64	2.34

29) Find the first fundamental harmonic of the Fourier series of  $f(x)$  given by the following table: [A/M10, N/D10, 12, M/J12].

X	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	$\pi$
Y	10	12	15	20	17	11	10

X	0	1	2	3	4	5
$f(x)$	9	18	24	28	26	20

30) Find the complex form of Fourier series for the function  $f(x) = e^{-x}$  in  $-1 < x < 1$ .

### UNIT-III APPLICATIONS OF PARTIAL DIFFERENTIAL EQUATIONS

#### PART-A

1) **Classify the following partial differential equation:**

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial y^2}$$

Solution:

$$\text{Given } \frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} = 0$$

$$\text{Here } A = 1, B = 0, C = -1$$

$$B^2 - 4AC = 0 - 4(1)(-1) = 4 > 0$$

Therefore, the given pde is hyperbolic.

2) **Classify the following second order partial differential equations:**

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2$$

Solution:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2$$

$$\text{Here } A = 1, B = 0, C = 1$$

$$B^2 - 4AC = (0)^2 - 4(1)(1) = 0 - 4 = -4 < 0$$

Therefore, the given pde is elliptic.

3) **Classify the following partial differential equations:**

$$y^2 u_{xx} + u_{yy} + u_x^2 + u_y^2 + 7 = 0$$

Solution:

$$y^2 u_{xx} + u_{yy} + u_x^2 + u_y^2 + 7 = 0$$

$$\text{Here } A = y^2, B = 0, C = 1$$

$$B^2 - 4AC = (0)^2 - 4(y^2)(1) = -4y^2$$

If  $y = 0$ ,  $B^2 - 4AC = 0$

Therefore, the given pde is parabolic.

If  $y > 0$  and  $y < 0$ ,  $B^2 - 4AC = -4 < 0$

Therefore, the given pde is elliptic.

4) In the wave equation  $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ , what does  $c^2$

stand for? [N/D11].

Solution:

The constant  $a^2$  in the wave equation  $u_{tt} = a^2 u_{xx}$

stands for  $\frac{Tension}{Mass}$

(i.e)  $a^2 = \frac{T}{M}$ .

5) What are the possible solutions of one dimensional wave equation? [N/D09,M/J14,N/D14].

Solution:

The possible solutions of one dimensional wave equation are

(i)

$$y(x, t) = (A_1 e^{\lambda x} + B_1 e^{-\lambda x}) (C_1 e^{\lambda at} + D_1 e^{-\lambda at})$$

(ii)

$$y(x, t) = (A_2 \cos \lambda x + B_2 \sin \lambda x) (C_2 \cos \lambda at + D_2 \sin \lambda at)$$

(iii)  $y(x, t) = (A_3 x + B_3) (C_3 t + D_3)$ .

6) State any two assumptions involved in deriving one dimensional wave equation.

Solution:

The assumptions involved in deriving one dimensional wave equation are

(i) The motion takes place entirely in one plane. This plane is chosen as the  $xy$  plane.

(ii) In this plane, each particle of the string moves in a direction perpendicular to the equilibrium position of the string.

(iii) The tension  $T$  caused by stretching the string before fixing it at the end points is constant at all times at all points of the deflected string.

(iv) The tension  $T$  is very large compared with the weight of the string and hence the gravitational force may be neglected.

(v)

**7) Write the initial conditions of the wave equation if the string has an initial displacement but no initial velocity.**

Solution:

The initial conditions of the wave equation if the string has an initial displacement but no initial velocity is given by

$$(i) \quad \frac{\partial y}{\partial t}(x, 0) = 0$$

$$(ii) \quad y(x, 0) = f(x).$$

**8) Write the boundary conditions and initial conditions for solving the vibration of string equation, if the string is subjected to initial displacement  $f(x)$  and initial velocity  $g(x)$ .**

Solution:

The boundary conditions and initial conditions for solving the vibration of string equation are

$$(i) \quad y(0, t) = 0$$

$$(ii) \quad y(l, t) = 0$$

$$(iii) \quad \frac{\partial y}{\partial t}(x, 0) = g(x)$$

$$(iv) \quad y(x, 0) = f(x).$$

**9) A taut string of length  $L$  cm fastened at both ends, is disturbed from its position of equilibrium by imparting to each of its points an initial velocity of magnitude  $kx(L-x)$  for  $0 < x < L$ . Formulate the problem mathematically.**

Solution:

The one dimensional wave equation is  $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$

The boundary conditions are given by

$$(i) y(0, t) = 0$$

$$(ii) y(L, t) = 0$$

$$(iii) y(x, 0) = 0$$

$$(iv) \frac{\partial y}{\partial t}(x, 0) = kx(L-x), 0 < x < L.$$

**10) A tightly stretched string with fixed end points  $x = 0$  and  $x = l$  is initially in a position given by**

$$y(x, 0) = y_0 \sin^3\left(\frac{\pi x}{l}\right).$$

**If it is released from rest in this position, write the boundary conditions. [A/M10].**

Solution:

The boundary conditions are

$$(i) y(0, t) = 0$$

$$(ii) y(l, t) = 0$$

$$(iii) \frac{\partial y}{\partial t}(x, 0) = 0$$

$$(iv) y(x, 0) = y_0 \sin^3\left(\frac{\pi x}{l}\right), 0 < x < l.$$

**11) Write the one dimensional heat equation.**

Solution:

The one dimensional heat equation is  $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$  where

$$\alpha^2 = \frac{k}{\rho c}.$$

**12) How many conditions are required to solve**

$$\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}.$$

Solution:

Three conditions are required to solve  $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}.$

**13) State any two laws which are assumed to derive one dimensional heat equation. [M/J14]**

Solution:

The laws which are assumed to derive one dimensional heat equation are

(i) Heat flows from a higher to lower temperature.  
(ii) The amount of heat required to produce a given temperature change in a body is proportional to the mass of the body and to the temperature change. This constant of proportionality is known as the specific heat ( $c$ ) of the conducting material.

(iii) The rate at which heat flows through any area is proportional to the area and to the temperature gradient normal to the area. This constant of proportionality is known as the thermal conductivity ( $k$ ) of the material.

**14) State the various possible solutions of one dimensional heat equation. [N/D10 , N/D14].**

Solution:

The various possible solutions of one dimensional heat equation are

$$(i) u(x, t) = (A_1 e^{\lambda x} + B_1 e^{-\lambda x}) e^{-\alpha^2 \lambda^2 t}$$

$$(ii) u(x, t) = (A_2 \cos \lambda x + B_2 \sin \lambda x) e^{-\alpha^2 \lambda^2 t}$$

$$(iii) u(x, t) = A_3 x + B_3.$$

**15) Define steady state condition on heat flow. [N/D13]**

Solution: The state in which the temperature at any point does not depend on  $t$ , but only on  $x$  is called steady state.

**16) What is the basic difference between the solutions of one dimensional wave equation and one dimensional heat equation. [M/J12]**

Solution:

The correct solution of one dimensional wave equation is of periodic in nature. But the solution of heat flow equation is not in periodic nature.

**17) In steady state conditions derive the solution of one dimensional heat flow equation.**

Solution:

The one dimensional heat flow equation is  $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$

.....(1)

In steady state conditions, the temperature at any particular point does not vary with time. (ie)  $u$  depends only on  $x$  and not on time  $t$ .

Hence the partial differential equation (1) in steady state becomes

$$\frac{d^2 u}{dx^2} = 0$$

Integrating we have

$$\frac{du}{dx} = a$$

Again integrating, we have

$$u(x) = ax + b$$

Therefore, in steady state condition, the solution of one dimensional heat flow equation is

$$u(x) = ax + b.$$

**18) The ends  $A$  and  $B$  of a rod  $20\text{ cm}$  long have the temperature at  $30^\circ\text{C}$  and  $80^\circ\text{C}$  until steady state prevails. Find the steady state temperature. [N/D14]**

Solution:

The steady state equation of one dimensional heat flow is

$$\frac{d^2 u}{dx^2} = 0 \text{ .....(1)}$$

The solution of (1) is  $u(x) = ax + b$  .....(2)

Here  $l = 20$

The boundary conditions are

$$(i) \quad u(0) = 30$$

$$(ii) \quad u(l) = 80$$

Applying condition (i) in (2), we have

$$u(0) = a(0) + b$$

$$30 = 0 + b$$

$$b = 30$$

Substituting  $b = 30$  in (2), we have

$$u(x) = ax + 30 \dots\dots\dots(3)$$

Applying condition (ii) in (3) and substituting  $l = 20$ , we have

$$u(l) = a(l) + 30$$

$$80 = a(20) + 30$$

$$20a = 50$$

$$a = \frac{5}{2}$$

Substituting  $a = \frac{5}{2}$  in (3), we have  $u(x) = \frac{5}{2}x + 30$ .

**19) An insulated rod of length 60 cm has its ends at A and B maintained at  $20^\circ C$  and  $80^\circ C$  respectively. Find the steady state solution of the rod. [N/D12,M/J12].**

Solution:

The steady state equation of one dimensional heat flow is

$$\frac{d^2u}{dx^2} = 0 \dots\dots(1)$$

$$\text{The solution of (1) is } u(x) = ax + b \dots\dots\dots(2)$$

Here  $l = 60$

The boundary conditions are

$$(i) \quad u(0) = 20$$

$$(ii) \quad u(l) = 80$$

Applying condition (i) in (2), we have

$$u(0) = a(0) + b$$

$$20 = 0 + b$$



$$b = 20$$

Substituting  $b = 20$  in (2), we have

$$u(x) = ax + 20 \dots\dots\dots(3)$$

Applying condition (ii) in (3), we have

$$u(l) = a(l) + 20$$

$$80 = a(60) + 20$$

$$60a = 60$$

$$a = 1$$

Substituting  $a = 1$  in (3), we have

$$u(x) = x + 20 .$$

**20) An insulated rod of length  $l$  cm has its ends  $A$  and  $B$  maintained at  $0^\circ C$  and  $80^\circ C$**

**respectively. Find the steady state solution of the rod.**

**[N/D13]**

Solution:

The steady state equation of one dimensional heat flow is

$$\frac{d^2u}{dx^2} = 0 \dots\dots(1)$$

$$\text{The solution of (1) is } u(x) = ax + b \dots\dots\dots(2)$$

The boundary conditions are

$$(i) u(0) = 0$$

$$(ii) u(l) = 80$$

Applying condition (i) in (2), we have

$$u(0) = a(0) + b$$

$$0 = b$$

$$b = 0$$

Substituting  $b = 0$  in (2), we have

$$u(x) = ax \dots\dots\dots(3)$$

Applying condition (ii) in (3), we have

$$u(l) = al$$

$$80 = al$$

$$a = \frac{80}{l}$$

Substituting  $a = \frac{80}{l}$  in (3), we have

$$u(x) = \frac{80}{l} x.$$

**21) Write down the governing equation of two dimensional steady state heat conduction.**

Solution:

$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$  is the governing equation of two dimensional steady state heat conduction.

**22) Write the steady state heat flow equation in two dimension in Cartesian equation and polar form. [M/J12].**

Solution:

The Cartesian equation of two dimensional heat flow is

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0.$$

The polar form of two dimensional heat flow is

$$r^2 \frac{\partial^2 u}{\partial r^2} + r \frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial \theta^2} = 0.$$

**23) Write down the three possible solutions of Laplace equation in two dimensions. [A/M10 ,N/D10 , A/M11]**

Solution:

The two solutions of the Laplace equation obtained by the method of separation of variables are

$$(i) \quad u(x, y) = (A_1 \cos \lambda x + B_1 \sin \lambda x) (C_1 e^{\lambda y} + D e^{-\lambda y})$$

$$(ii) \quad u(x, y) = (A_2 e^{\lambda x} + B_2 e^{-\lambda x}) (C_2 \cos \lambda y + D_2 \sin \lambda y)$$

$$(iii) \quad u(x, y) = (A_3 x + B_3) (C_3 y + D_3).$$

24) A plate is bounded by the lines  $x = 0$ ,  $y = 0$ ,  $x = l$  and  $y = l$ . Its faces are insulated. The edge coinciding with  $x$ -axis is kept at  $100^\circ\text{C}$ . The edge coinciding with  $y$ -axis is kept at  $50^\circ\text{C}$ . The other two edges are kept at  $0^\circ\text{C}$ . Write the boundary conditions that are needed for solving two dimensional heat flow equation. [N/D11, N/D12]

Solution: The boundary conditions are

$$(i) u(x, 0) = 100^\circ\text{C}, 0 < x < l$$

$$(ii) u(0, y) = 50^\circ\text{C}, 0 < y < l$$

$$(iii) u(x, l) = 0^\circ\text{C}, 0 < x < l$$

$$(iv) u(l, y) = 0^\circ\text{C}, 0 < y < l$$

25) Write down the two dimensional heat equations both in transient and steady states. [M/J13]

Solution: The two dimensional heat equation in transient state is

$$\frac{\partial u}{\partial t} = \alpha^2 \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \text{ and the two dimensional heat equation in}$$

$$\text{steady state is } \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0.$$

### PART-B

1) A uniform string is stretched and fastened to two points ' $l$ ' apart. Motion is started by displacing the string into the form of the curve  $y = kx(l - x)$  and then releasing it from this position at time  $t = 0$ . Find the displacement of the point of the string at a distance  $x$  from one end at time  $t$ . [A/M11, N/D13].

2) A tightly stretched string with fixed end points  $x = 0$  and  $x = l$  is initially in a position given by  $y(x, 0) = y_0 \sin^3\left(\frac{\pi x}{l}\right)$ . It is released from rest from this position. Find the displacement at any time ' $t$ '. [N/D12].

3) A tightly stretched string of length  $l$  has its ends fastened at  $x = 0$  and  $x = l$ . The midpoint of the string is then taken to a

height  $h$  and then released from rest in that position. Obtain an expression for the displacement of the string at any subsequent time. [A/M10].

4) A tightly stretched string with fixed end points  $x = 0$  and  $x = l$  is initially at rest in its equilibrium position. If it is set vibrating by giving each point a velocity  $kx(l - x)$ . Find the displacement of the string at any time. [M/J13, N/D14]

5) A tightly stretched string with fixed end points  $x=0$  and  $x=l$  is initially at rest in its equilibrium position. If it is set vibrating giving each point a initial velocity  $3x(l - x)$ , find the displacement. [N/D09].

6) A tightly stretched string with fixed end points  $x=0$  and  $x=10$  is initially at rest in its equilibrium position. If it is set vibrating giving each point a velocity  $3x(10 - x)$ , find the displacement  $y$  at any time and at any distance from end  $x=0$ .

7) A tightly stretched string of length  $l$  is initially at rest in its equilibrium position and each of its points is given the velocity  $v = v_0 \sin^3 \frac{\pi x}{l}$ , find the displacement of the string at any subsequent time. [N/D11, N/D14].

8) If a string of length  $l$  is initially at rest in its equilibrium position and each of its points is given a velocity  $V$  such that

$$v = \begin{cases} cx; 0 < x < \frac{l}{2} \\ c(l-x); \frac{l}{2} < x < l \end{cases}, \text{ find the displacement at any time } t.$$

9) An elastic string of length  $2l$  fixed at both ends is disturbed from its position at equilibrium position by imparting to each point an initial velocity of magnitude  $k(2lx - x^2)$ . Find the displacement function  $y(x, t)$ .

10) A string is stretched between two fixed points at a distance  $2l$  apart and the points of the string are given initial velocities  $v$

$$\text{where } v = \begin{cases} \frac{cx}{l}; 0 < x < l \\ \frac{c}{l}(2l - x); l < x < 2l \end{cases}, \quad x \text{ being the distance from one}$$

end point. Find the displacement of the string at any subsequent time.

11) Find the solution to the equation  $\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}$  that satisfies the conditions  $u(0, t) = 0$ ,  $u(l, t) = 0$ , for  $t > 0$  and

$$u(x, 0) = \begin{cases} x, & 0 \leq x \leq \frac{l}{2} \\ l - x, & \frac{l}{2} \leq x \leq l \end{cases}. \quad [\text{N/D13}]$$

12) An insulated rod of length  $L$  has its ends  $A$  and  $B$  maintained at  $0^\circ \text{C}$  and  $100^\circ \text{C}$  respectively, until steady state conditions prevail. If  $B$  is suddenly reduced to  $0^\circ \text{C}$  and that at  $A$  is maintained at  $0^\circ \text{C}$ , find the temperature at a distance  $x$  from  $A$  at time  $t$ .

13) A rod 30 cm long, has its ends A and B kept at  $20^\circ \text{C}$  and  $80^\circ \text{C}$  respectively, until steady state conditions prevail. The temperature at each end is then suddenly reduced to  $0^\circ \text{C}$  and kept so. Find the resulting temperature function  $U(x, t)$  taking  $x=0$  at A. [N/D09].

14) The ends A and B of a rod  $l \text{ cm}$  long have the temperatures  $40^\circ \text{C}$  and  $90^\circ \text{C}$  until steady state prevails. The temperature at A is suddenly raised to  $90^\circ \text{C}$  and at the same time that at B is lowered to  $40^\circ \text{C}$ . Find the temperature distribution in the rod at time  $t$ . Also show that the temperature at the midpoint of the rod remains unaltered for all time, regardless of the material of the rod.

15) Find the steady state temperature at any point of a square plate if two adjacent edges are kept at  $0^{\circ}C$  and the others at  $100^{\circ}C$ .

16) A rectangular plate is bounded by the lines  $x=0, y=0, x=a, y=b$ . Its surfaces are insulated. The temperature along  $x=0$  and  $y=0$  are kept at  $0^{\circ}C$  and the others at  $100^{\circ}C$ . Find the steady state temperature at any point of the plate.

17) A square plate is bounded by the lines  $x=0, x=a, y=0$  and  $y=b$ . Its surfaces are insulated and the temperature along  $y=b$  is kept at  $100^{\circ}C$ , while the temperature along other three edges are at  $0^{\circ}C$ . Find the steady state temperature at any point in the plate.[N/D14]

18) The boundary value problem governing the steady state temperature distribution in a flat thin square plate is given by

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad 0 < x < a, \quad 0 < y < a, \quad u(x, 0) = 0,$$

$$u(x, a) = 4 \sin^3 \left[ \frac{\pi x}{a} \right], \quad 0 < x < a \quad \text{and}$$

$u(0, y) = 0, u(a, y) = 0, 0 < y < a$ . Find the steady state temperature distribution in the plate.

19) Find the steady state temperature distribution in a rectangular plate of sides  $a$  and  $b$  insulated at the lateral surface and satisfying the boundary conditions  $u(0, y) = u(a, y) = 0$  for  $0 \leq y \leq b$ ,  $u(x, b) = 0$  and  $u(x, 0) = x(a - x)$  for  $0 \leq x \leq a$ . [N/D12].

20) A square plate is bounded by the lines  $x=0, y=0, x=20$  and  $y=20$ . Its faces are insulated. The temperature along the upper horizontal edge is given by  $u(x, 20) = x(20 - x)$ ,  $0 < x < 20$  while the other three edges are kept at  $0^{\circ}C$ . Find the steady state temperature distribution in the plate. [N/D10, N/D11, N/D14].

21) An infinitely long rectangular plate with insulated surfaces is 10cm wide. The two long edges and one short edge are kept at  $0^{\circ}C$ , while the other short edge  $x = 10$  is kept at temperature

$$u = \begin{cases} 20y & , 0 \leq y \leq 5 \\ 20(10 - y) & , 5 \leq y \leq 10 \end{cases} . \text{ Find the steady state}$$

temperature distribution in the plate.

22) A rectangular plate with insulated surface is 10cm so long compared to its width that it may be considered infinite length. If the temperature along short edge  $y = 0$  is given

$$u(x,0) = 8 \sin \left[ \frac{\pi x}{10} \right] \text{ when } 0 < x < 10, \text{ while the two long}$$

edges  $x = 0$  and  $x = 10$  as well as the other short edge are kept at  $0^{\circ}C$ , find the steady state temperature function  $u(x, y)$ . [M/J14]

23) A rectangular plate with insulated surface is 10cm wide and so long compared to its width that it may be considered infinite in length without introducing appreciable error. The temperature at

$$\text{short edge } y = 0 \text{ is given by } u = \begin{cases} 20x, 0 \leq x \leq 5 \\ 20(10 - x), 5 \leq x \leq 10 \end{cases} \text{ and}$$

all the other three edges are kept at  $0^{\circ}C$ . Find the steady state temperature at any point in the plate. [A/M10, M/J13]

24) A rectangular plate with insulated surfaces is 8cm wide and so long compared to its width that it may be considered as an infinite plate. If the temperature along short edge  $y = 0$  is

$$u(x, 0) = 100 \sin \frac{\pi x}{8}, 0 < x < 8, \text{ while two long edges } x=0 \text{ and}$$

$x=8$  as well as the other short edge are kept at  $0^{\circ}C$ . Find the steady state temperature at any point of the plate.

25) A long rectangular plate with insulated surface is  $l$  cm wide. If the temperature along one short edge ( $y = 0$ ) is

$$u(x, 0) = k(lx - x^2) \text{ degrees, for } 0 < x < l, \text{ while the other two long edges } x=0 \text{ and } x=l \text{ as well as the other short edge are kept at } 0^{\circ}C, \text{ find the steady state temperature function } u(x, y).$$

[M/J12].

## UNIT-IV FOURIER TRANSFORMS

### PART-A

**1) State the Fourier integral theorem. [M/J14,A/M15]**

Solution: If  $f(x)$  is piecewise continuous, has piecewise continuous derivatives in every finite interval in  $(-\infty, \infty)$  and absolutely integrable in  $(-\infty, \infty)$ , then

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(t) e^{is(x-t)} dt ds \quad \text{or} \quad \text{equivalently}$$

$$f(x) = \frac{1}{2\pi} \int \int f(t) \cos s(x-t) dt ds.$$

**2) Write the Fourier transform pair. [N/D10, N/D11].**

Solution: The Fourier transform pair is

$$F(s) = F[f(x)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{isx} dx \quad \text{and}$$

$$f(x) = F^{-1}[F(s)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(s) e^{-isx} ds$$

**3) Define self reciprocal with respect to Fourier transform. [N/D13]**

Solution: If  $f(s)$  is the Fourier transform of  $f(x)$ , then  $f(x)$  is said to be self reciprocal under Fourier transform.

**4) Find the Fourier transform of**

$$f(x) = \begin{cases} e^{ikx}, & a < x < b \\ 0, & x < a \text{ and } x > b \end{cases} \quad \text{[N/D09]}$$

Solution:  $F[f(x)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{isx} dx$

$$F[f(x)] = \frac{1}{\sqrt{2\pi}} \int_a^b e^{ikx} e^{isx} dx = \frac{1}{\sqrt{2\pi}} \int_a^b e^{i(s+k)x} dx$$



$$= \frac{1}{\sqrt{2\pi}} \left[ \frac{e^{i(s+k)x}}{i(s+k)} \right] = \frac{1}{\sqrt{2\pi}} \left[ \frac{e^{i(s+k)b} - e^{i(s+k)a}}{i(s+k)} \right].$$

**5) If  $F(s)$  is the Fourier transform of  $f(x)$ , write the formula for the Fourier transform of  $f(x)\cos ax$  in terms of  $F$ .**

Solution: By definition,  $F[f(x)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{isx} dx$

$$F[f(x)\cos ax] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)\cos ax e^{isx} dx$$

$$F[f(x)\cos ax] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) \left( \frac{e^{iax} + e^{-iax}}{2} \right) e^{isx} dx$$

$$= \frac{1}{2} \left[ \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{iax} e^{isx} dx + \int_{-\infty}^{\infty} f(x) e^{-iax} e^{isx} dx \right]$$

$$= \frac{1}{2} \left[ \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{i(s+a)x} dx + \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{i(s-a)x} dx \right]$$

$$= \frac{1}{2} [F(s+a) + F(s-a)]$$

$$(ie) \quad F[f(x)\cos ax] = \frac{1}{2} [F(s+a) + F(s-a)].$$

**6) State the shifting property on Fourier transform.**

Solution:

If  $F(s)$  is the Fourier transform of  $f(x)$ , then  $F(s)e^{ias}$  will be the Fourier transform of  $f(x-a)$ . (ie)  $F[f(x-a)] = e^{ias} F(s)$ .

**7) Find (a)  $F[x^n f(x)]$  and (b)  $F\left[\frac{d^n}{dx^n} f(x)\right]$  in terms of**

**Fourier transform of  $f(x)$ .**

Solution:

(a) By definition,

$$F(s) = F[f(x)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{isx} dx \quad \text{----- (1)}$$

Differentiating both sides of (1) with respect to  $S$ , we have

$$\begin{aligned} \frac{d}{ds} F(s) &= \frac{d}{ds} \left[ \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{isx} dx \right] \\ &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) \frac{\partial}{\partial s} (e^{isx}) dx \\ &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{isx} (ix) dx = i \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} (x f(x)) e^{isx} dx \\ (\text{ie}) \quad \frac{d}{ds} F(s) &= i F[x f(x)]. \end{aligned}$$

Differentiating both sides of (1)  $n$  times with respect to  $s$ , we have

$$\begin{aligned} \frac{d^n}{ds^n} F[s] &= i^n F[x^n f(x)] \\ F[x^n f(x)] &= \frac{1}{i^n} \frac{d^n}{ds^n} F(s) \\ F[x^n f(x)] &= (-i)^n \frac{d^n}{ds^n} F(s) \end{aligned}$$

(b) The Fourier transform of  $f'(x)$  is  $-is F(s)$ , if  $f(x) \rightarrow 0$  as  $x \rightarrow \pm \infty$

$$\begin{aligned} F[f(x)] &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{isx} dx \\ F[f'(x)] &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f'(x) e^{isx} dx \\ F[f'(x)] &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{isx} d(f(x)) \end{aligned}$$

$$u = e^{i s x} \quad dv = d(f(x))$$

$$du = i s e^{i s x} dx \quad v = f(x)$$

$$F[f(x)] = \frac{1}{\sqrt{2\pi}} \left[ \left( e^{i s x} f(x) \right)_{-\infty}^{\infty} - \int_{-\infty}^{\infty} f(x) i s e^{i s x} dx \right]$$

$$= \frac{1}{\sqrt{2\pi}} (0 - 0) - i s \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{i s x} dx$$

$$= -i s F[f(x)] = -i s F(s)$$

In general,  $F\left[\frac{d^n}{dx^n} f(x)\right] = (-i s)^n F[f(x)]$  assuming that

$$f(x), f'(x), \dots, f^{n-1}(x) \text{ all tends to zero as } x \rightarrow \pm \infty.$$

**8) What is the Fourier transform of  $f(x-a)$  if the Fourier transform of  $f(x)$  is  $F(s)$ .**

**[A/M 10, M/J 12, N/D13]. Prove that**  
 $F[f(x-a)] = e^{i s a} F(s)$ . **[N/D14]**

Solution:

$$\text{By definition, } F[f(x)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{i s x} dx$$

$$F[f(x-a)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x-a) e^{i s x} dx$$

Put

$$x-a=t \quad \text{when } x=-\infty, t=-\infty$$

$$x=t+a \quad \text{when } x=\infty, t=\infty$$

$$dx=dt$$

$$F[f(x-a)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{i s (t+a)} dt$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{i s t} e^{i s a} dt$$

$$= e^{ias} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{ist} dt = e^{ias} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{isx} dx$$

$$F[f(x-a)] = e^{ias} F[f(x)] = e^{ias} F(s)$$

**9) State the Fourier transforms of the derivatives of a function.**

Solution:

If the Fourier transform of  $f(x)$  is  $F(s)$ , then the Fourier transform of the derivatives

of a function  $f(x)$  (ie)  $F[f'(x)] = -is F(s)$ , if  $f(x) \rightarrow 0$  as  $x \rightarrow \pm \infty$ .

10) If  $F[f(x)] = F(s)$ , then give the value of  $F[f(ax)]$ .

(OR) State and prove the change of scale property of Fourier transform. [A/M 11, M/J13].

Solution:

By definition,

$$F[f(x)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{isx} dx$$

$$F[f(ax)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(ax) e^{isx} dx$$

when

$a > 0$ ,

put  $ax = t$  when  $x = -\infty, t = -\infty$

$x = \frac{t}{a}$  when  $x = \infty, t = \infty$

$$dx = \frac{dt}{a}$$

$$F[f(ax)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{is\left(\frac{t}{a}\right)} \frac{dt}{a}$$

$$= \frac{1}{a} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{i\left(\frac{s}{a}\right)t} dt$$

$$= \frac{1}{a} F\left[\frac{s}{a}\right] \text{-----(1)}$$

when  $a < 0$ ,

put  $ax = t$  when  $x = -\infty, t = -\infty$

$x = \frac{t}{a}$  when  $x = \infty, t = \infty$

$$dx = \frac{dt}{a}$$

$$\begin{aligned} F[f(ax)] &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{is\left(\frac{t}{a}\right)} \frac{dt}{a} \\ &= \frac{1}{a} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{i\left(\frac{s}{a}\right)t} dt \\ &= -\frac{1}{a} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{i\left(\frac{s}{a}\right)t} dt = -\frac{1}{a} F\left[\frac{s}{a}\right] \text{-----(2)} \end{aligned}$$

From (1) and (2), we have

$$F[f(ax)] = \frac{1}{|a|} F\left(\frac{s}{a}\right).$$

**11) If  $F\{f(x)\} = F(s)$ , then find  $F\{e^{iax} f(x)\}$ . [N/D14]**

Solution: By definition,  $F[f(x)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{isx} dx$

$$F[e^{iax} f(x)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{iax} f(x) e^{isx} dx$$

$$F[e^{iax} f(x)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{i(s+a)x} dx = F(s+a)$$

**12) Find the Fourier transform of  $e^{-\alpha|x|}$ ,  $\alpha > 0$ . [N/D12].**

Solution: By definition,

$$F[f(x)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{isx} dx$$

$$F[e^{-\alpha|x|}] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\alpha|x|} e^{isx} dx$$

$$\begin{aligned} F[e^{-\alpha|x|}] &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\alpha|x|} (\cos sx + i \sin sx) dx \\ &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\alpha|x|} \cos sx dx + i \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\alpha|x|} \sin sx dx \\ &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\alpha|x|} \cos sx dx + i(0) \quad (\text{since the second integrand} \end{aligned}$$

is an odd function)

$$\begin{aligned} &= \frac{1}{\sqrt{2\pi}} 2 \int_0^{\infty} e^{-\alpha x} \cos sx dx = \sqrt{\frac{2}{\pi}} \int_0^{\infty} e^{-\alpha x} \cos sx dx \\ &= \sqrt{\frac{2}{\pi}} \left[ \frac{e^{-\alpha x}}{(-\alpha)^2 + (s)^2} (-\alpha \cos sx + s \sin sx) \right]_0^{\infty} \\ &= \sqrt{\frac{2}{\pi}} \left[ 0 - \frac{1}{\alpha^2 + s^2} (-\alpha + 0) \right] \\ &= \sqrt{\frac{2}{\pi}} \frac{\alpha}{\alpha^2 + s^2}. \end{aligned}$$

**13) Find the Fourier transform of**  $f(x) = \begin{cases} 1 & , \text{if } |x| < 1 \\ 0 & , \text{if } |x| > 1 \end{cases}$

**.[N/D14]**

Solution: By definition,  $F[f(x)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{isx} dx$

$$\begin{aligned} &= \frac{1}{\sqrt{2\pi}} \left[ \int_{-\infty}^{-1} f(x) e^{isx} dx + \int_{-1}^1 f(x) e^{isx} dx + \int_1^{\infty} f(x) e^{isx} dx \right] \\ &= \frac{1}{\sqrt{2\pi}} \left[ \int_{-\infty}^{-1} (0) e^{isx} dx + \int_{-1}^1 (1) e^{isx} dx + \int_1^{\infty} (0) e^{isx} dx \right] \end{aligned}$$

$$\begin{aligned}
&= \frac{1}{\sqrt{2\pi}} \int_{-1}^1 e^{isx} dx \\
&= \frac{1}{\sqrt{2\pi}} \int_{-1}^1 (\cos sx + i \sin sx) dx \\
&= \frac{1}{\sqrt{2\pi}} \int_{-1}^1 \cos sx dx + i \frac{1}{\sqrt{2\pi}} \int_{-1}^1 \sin sx dx \\
&= \frac{1}{\sqrt{2\pi}} 2 \int_0^1 \cos sx dx + i(0) = \frac{2}{\sqrt{2\pi}} \left[ \frac{\sin sx}{s} \right]_0^1 \\
&= \sqrt{\frac{2}{\pi}} \left[ \frac{\sin s}{s} - 0 \right] = \sqrt{\frac{2}{\pi}} \frac{\sin s}{s}.
\end{aligned}$$

**14) Write down the Fourier cosine transform pair.**

Solution: Fourier cosine transform pair is

$$F_C[f(x)] = F_C(s) = \sqrt{\frac{2}{\pi}} \int_0^\infty f(x) \cos sx dx$$

$$\text{and } f(x) = F_C^{-1}(F(s)) = \sqrt{\frac{2}{\pi}} \int_0^\infty F_C(s) \cos sx ds.$$

**15) If  $F_C(s)$  is the Fourier cosine transform of  $f(x)$ , prove that Fourier cosine transform of**

$$f(ax) \text{ is } \frac{1}{a} F_C\left(\frac{s}{a}\right). \text{ [A/M11].}$$

Solution:

$$F_C[f(ax)] = F_C(s) = \sqrt{\frac{2}{\pi}} \int_0^\infty f(ax) \cos sx dx$$

$$\text{put } ax=t \quad \text{when } x=0, t=0$$

$$x = \frac{t}{a} \quad \text{when } x=\infty, t=\infty$$

$$dx = \frac{dt}{a}$$

$$\begin{aligned}
&= \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(t) \cos s \left( \frac{t}{a} \right) \frac{dt}{a} = \frac{1}{a} \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(t) \cos \left( \frac{s}{a} \right) t \, dt \\
&= \frac{1}{a} \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(x) \cos \left( \frac{s}{a} \right) x \, dx \quad (\text{ie}) \\
F_c[f(ax)] &= \frac{1}{a} F_c \left( \frac{s}{a} \right)
\end{aligned}$$

**16) If  $F_s(s)$  is the Fourier sine transform of  $f(x)$ , show that  $F_s[f(x) \cos ax] = \frac{1}{2} [F_s(s+a) + F_s(s-a)]$ .**

Solution:

$$\begin{aligned}
F_s(f(x) \cos ax) &= \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(x) \cos ax \sin sx \, dx \\
&= \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(x) \sin sx \cos ax \, dx \\
&= \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(x) \left[ \frac{\sin(s+a)x + \sin(s-a)x}{2} \right] dx \\
&= \frac{1}{2} \left[ \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(x) \sin(s+a)x \, dx + \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(x) \sin(s-a)x \, dx \right] \\
&= \frac{1}{2} [F_s(s+a) + F_s(s-a)] .
\end{aligned}$$

**17) Find the Fourier cosine transform of  $e^{-x}$ .**

Solution:

$$\begin{aligned}
F_c(e^{-x}) &= \sqrt{\frac{2}{\pi}} \int_0^{\infty} e^{-x} \cos sx \, dx \\
&= \sqrt{\frac{2}{\pi}} \left[ \frac{e^{-x}}{(-1)^2 + s^2} (-\cos sx + s \sin sx) \right]_0^{\infty}
\end{aligned}$$



$$= \sqrt{\frac{2}{\pi}} \left[ 0 - \frac{1}{1+s^2}(-1+0) \right] = \sqrt{\frac{2}{\pi}} \frac{1}{1+s^2}.$$

**18) Find the Fourier sine transform of  $\frac{1}{x}$ . [N/D09, M/J14].**

Solution:  $F_s \left[ \frac{1}{x} \right] = \sqrt{\frac{2}{\pi}} \int_0^\infty \frac{1}{x} \sin sx \, dx$

put  $sx = \theta$  when  $x=0, \theta=0$

$x = \frac{\theta}{s}$  when  $x=\infty, \theta=\infty$

$dx = \frac{d\theta}{s}$

$$= \sqrt{\frac{2}{\pi}} \int_0^\infty \frac{1}{\left(\frac{\theta}{s}\right)} \sin \theta \frac{d\theta}{s} = \sqrt{\frac{2}{\pi}} \int_0^\infty \frac{s}{\theta} \frac{\sin \theta}{s} d\theta$$

$$= \sqrt{\frac{2}{\pi}} \int_0^\infty \frac{\sin \theta}{\theta} d\theta = \sqrt{\frac{2}{\pi}} \frac{\pi}{2} = \sqrt{\frac{\pi}{2}}.$$

**19) Find the Fourier sine transform of  $e^{-ax}$ ,  $a > 0$ . [N/D10, M/J12].**

Solution:

$$F_s(e^{-ax}) = \sqrt{\frac{2}{\pi}} \int_0^\infty e^{-ax} \sin sx \, dx$$

$$= \sqrt{\frac{2}{\pi}} \left[ \frac{e^{-ax}}{(-a)^2 + s^2} (-a \sin sx - s \cos sx) \right]_0^\infty$$

$$= \sqrt{\frac{2}{\pi}} \left[ 0 - \frac{1}{a^2 + s^2} (0 - s) \right] = \sqrt{\frac{2}{\pi}} \frac{s}{s^2 + a^2}.$$

**20) Find the Fourier sine transform of  $e^{-3x}$ . [M/J13].**

Solution:

$$\begin{aligned}
 F_s(e^{-3x}) &= \sqrt{\frac{2}{\pi}} \int_0^{\infty} e^{-3x} \sin sx \, dx \\
 &= \sqrt{\frac{2}{\pi}} \left[ \frac{e^{-3x}}{(-3)^2 + s^2} (-3 \sin sx - s \cos sx) \right]_0^{\infty} \\
 &= \sqrt{\frac{2}{\pi}} \left[ 0 - \frac{1}{9 + s^2} (0 - s) \right] = \sqrt{\frac{2}{\pi}} \frac{s}{s^2 + 9}.
 \end{aligned}$$

**21) Find the Fourier cosine transform of  $e^{-ax}$ ,  $x > 0$ .**

**[A/M10].**

Solution:

$$\begin{aligned}
 F_c(e^{-ax}) &= \sqrt{\frac{2}{\pi}} \int_0^{\infty} e^{-ax} \cos sx \, dx \\
 &= \sqrt{\frac{2}{\pi}} \left[ \frac{e^{-ax}}{(-a)^2 + s^2} (-a \cos sx + s \sin sx) \right]_0^{\infty} \\
 &= \sqrt{\frac{2}{\pi}} \left[ 0 - \frac{1}{a^2 + s^2} (-a + 0) \right] = \sqrt{\frac{2}{\pi}} \frac{a}{s^2 + a^2}.
 \end{aligned}$$

**22) Find the Fourier sine transform of  $f(x)=1$  in  $(0, l)$ .**

Solution:

$$\begin{aligned}
 F_s[f(x)] &= \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(x) \sin sx \, dx = \sqrt{\frac{2}{\pi}} \int_0^l f(x) \sin sx \, dx \\
 &= \sqrt{\frac{2}{\pi}} \int_0^l (1) \sin sx \, dx = \sqrt{\frac{2}{\pi}} \left[ -\frac{\cos sx}{s} \right]_0^l \\
 &= \sqrt{\frac{2}{\pi}} \left[ -\frac{\cos sl}{s} + \frac{1}{s} \right] = \sqrt{\frac{2}{\pi}} \left[ \frac{1 - \cos sl}{s} \right].
 \end{aligned}$$

**23) State Convolution theorem on Fourier transforms.**

**[N/D12].**

Solution:

If  $F(s)$  and  $G(s)$  are the Fourier transforms of  $f(x)$  and  $g(x)$  respectively, then the Fourier transform of the convolution of  $f(x)$  and  $g(x)$  is the product of their Fourier transforms.

(ie)  $F[f(x) * g(x)] = F[f(x)] \cdot F[g(x)]$ .

**24) State Parseval's identity on Fourier transform. [N/D11].**

Solution: If  $F(s)$  is the Fourier transform of  $f(x)$ , then

$$\int_{-\infty}^{\infty} |f(x)|^2 dx = \int_{-\infty}^{\infty} |F(s)|^2 ds.$$

**25) Solve the integral equation  $\int_0^{\infty} f(x) \cos \lambda x dx = e^{-\lambda}$ .**

Solution: Given  $\int_0^{\infty} f(x) \cos \lambda x dx = e^{-\lambda}$

Multiplying both sides by  $\sqrt{\frac{2}{\pi}}$ , we have

$$\sqrt{\frac{2}{\pi}} \int_0^{\infty} f(x) \cos \lambda x dx = \sqrt{\frac{2}{\pi}} e^{-\lambda}$$

$$F_c[f(x)] = \sqrt{\frac{2}{\pi}} e^{-\lambda}$$

$$f(x) = F_c^{-1} \left[ \sqrt{\frac{2}{\pi}} e^{-\lambda} \right] = \sqrt{\frac{2}{\pi}} \int_0^{\infty} \sqrt{\frac{2}{\pi}} e^{-\lambda} \cos x \lambda d\lambda$$

$$= \frac{2}{\pi} \int_0^{\infty} e^{-\lambda} \cos x \lambda d\lambda$$

$$= \frac{2}{\pi} \left[ \frac{e^{-\lambda}}{(-1)^2 + x^2} (-\cos x \lambda + x \sin x \lambda) \right]_0^{\infty}$$

$$= \frac{2}{\pi} \left[ 0 - \frac{1}{1+x^2} (-1+0) \right] = \frac{2}{\pi} \frac{1}{1+x^2}.$$

## **PART-B**

- 1) Find the Fourier integral representation of  $f(x)$  defined

$$\text{as } f(x) = \begin{cases} 0 & \text{for } x < 0 \\ \frac{1}{2} & \text{for } x = 0 \\ e^{-x} & \text{for } x > 0 \end{cases}.$$

[N/D 10, M/J 12].

- 2) Find the Fourier transform of  $f(x) = \begin{cases} 1; & |x| < 1 \\ 0; & \text{otherwise} \end{cases}.$

$$\text{Hence prove that } \int_0^{\infty} \frac{\sin x}{x} dx = \int_0^{\infty} \frac{\sin^2 x}{x^2} dx = \frac{\pi}{2}.$$

- 3) Find the Fourier transform of  $e^{-a|x|}$ , if  $a > 0$ . Deduce that

$$\int_0^{\infty} \frac{dx}{(x^2 + a^2)^2} = \frac{\pi}{4a^3} \text{ if } a > 0.$$

- 4) Find the Fourier transform of  $f(x)$  defined by

$$f(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| > a \end{cases} \text{ and hence find the value of } \int_0^{\infty} \frac{\sin x}{x} dx \text{ using}$$

Parseval's identity prove that  $\int_0^{\infty} \left[ \frac{\sin t}{t} \right]^2 dt = \frac{\pi}{2}$ . [A/M 11, M/J13].

- 5) Find the Fourier transform of  $f(x) = \begin{cases} a^2 - x^2, & |x| < a \\ 0, & |x| > a > 0 \end{cases}$

Hence evaluate (i)  $\int_0^{\infty} \left( \frac{x \cos x - \sin x}{x^3} \right) \cos \frac{x}{2} dx$  and

$$(ii) \int_0^{\infty} \left( \frac{x \cos x - \sin x}{x^3} \right)^2 dx.$$

- 6) Find the Fourier transform of  $f(x) = \begin{cases} 1 - |x| & ; \text{ if } |x| < 1 \\ 0 & ; \text{ if } |x| \geq 1 \end{cases}$

. Hence deduce that  $\int_0^\infty \left( \frac{\sin t}{t} \right)^4 dt = \frac{\pi}{3}$  and  $\int_0^\infty \left( \frac{\sin t}{t} \right)^2 dt = \frac{\pi}{2}$ .

[N/D 11 , N/D12 , N/D14].

- 7) Find the Fourier transform of  $e^{-a|x|}$ ,  $a > 0$  and hence deduce that

$$(1) \int_0^\infty \frac{\cos xt}{a^2 + t^2} dt = \frac{\pi}{2a} e^{-a|x|}$$

$$(2) F\{xe^{-a|x|}\} = i \sqrt{\frac{2}{\pi}} \frac{2as}{(s^2 + a^2)}, \text{ here } F \text{ stands for}$$

Fourier transform.

[M/J14,N/D14]

- 8) Show that the Fourier transform of  $e^{-\frac{x^2}{2}}$  is  $e^{-\frac{s^2}{2}}$ .  
[A/M10,N/D11,M/J13].

- 9) Find the Fourier sine transform of

$$f(x) = \begin{cases} \sin x & ; 0 < x \leq \pi \\ 0 & ; \pi \leq x < \infty \end{cases}.$$

- 10) Find the Fourier cosine transform of  $e^{-4x}$ . Deduce that

$$\int_0^\infty \frac{\cos 2x}{x^2 + 16} dx = \frac{\pi}{8} e^{-8} \text{ and } \int_0^\infty \frac{x \sin 2x}{x^2 + 16} dx = \frac{\pi}{2} e^{-8}.$$

- 11) Find the Fourier sine transform of  $xe^{-\frac{x^2}{2}}$ .

- 12) Prove that  $f(x) = e^{-\frac{x^2}{2}}$  is self reciprocal under the Fourier cosine transform.

- 13) Prove that  $\frac{1}{\sqrt{x}}$  is self reciprocal under Fourier sine and cosine transforms. [N/D09].

14) Find the Fourier sine and cosine transform of  $e^{-ax}$  and hence find the Fourier sine

transform of  $\frac{x}{x^2+a^2}$  and Fourier cosine transform of

$$\frac{1}{x^2+a^2}.$$

15) Find the Fourier sine and cosine transform of

$$f(x) = \begin{cases} x & ; 0 < x < 1 \\ 2-x & ; 1 < x < 2 \\ 0 & ; x > 2 \end{cases} . \text{ [N/D10, A/M11].}$$

16) Find the Fourier sine transform of  $\frac{e^{-ax}}{x}$  where  $a > 0$ .

17) Find the Fourier cosine transform of  $e^{-a^2 x^2}$ . Hence evaluate the Fourier sine transform of  $xe^{-a^2 x^2}$ .

18) Find Fourier sine and cosine transform of  $x^{n-1}$  and hence

prove that  $\frac{1}{\sqrt{x}}$  is self reciprocal under Fourier sine and cosine transforms. [M/J12].

19) Find the Fourier sine transform of  $e^{-ax}$ ,  $a > 0$ . Hence find  $F[xe^{-ax}]$ .

20) Find Fourier sine transform of  $f(x)$  defined as

$$f(x) = \begin{cases} \sin x & ; 0 < x < a \\ 0 & ; x > a \end{cases}.$$

21) Prove that  $e^{-\frac{x^2}{2}}$  is self reciprocal under Fourier cosine transform.

22) Find the function whose Fourier sine transform is

$$\frac{e^{-as}}{s} \quad (a > 0). \text{ [N/D13]}$$

- 23) Find Fourier sine transform and Fourier cosine transform of  $e^{-ax}, a > 0$ . Hence evaluate  $\int_0^{\infty} \frac{x^2}{(a^2 + x^2)^2} dx$  and  $\int_0^{\infty} \frac{dx}{(x^2 + a^2)(x^2 + b^2)}$ .
- 24) State and prove convolution theorem for Fourier transforms. [N/D11, M/J12].
- 25) Derive the Parseval's identity for Fourier transforms. [N/D10, M/J12].

## UNIT-V Z-TRANSFORMS AND DIFFERENCE EQUATIONS

### PART-A

1) Define Z-transform of the sequence  $\{f(n)\}$ .

Solution:

Let  $\{f(n)\}$  be a sequence defined for  $n = 0, \pm 1, \pm 2, \dots$ , then the two-sided Z-transform of the sequence  $f(n)$  is defined as

$$Z\{f(n)\} = F[z] = \sum_{n=-\infty}^{\infty} f(n)z^{-n}, \text{ where } z \text{ is a complex}$$

variable.

If  $\{f(n)\}$  is a casual sequence, then the z-transform reduces to one-sided Z-transform and its definition is

$$Z\{f(n)\} = F[z] = \sum_{n=0}^{\infty} f(n)z^{-n}.$$

2) State and prove initial value theorem in Z-transform. [N/D14]

Solution:

Statement: If  $Z[f(t)] = F[z]$ , then  $f(0) = \lim_{z \rightarrow \infty} z F[z]$ .

$$\begin{aligned} \text{Proof: } F[z] &= Z[f(t)] = \sum_{n=0}^{\infty} f(nT)z^{-n} \\ &= \sum_{n=0}^{\infty} f(nT)z^{-n} = \sum_{n=0}^{\infty} \frac{f(nT)}{z^n} \end{aligned}$$

$$= f(0.T) + \frac{f(1.T)}{z} + \frac{f(2.T)}{z^2} + \dots + \infty$$

$$= f(0) + \frac{1}{z} f(T) + \frac{1}{z^2} f(2T) + \dots + \infty$$

Taking limit as  $z \rightarrow \infty$  on both sides, we have

$$\lim_{z \rightarrow \infty} F[z] = \lim_{z \rightarrow \infty} \left[ f(0) + \frac{1}{z} f(T) + \frac{1}{z^2} f(2T) + \dots + \infty \right]$$

$$\lim_{z \rightarrow \infty} F[z] = f(0)$$

$$(i.e) f(0) = \lim_{z \rightarrow \infty} F[z].$$

### 3) State the final value theorem in Z-transform.

Solution: If  $Z[f(t)] = F[z]$ , then  $\lim_{t \rightarrow \infty} f(t) = \lim_{z \rightarrow 1} (z-1)F[z]$ .

### 4) Find $Z\{n\}$ . [M/J13, N/D14]

$$\text{Solution: } Z\{n\} = \sum_{n=0}^{\infty} n z^{-n} = \sum_{n=0}^{\infty} \frac{n}{z^n}$$

$$= 0 + \frac{1}{z} + \frac{2}{z^2} + \frac{3}{z^3} + \dots$$

$$= \frac{1}{z} \left[ 1 + \frac{2}{z} + \frac{3}{z^2} + \dots \right] = \frac{1}{z} \left[ 1 + 2 \left( \frac{1}{z} \right) + 3 \left( \frac{1}{z} \right)^2 + \dots \right]$$

$$= \frac{1}{z} \left[ 1 - \frac{1}{z} \right]^{-2} = \frac{1}{z} \left[ \frac{z-1}{z} \right]^{-2} = \frac{1}{z} \cdot \frac{z^2}{(z-1)^2} = \frac{z}{(z-1)^2}$$

$$Z\{n\} = \frac{z}{(z-1)^2}.$$

### 5) Find $Z\left\{\frac{1}{n}\right\}$ . [N/D13]

$$\text{Solution: } Z\left\{\frac{1}{n}\right\} = \sum_{n=0}^{\infty} \frac{1}{n} z^{-n} = \sum_{n=1}^{\infty} \frac{1}{n z^n}$$



$$= \frac{1}{z} + \frac{1}{2z^2} + \frac{1}{3z^3} + \dots$$

$$= \frac{\left(\frac{1}{z}\right)}{1} + \frac{\left(\frac{1}{z}\right)^2}{2} + \frac{\left(\frac{1}{z}\right)^3}{3} + \dots$$

$$= -\log\left(1 - \frac{1}{z}\right) = -\log\left(\frac{z-1}{z}\right) = \log\left(\frac{z-1}{z}\right)^{-1} = \log\left(\frac{z}{z-1}\right)$$

$$Z\left\{\frac{1}{n}\right\} = \log\left(\frac{z}{z-1}\right).$$

6) Find  $Z\left\{\frac{a^n}{n!}\right\}$  in Z-transform. [N/D 09, M/J 12].

Solution:  $Z\left\{\frac{a^n}{n!}\right\} = \sum_{n=0}^{\infty} \frac{a^n}{n!} z^{-n} = \sum_{n=0}^{\infty} \frac{a^n}{n! z^n} = \sum_{n=0}^{\infty} \frac{\left(\frac{a}{z}\right)^n}{n!}$

$$= 1 + \frac{\left(\frac{a}{z}\right)}{1!} + \frac{\left(\frac{a}{z}\right)^2}{2!} + \frac{\left(\frac{a}{z}\right)^3}{3!} + \dots = e^{\frac{a}{z}}.$$

7) Find the Z-transform of  $\frac{1}{n!}$ . [N/D11]

Solution:  $Z\left\{\frac{1}{n!}\right\} = \sum_{n=0}^{\infty} \frac{1}{n!} z^{-n} = \sum_{n=0}^{\infty} \frac{1}{n! z^n} = \sum_{n=0}^{\infty} \frac{\left(\frac{1}{z}\right)^n}{n!}$

$$= 1 + \frac{\left(\frac{1}{z}\right)}{1!} + \frac{\left(\frac{1}{z}\right)^2}{2!} + \frac{\left(\frac{1}{z}\right)^3}{3!} + \dots = e^{\frac{1}{z}}.$$

8) Find the Z-transform of  $3^n$ .

Solution:  $Z \{3^n\} = \sum_{n=0}^{\infty} 3^n z^{-n} = \sum_{n=0}^{\infty} \frac{3^n}{z^n} = \sum_{n=0}^{\infty} \left(\frac{3}{z}\right)^n$

$$= 1 + \frac{3}{z} + \left(\frac{3}{z}\right)^2 + \left(\frac{3}{z}\right)^3 + \dots$$

$$= \left(1 - \frac{3}{z}\right)^{-1} = \left(\frac{z-3}{z}\right)^{-1} = \frac{z}{z-3}$$

**9) Find the Z-transform of  $a^n$  . [A/M11,N/D12].**

Solution:  $Z \{a^n\} = \sum_{n=0}^{\infty} a^n z^{-n} = \sum_{n=0}^{\infty} \frac{a^n}{z^n} = \sum_{n=0}^{\infty} \left(\frac{a}{z}\right)^n$

$$= 1 + \frac{a}{z} + \left(\frac{a}{z}\right)^2 + \left(\frac{a}{z}\right)^3 + \dots$$

$$= \left(1 - \frac{a}{z}\right)^{-1} = \left(\frac{z-a}{z}\right)^{-1} = \frac{z}{z-a}$$

**10) Find the Z-transform of  $\sin \frac{n\pi}{2}$  . [A/M10].**

Solution:  $Z \left\{ \sin \frac{n\pi}{2} \right\} = \sum_{n=0}^{\infty} \sin \frac{n\pi}{2} z^{-n} = \sum_{n=0}^{\infty} \frac{\sin \frac{n\pi}{2}}{z^n}$

$$= \frac{0}{1} + \frac{\sin \frac{\pi}{2}}{z} + \frac{\sin \pi}{z^2} + \frac{\sin \frac{3\pi}{2}}{z^3} + \frac{\sin 2\pi}{z^4} + \dots$$

$$= \frac{1}{z} - \frac{1}{z^3} + \frac{1}{z^5} - \dots$$

$$= \frac{1}{z} \left[ 1 - \frac{1}{z^2} + \frac{1}{z^4} - \dots \right] = \frac{1}{z} \left[ 1 - \frac{1}{z^2} + \left(\frac{1}{z^2}\right)^2 - \dots \right]$$

$$= \frac{1}{z} \left[ 1 + \frac{1}{z^2} \right]^{-1} = \frac{1}{z} \left[ \frac{z^2 + 1}{z^2} \right]^{-1} = \frac{1}{z} \cdot \frac{z^2}{z^2 + 1} = \frac{z}{z^2 + 1}$$

**11) Find the value of  $Z[f(n)]$  where  $f(n) = n a^n$ .**

Solution:

$$\begin{aligned} Z[f(n)] &= Z\{n a^n\} = \sum_{n=0}^{\infty} n a^n z^{-n} = \sum_{n=0}^{\infty} \frac{n a^n}{z^n} = \sum_{n=0}^{\infty} n \left(\frac{a}{z}\right)^n \\ &= (0) \left(\frac{a}{z}\right)^0 + (1) \left(\frac{a}{z}\right)^1 + 2 \left(\frac{a}{z}\right)^2 + 3 \left(\frac{a}{z}\right)^3 + \dots \\ &= \frac{a}{z} \left[ 1 + 2 \left(\frac{a}{z}\right) + 3 \left(\frac{a}{z}\right)^2 + \dots \right] \end{aligned}$$

$$= \frac{a}{z} \left[ 1 - \frac{a}{z} \right]^{-2} = \frac{a}{z} \left[ \frac{z-a}{z} \right]^{-2} = \frac{a}{z} \cdot \frac{z^2}{(z-a)^2} = \frac{a z}{(z-a)^2}$$

**12) Find the Z-transform of  $(n+2)$ .**

Solution:  $Z[n+2] = Z[n] + 2Z[1]$

$$= \frac{z}{(z-1)^2} + \frac{2z}{z-1}$$

**13) Find the Z-transform of  $(n+1)(n+2)$ .**

Solution:  $Z[(n+1)(n+2)] = Z[n^2 + 3n + 2]$

$$\begin{aligned} &= Z[n^2] + 3Z[n] + 2Z[1] \\ &= \frac{z(z+1)}{(z-1)^3} + \frac{3z}{(z-1)^2} + \frac{2z}{z-1} \end{aligned}$$

**14) Express  $Z\{f(n+1)\}$  in terms of  $F(z)$ .**

Solution:

$$Z\{f(n+1)\} = \sum_{n=0}^{\infty} f(n+1)z^{-n}$$

$$n+1 = k \Rightarrow n = k-1$$

$$= \sum_{k=1}^{\infty} f(k)z^{-(k-1)} \quad n=0, k=1 \quad n=\infty, k=\infty$$

$$= \sum_{k=1}^{\infty} f(k)z^{-k} z = z \sum_{k=1}^{\infty} f(k)z^{-k}$$

$$= z \left[ \sum_{k=0}^{\infty} f(k)z^{-k} - f(0) \right] = z[F(z) - f(0)]$$

$$Z\{f(n+1)\} = z[F(z) - f(0)].$$

**15) Find  $Z[e^{-iat}]$  using Z-transform.**

Solution:  $Z[e^{-iat}] = Z[e^{-iat} \cdot 1] = [Z(1)]_{z \rightarrow ze^{iat}}$

$$= \left[ \frac{z}{z-1} \right]_{z \rightarrow ze^{iat}} = \frac{ze^{iat}}{ze^{iat}-1}.$$

**16) If  $F(z) = \frac{z^2}{\left(z - \frac{1}{2}\right)\left(z - \frac{1}{4}\right)\left(z - \frac{3}{4}\right)}$ , find  $f(0)$ . [N/D09].**

Solution: Given  $F(z) = \frac{z^2}{\left(z - \frac{1}{2}\right)\left(z - \frac{1}{4}\right)\left(z - \frac{3}{4}\right)}$

By initial value theorem, we have

$$f(0) = \lim_{z \rightarrow \infty} F(z) = \lim_{z \rightarrow \infty} \frac{z^2}{\left(z - \frac{1}{2}\right)\left(z - \frac{1}{4}\right)\left(z - \frac{3}{4}\right)}$$

$$\begin{aligned}
&= \lim_{z \rightarrow \infty} \frac{z^2}{z \left(1 - \frac{1}{2z}\right) \cdot z \left(1 - \frac{1}{4z}\right) \left(z - \frac{3}{4}\right)} \\
&= \lim_{z \rightarrow \infty} \frac{1}{\left(1 - \frac{1}{2z}\right) \left(1 - \frac{1}{4z}\right) \left(z - \frac{3}{4}\right)} = \frac{1}{\infty} = 0.
\end{aligned}$$

**17) Obtain**  $Z^{-1} \left[ \frac{z}{(z+1)(z+2)} \right]$ . [M/J13]

Solution: Let  $X(z) = \frac{z}{(z+1)(z+2)}$

$$\frac{X(z)}{z} = \frac{1}{(z+1)(z+2)}$$

$$\frac{1}{(z+1)(z+2)} = \frac{A}{z+1} + \frac{B}{z+2}$$

$$1 = A(z+2) + B(z+1)$$

Put  $z = -1$

$$1 = A(-1+2) + B(0) \Rightarrow A = 1$$

Put  $z = -2$

$$1 = A(0) + B(-2+1) \Rightarrow B = -1$$

Therefore,  $\frac{1}{(z+1)(z+2)} = \frac{1}{z+1} - \frac{1}{z+2}$

$$\frac{X(z)}{z} = \frac{1}{z+1} - \frac{1}{z+2}$$

$$X(z) = \frac{z}{z+1} - \frac{z}{z+2}$$

$$Z\{x(n)\} = \frac{z}{z+1} - \frac{z}{z+2}$$

$$x(n) = Z^{-1} \left( \frac{z}{z+1} \right) - Z^{-1} \left( \frac{z}{z+2} \right)$$

$$x(n) = (-1)^n - (-2)^n, \quad n = 0, 1, 2, 3, \dots$$

**18) Find the inverse Z-transform of  $\frac{z}{(z+1)^2}$ . [N/D13]**

Solution: Let  $X(z) = \frac{z}{(z+1)^2}$

$$X(z)z^{n-1} = \frac{z \cdot z^{n-1}}{(z+1)^2} = \frac{z^n}{(z+1)^2}$$

$z = -1$  is a pole of order 2

$$x(n) = \sum R \text{ where } \sum R \text{ is the sum of residue of } X(z)z^{n-1}$$

$$\begin{aligned} \text{Res}[X(z)z^{n-1}] &= \lim_{z \rightarrow -1} \frac{1}{(2-1)!} \frac{d}{dz} \left[ (z+1)^2 \frac{z^n}{(z+1)^2} \right] = \lim_{z \rightarrow -1} \frac{d}{dz} [z^n] = \lim_{z \rightarrow -1} n z^{n-1} \\ &= n(-1)^{n-1} = -n(-1)^n. \end{aligned}$$

**19) State convolution theorem of Z-transform. [M/J14,A/M15]**

Solution: If  $w(n)$  is the convolution of two sequences  $x(n)$  and  $y(n)$ , then

$$Z[w(n)] = W(z) = Z[x(n)] \cdot Z[y(n)].$$

**20) Form a difference equation by eliminating arbitrary constant from  $u_n = A2^{n+1}$ . [N/D11].**

Solution: Given  $u_n = A2^{n+1}$

$$u_n = A2^n \cdot 2$$

$$u_n = 2A2^n \dots\dots\dots(1)$$

$$u_{n+1} = A2^{n+2}$$

$$u_{n+1} = A2^n \cdot 2^2$$

$$u_{n+1} = 4A2^n \dots\dots\dots(2)$$

Eliminating  $A$  from (1) and (2), we have

$$\begin{vmatrix} u_n & 2 \\ u_{n+1} & 4 \end{vmatrix} = 0 \Rightarrow 4u_n - 2u_{n+1} = 0 \Rightarrow u_{n+1} - 2u_n = 0.$$

**21) Form the difference equation generated by  $y_n = a + b2^n$ .**

**[A/M10].**

Solution: Given  $y_n = a + b2^n$  -----(1)

$$y_{n+1} = a + b2^{n+1} = a + b2^n 2$$

$$y_{n+1} = a + 2b2^n \text{ -----(2)}$$

$$y_{n+2} = a + b2^{n+2} = a + b2^n 4$$

$$y_{n+2} = a + 4b2^n \text{ -----(3)}$$

Eliminating  $a$  and  $b$  from (1), (2) and (3), we have

$$\begin{vmatrix} y_n & 1 & 1 \\ y_{n+1} & 1 & 2 \\ y_{n+2} & 1 & 4 \end{vmatrix} = 0$$

$$y_n(4-2) - y_{n+1}(4-1) + y_{n+2}(2-1) = 0$$

$$2y_n - 3y_{n+1} + y_{n+2} = 0$$

$$y_{n+2} - 3y_{n+1} + 2y_n = 0$$

**22) Solve  $y(n+1) - 2y(n) = 0$  given  $y(0) = 2$ . [N/D12]**

Solution: Given  $y(n+1) - 2y(n) = 0$

Taking Z-transform on both sides of the above equation, we have

$$z[y(n+1)] - 2Z[y(n)] = 0$$

$$zY(z) - zy(0) - 2Y(z) = 0$$

$$zY(z) - z(2) - 2Y(z) = 0$$

$$(z-2)Y(z) = 2z$$

$$Y(z) = \frac{2z}{z-2} \quad \text{implies} \quad Z[y(n)] = \frac{2z}{z-2}$$

$$y(n) = 2Z^{-1}\left[\frac{z}{z-2}\right] = 2 \cdot 2^n = 2^{n+1}.$$

**23) Define unit step sequence. Write its Z-transform.**

Solution: The unit step sequence  $u(n)$  is defined as

$$u(n) = \begin{cases} 1 & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}. \quad \text{Its Z-transform is given by}$$

$$Z[u(n)] = \frac{z}{z-1}.$$

**24) Prove that  $Z[a^n f(n)] = \bar{f}\left(\frac{z}{a}\right)$ . [N/D14]**

Solution:

By

definition,

$$\begin{aligned} Z[a^n f(n)] &= \sum_{n=0}^{\infty} a^n f(n) z^{-n} = \sum_{n=0}^{\infty} f(n) \frac{z^{-n}}{a^{-n}} \\ &= \sum_{n=0}^{\infty} f(n) \left(\frac{z}{a}\right)^{-n} = \bar{f}\left(\frac{z}{a}\right). \end{aligned}$$

**25) Find the Z-transform of  $n^2$ . [M/J14]**

Solution:

$$\begin{aligned} Z[n^2] &= Z[n \cdot n] = -z \frac{d}{dz} Z[n] = -z \frac{d}{dz} \left[ \frac{z}{(z-1)^2} \right] \\ &= -z \left[ \frac{(z-1)^2(1) - z \cdot 2(z-1)}{(z-1)^4} \right] = -z \left[ \frac{(z-1)(z-1-2z)}{(z-1)^4} \right] \\ &= -z \left[ \frac{-z-1}{(z-1)^3} \right] = \frac{z(z+1)}{(z-1)^3}. \end{aligned}$$

**PART-B**



1) State and prove the second shifting theorem in Z-transform. [M/J13]

2) State and prove final value theorem in Z-transform. [N/D14]

3) If  $Z[f(n)] = F(z)$ , find  $Z[f(n-k)]$  and  $Z[f(n+k)]$ . [N/D11]

4) Find  $Z(n(n-1)(n-2))$ . [M/J12].

5) Prove that  $Z\left[\frac{1}{n+1}\right] = z \log\left[\frac{z}{z-1}\right]$ .

6) Find the Z-transform of  $\frac{1}{n(n-1)}$ .

7) Find the Z-transform of  $f(n) = \frac{2n+3}{(n+1)(n+2)}$ .

8) Find the Z-transform of  $\{a^n\}$  and  $\{na^n\}$ .

9) Find the Z-transforms of  $\sin^2\left(\frac{n\pi}{4}\right)$  and

$\cos\left(\frac{n\pi}{2} + \frac{n\pi}{4}\right)$ . [N/D 12]

10) Find  $Z(n a^n \sin n\theta)$ . [A/M11].

11) Find the  $Z$  - transform of  $\cos n\theta$  and  $\sin n\theta$  . Hence deduce the  $Z$  -transforms of  $\cos (n+1)\theta$  and  $a^n \sin n\theta$  . [N/D10].

12) Find  $Z^{-1}\left[\frac{z^3}{(z-1)^2(z-2)}\right]$  using partial fraction.

13) Find  $Z^{-1}\left[\frac{z(z^2-z+2)}{(z+1)(z-1)^2}\right]$  by using the method of partial fraction.

14) Find the inverse Z-transform of  $\frac{z^3-20z}{(z-2)^3(z-4)}$  . [N/D09].

15) Find the inverse Z-transform of  $\frac{z(z+1)}{(z-1)^3}$  by residue method. [N/D10].

16) Find the inverse Z-transform of  $\frac{z^3+3z}{(z-1)^2(z^2+1)}$  .

17) Find  $Z^{-1}\left[\frac{z(z^2-z+2)}{(z+1)(z-1)^2}\right]$  and

$Z^{-1}\left[\frac{z}{(z-1)(z-2)}\right]$  .[A/M10].

18) Find  $Z^{-1}\left[\frac{z^3}{(z-2)^2(z-3)}\right]$  .

19) Find the inverse Z-transform of  $\frac{z^2+z}{(z-1)(z^2+1)}$  , using partial fraction. [N/D14]

20) If  $U(z) = \frac{2z^2+5z+14}{(z-1)^4}$  , evaluate  $u_2$  and  $u_3$  .

21) State and prove convolution theorem on Z-transformation.

Find  $Z^{-1} \left[ \frac{z^2}{(z-a)(z-b)} \right]$ . [M/J13]

22) Find the inverse Z-transform of  $\frac{8z^2}{(2z-1)(4z-1)}$  by using convolution theorem. [M/J12,N/D14]

23) Using convolution theorem, find the inverse Z-transform of  $\left( \frac{z}{z-4} \right)^3$ . [A/M10].

24) Using Z-transform method solve  $y_{n+2} + y_n = 2$  given that  $y_0 = y_1 = 0$ .

25) Using Z-transform solve difference equation  $y(n+2) - 4y(n+1) + 4y(n) = 0$  given that  $y(0) = 1$ ,  $y(1) = 0$ .

26) Solve the difference equation  $y(n+3) - 3y(n+1) + 2y(n) = 0$  given that  $y(0) = 4$ ,  $y(1) = 0$  and  $y(2) = 8$ , by the method of Z-transform. [A/M11,N/D12,N/D14].

27) Solve  $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$  given  $y_0 = y_1 = 0$ , using Z-transform. [N/D 09 ,N/D12].

28) Using Z-transform, solve  $y_{n+2} + 4y_{n+1} - 5y_n = 24n - 8$  given that  $y_0 = 3$  and  $y_1 = -5$ .

29) Solve  $u_{n+2} - 3u_{n+1} + 2u_n = 4^n$ , given that

$$u_0 = 0, u_1 = 1. \text{ [M/J14]}$$

30)

31) Using Z-transforms solve  $u_{n+2} - 3u_{n+1} + 2u_n = 0$

given that  $u_0 = 0, u_1 = 1$ . [N/D14]

# **ME8391 ENGINEERING THERMODYNAMICS**

## **OBJECTIVE:**

- To familiarize the students to understand the fundamentals of thermodynamics and to perform thermal analysis on their behavior and performance.
- (Use of Standard and approved Steam Table, Mollier Chart, Compressibility Chart and Psychrometric Chart permitted)

**UNIT 1      BASIC CONCEPTS AND FIRST LAW****9 + 6**

Basic concepts - concept of continuum, comparison of microscopic and macroscopic approach. Path and point functions. Intensive and extensive, total and specific quantities. System and their types. Thermodynamic Equilibrium State, path and process. Quasi-static, reversible and irreversible processes. Heat and work transfer, definition and comparison, sign convention. Displacement work and other modes of work .P-V diagram. Zeroth law of thermodynamics – concept of temperature and thermal equilibrium– relationship between temperature scales –new temperature scales. First law of thermodynamics –application to closed and open systems – steady and unsteady flow processes.

**UNIT II      SECOND LAW AND AVAILABILITY ANALYSIS    9 + 6**

Heat Reservoir, source and sink. Heat Engine, Refrigerator, Heat pump. Statements of second law and its corollaries. Carnot cycle Reversed Carnot cycle, Performance. Clausius inequality. Concept of entropy, T-s diagram, Tds Equations, entropy change for - pure substance, ideal gases - different processes, principle of increase in entropy. Applications of II Law. High and low grade energy. Available and non-available energy of a source and finite body. Energy and irreversibility. Expressions for the energy of a closed system and open systems. Energy balance and entropy generation. Irreversibility. I and II law Efficiency.

**UNIT III      PROPERTIES OF PURE SUBSTANCE AND STEAM  
POWER CYCLE****9+6**

Formation of steam and its thermodynamic properties, p-v, p-T, T-v, T-s, h-s diagrams. p-v-T surface. Use of Steam Table and Mollier Chart. Determination of dryness fraction. Application of I and II law for pure substances. Ideal and actual Rankine cycles, Cycle Improvement Methods - Reheat and Regenerative cycles, Economiser, preheater, Binary and Combined cycles.

**UNIT IV      IDEAL AND REAL GASES, THERMODYNAMIC  
RELATIONS****9+6**

Properties of Ideal gas- Ideal and real gas comparison- Equations of state for ideal and real gases- Reduced properties. Compressibility factor-.Principle of Corresponding states. -Generalised Compressibility Chart and its use-. Maxwell relations, Tds Equations, Difference and ratio of heat capacities, Energy equation, Joule-

Thomson Coefficient, Clausius Clapeyron equation, Phase Change Processes. Simple Calculations.

## **UNIT V GAS MIXTURES AND PSYCHROMETRY**

**9+6**

Mole and Mass fraction, Dalton's and Amagat's Law. Properties of gas mixture – Molar mass, gas constant, density, change in internal energy, enthalpy, entropy and Gibbs function. Psychrometric properties, Psychrometric charts. Property calculations of air vapour mixtures by using chart and expressions. Psychrometric process – adiabatic saturation, sensible heating and cooling, humidification, dehumidification, evaporative cooling and adiabatic mixing. Simple Applications

**TOTAL : 75 PERIODS**

### **COURSE OUTCOMES:**

**Upon the completion of this course, the students will be able to**

<b>CO 1</b>	Apply the first law of thermodynamics for simple open and closed systems under steady and unsteady conditions.
<b>CO 2</b>	Apply second law of thermodynamics to open and closed systems and calculate entropy and availability.
<b>CO 3</b>	Apply Rankine cycle to steam power plant and compare few cycle improvement methods
<b>CO 4</b>	Derive simple thermodynamic relations of ideal and real gases
<b>CO 5</b>	Calculate the properties of gas mixtures and moist air and its use in psychrometric processes.

## **UNIT-I BASIC CONCEPTS AND FIRST LAW**

### **PART A**

**1. What is the difference between the classical and the statistical approaches in thermodynamics? / What is microscopic approach in thermodynamics? [AU Dec 2013] [AU Nov 2009]**

In classical / macroscopic approach, the events occurring at molecular level are not taken into account in arriving at the behavior of the system. In statistical/ microscopic approach, the behavior of a system is arrived at from the events at molecular level.

**2. What is meant by Continuum? Identify its importance? [AU Nov 2009]**

In thermodynamics, material in a system is considered to be continuum that is, it is continuously distributed throughout the system. This assumption allows us to describe a system using only a few measurable properties, thereby making the analysis easier.

**3. Distinguish between the terms “state and process” of thermodynamics. [AU Dec2011, May 2012]**

State is a condition of the system when it has definite values for all its properties. When one or more properties change in an operation, it is said to undergo a process.

**4. What is a quasi-static process? Give an example.[AU Nov 2010],[AU Dec 2012]**

A process in which the system departs from equilibrium state only by a very small extent is quasi-equilibrium process. Slow compression and slow expansions of a system of gases are quasi-equilibrium processes.

**5. What is the requirement for thermal equilibrium? Which law governs it? [AU Nov 2009]**

If two systems which are in mechanical, chemical and phase equilibrium do not exchange heat with each other, they will be under thermal equilibrium. Zeroth law governs thermal equilibrium.

**6. State Zeroth law of thermodynamics. Why is it so called?[AU Nov 2009] [AU Apr 2015]**

If a body A is in thermal equilibrium with a body B, and also separately with a body C, then B and C are in thermal equilibrium with each other. It is called so, as it is more basic compared to First law, which was established prior to it.



**7. List any five physical properties of matter which can be used for measurement of temperature. [AU Apr 2015]**

Pressure (in constant volume gas thermometer), Volume (in constant pressure gas thermometer), Length (in mercury-in-glass thermometer), emf (in thermocouple) and resistance (in electric resistance thermometer).

**8. Define: Thermodynamic equilibrium? [AU Nov 2014] [AU May 2014]**

It is a state of balance; a system is said to be thermodynamic equilibrium if the conditions for thermal, mechanical and chemical equilibrium are satisfied.

**9. What are point and path function? Give some examples. [AU Nov 2010] [AU May 2014]**

**Path function:** The function whose value is dependent on the path of the process. e.g. work transfer and heat transfer .

**Point function:** The function the change in whose value is independent on the path of the process.e.g. pressure, temperature, etc.

**10. Classify the following as point or path function: Heat, Enthalpy, Displacement work, Entropy. [AU Nov 2009]**

Heat and Displacement work – Path functions, Enthalpy and Entropy – point functions

**11. What do you understand by flow work? Is it different from displacement work? [AU May 2009,2010]**

Work transfer process involving open systems or control volumes where flow of mass across the boundary occurs is known as flow work. e.g. Compression of air in an air-compressor. A work transfer in which one or more boundaries of the system is moved is called displacement work or moving boundary work. It is also called  $pdV$  work. e.g. compression of a gas in a piston-cylinder arrangement.

**12. Define extensive property. [AU Dec 2013]**

The property which depends on mass or size of the system is an extensive property.

**13. Distinguish between “flow process and non-flow process”. [AU Dec 2012]**

Process undergone by a closed system is a non-flow process and processes undergone by open systems are flow process.

**14. State the first law of thermodynamics for a non-flow process and for a cycle. [AU May 2012]**

State First Law of Thermodynamics for a closed system undergoing  
(a) a cycle b) a process

(a)  $Q = W$  (b)  $Q - W = E$

**Note:** non-flow process is a process undergone by a closed system.

**15. Show that energy of an isolated system is always constant. [AU Dec 2011]**

For an isolated system,  $Q = 0$  and  $W = 0$ , Therefore according to first law,  $E = 0$  and hence energy is constant.

**16. What is the difference between adiabatic and isentropic processes? [AU May 2013]**

Adiabatic process is a process without heat transfer. Isentropic process is a process with no change in entropy. A reversible adiabatic process is isentropic.

**17. What is meant by “Hyperbolic” process? [AU May 2011]**

A process in which  $pV = \text{constant}$  is known as hyperbolic process.

**18. Distinguish between stored energies and interaction energies. [AU Nov 2010] Distinguish between “Macroscopic Energy and Microscopic Energy”. [AU Dec 2012]**

Stored energies / Microscopic energies are energy stored in the system such as Kinetic Energy, Potential Energy and internal energy ( $U$ ). Interaction energies / Macroscopic are energy in transit such as heat and work.

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**19. Which property of a system increases when heat is transferred: [a] At constant volume, [b] At constant pressure [AU May 2010]**

[a] Pressure and Temperature increases at constant volume

[b] Volume and Temperature increases at constant pressure

**20. Define flow energy. [AU May 2013]**

Energy required to introduce a quantity of fluid in a pipe section is flow energy. It is equal to  $p \cdot v$ .

**21. What is a steady flow process?**

Steady flow means that the rates of flow of mass and energy across the control surface are constant.

**22. Define Zeroth law of Thermodynamics. [Nov/Dec 2009]**

When a body A is in thermal equilibrium with body B and also separately with a body C, then Body B and C will be in thermal equilibrium with each other.

**23. Indicate the practical application of steady flow energy equation.**

1. Turbine 2. Nozzle 3. Condenser, 4. Compressor.

**24. Define state. [Nov/Dec 2011, April /May 2012]**

The condition of the system at particular time.

**25. Define the term process and path. [Nov/Dec 2011, April /May 2012]**

**Process:** Any change that a system undergoes from one equilibrium state to another is called a process.

**Path:** Series of states through which a system passes during a process is called the path.

**PART-B**

**PROBLEMS ON CLOSED SYSTEM (NON FLOW PROCESS)**

1) 5 kg of a gas was heated from a temperature of  $100^{\circ}\text{C}$  at constant volume till its pressure becomes three times its original pressure. For this process calculate (i) Heat transferred (ii) Change in internal energy and (iii) Change in enthalpy. Assume  $C_p = 1 \text{ kJ/kgK}$  and  $C_v = 0.71 \text{ kJ/kgK}$ . **(13)**

2) A gas whose pressure, volume and temperature are 5 bar,  $0.23 \text{ m}^3$  and  $185^{\circ}\text{C}$  respectively, has its state change at constant pressure until its temperature becomes  $70^{\circ}\text{C}$ . Determine (i) Work done (ii) Change in Internal energy (iii) Heat transferred during the process. Take  $R = 0.29 \text{ kJ/kgK}$  and  $C_p = 1.005 \text{ kJ/kgK}$ . **(13)**

3) 1 kg of air at 11 bar and  $80^{\circ}\text{C}$  is expanded to 10 times the original volume by (i) Isothermal process (ii) Isentropic (reversible adiabatic) process. Determine the work done in each of the cases. Plot these on a common PV diagram. Take  $R = 0.287 \text{ kJ/kgK}$  and  $\gamma = 1.4$ . **(13)**

4) A quantity of gas occupies a volume of  $0.28 \text{ m}^3$  at a pressure of  $120 \text{ kN/m}^2$  and temperature of  $250^\circ\text{C}$ . The gas is compressed isothermally to a pressure of  $600 \text{ kN/m}^2$  and then expanded adiabatically to its initial volume. Determine (i) heat received or rejected during compression (ii) mass of the gas and (iii) Change in internal energy during expansion. Assume  $\gamma = 1.4$  and  $C_p = 1 \text{ kJ/kgK}$ . **(13)**

5) A stationary fluid system goes through a cycle of 3 process. Initially isochoric heat addition of  $235 \text{ kJ/kg}$  is added to the system and then due to adiabatic expansion the system is brought back to the initial pressure with loss of  $70 \text{ kJ/kg}$  in internal energy and then due to isobaric compression, the system is brought back to the initial volume with heat rejection of  $200 \text{ kJ/kg}$ . For this cycle, prepare a balance sheet on  $Q$ ,  $W$ , change in  $U$ . **(13)**

6) When a system is taken from state  $l$  to state  $m$  as shown in figure along path  $lqm$ .  $168 \text{ kJ}$  of heat flows into the system and the system does  $64 \text{ kJ}$  of work. (i) how much will be the heat that flows into the system along the path  $lnm$ , if work done is  $21 \text{ kJ}$ . (ii) When the system is returned from  $m$  to  $l$  along the curved path, the work done on the system is  $42 \text{ kJ}$ . Does the system absorb or liberate heat and how much of heat is absorbed or liberated? (iii) If  $U_l = 0$  and  $U_m = 84 \text{ kJ}$ . Find the heat absorbed in the process  $ln$  and  $lm$ . **(13)**

7) A gas undergoes a thermodynamic cycle consisting of the following processes.

Process 1-2: Constant pressure  $p = 1.4 \text{ bar}$ ,  $V_1 = 0.028 \text{ m}^3$ ,  $W_{12} = 10.5 \text{ kJ}$

Process 2-3: Compression with  $pV = \text{constant}$ ,  $U_3 = U_2$

Process 3-1: Constant volume,  $U_1 - U_3 = -26.4 \text{ kJ}$ .

There are no significant changes in KE and PE.

- i (i) Sketch the cycle on a  $p$ - $V$  diagram
- ii (ii) Calculate the net Work in the cycle in  $\text{kJ}$ .
- iii (iii) Calculate the heat transfer for the process 1-2
- iv (iv) Show that  $\Sigma Q = \Sigma W_{\text{cycle}}$

### **PROBLEMS ON OPEN SYSTEM (FLOW PROCESS)**

8) Air at a temperature of  $15^\circ\text{C}$  passes through a heat exchanger at a velocity of  $30 \text{ m/s}$  where its temperature is raised to  $800^\circ\text{C}$ . It then enters to a turbine with the same velocity of  $30 \text{ m/s}$  and expands until the temperature falls to  $650^\circ\text{C}$ . On leaving the turbine the air is taken at a velocity of  $60 \text{ m/s}$  to a nozzle where it expands until the temperature fallen to  $500^\circ\text{C}$ . If the air flow rate is  $2 \text{ kg/s}$ , calculate (i) the rate of heat transfer to the air in the heat exchanger. (ii) The power output from the turbine assuming no heat loss. (iii) The velocity at exit from the nozzle assuming no heat loss. Take the

enthalpy of air as  $h = c_p t$ , where  $C_p$  is the specific heat = 1.005 kJ/kgK and  $t$ , temperature in °C. (13)

9) In a steady flow apparatus, 135 kJ of work is done by each kg of fluid. The specific volume of the fluid, pressure, and velocity at the inlet are 0.37 m<sup>3</sup>/kg, 600 kPa, and 16 m/s. the inlet is 32 m above the floor, and the discharge pipe is at the floor level. The discharge conditions are 0.62 m<sup>3</sup>/kg, 100 kPa, and 270 m/s. the total heat loss between the inlet and discharge is 9 kJ/kg of fluid. In flowing through this apparatus, does the specific internal energy increase or decrease, and by how much? (13)

10) In an isentropic flow through a nozzle, air flows at the rate of 600 kg/hr. At inlet to the nozzle, pressure is 2 MPa and temperature is 127°C. The exit pressure is 0.5 MPa. Initial air velocity is 300 m/s. Determine (i) Exit velocity of air and (ii) Inlet and exit area of nozzle. (13)

11) In a turbo machine handling an incompressible fluid with a density of 1000 kg/m<sup>3</sup> the conditions of the fluid at the rotor entry and exit are given below:

	Inlet	Exit
Pressure	1.15 MPa	0.05 MPa
Velocity	30 m/s	15.5 m/s
Height above datum	10 m	2 m

If the volume flow rate of the fluid is 40 m<sup>3</sup>/s, estimate the net energy transfer from the fluid as work.

### PART-C

1. 25 People attended a farewell party in room of size 10 x 8m and has a 5m ceiling. Each person gives up about 350 kJ of heat / hour. Assuming that the room is completely sealed off and insulated, calculate the air temperature rise occurring within 10 minutes. Assume  $C_p$ ,  $C_v$  and  $R$  and each person occupies a volume of 0.05m<sup>3</sup>. (15)

2. Air flows steadily at the rate of 0.5 kg/sec through an air compressor, entering at 7m/sec velocity, 100 kPa pressure and 0.95 m<sup>3</sup>/kg, volume leaving at 5m/sec, 700kPa and 0.19 m<sup>3</sup>/kg. The internal energy of the rate leaving is 90kJ/kg greater than that of the air entering. Cooling water in compressor jackets absorbs heat from the air at the rate of 58kW (a) Compute the rate of shaft work input to air in kW. (b) Find the ratio of the inlet pipe diameter to the outlet pipe diameter. (15)

3. A room for 4 people has 2 fans each consuming 0.18kW power, and three 100W lamps. Ventilation air at the rate of 80kg/hr enters with an enthalpy of 84 kJ/kg and leaves with an enthalpy of 59kJ/kg. If each person puts out heat at the rate of 630kJ/hr. determine the

rate at which heat is removed by a room cooler, so that a steady state is maintained in the room. (15)

## **UNIT II**

### **SECOND LAW AND AVAILABILITY**

#### **PART - A**

**1. Define Kelvin Planck Statement. [AU May 2014] [AU May 2013] [AU Nov 2009]**

Kelvin- Planck states that it is impossible to construct a heat engine working on cyclic process, whose only purpose is to convert all the heat energy given to it in an equal amount of work

**2. Define Clausius statement [AU Dec 2013]**

It states that heat can flow from hot body to cold body without any external aid but heat cannot flow from cold body to hot body without any external aid.

**3. List the limitations of First law of Thermodynamics. [AU Dec 2012]**

First law does not indicate whether a process is possible or not. It does not give any information regarding the extent of conversion of heat into work.

**4. List the causes of Irreversibility [AU Nov 2010] [AU May 2011]**

Lack of equilibrium during the process (ex: Heat transfer through a finite temperature difference) Involvement of dissipative effects. (ex: Free Expansion)

**5. What is meant by dead state? [AU May 2013]**

The state at which the system will be incapable of delivering a work output.

**6. Isentropic process need not be necessarily an adiabatic process – justify [AU May 2010]**

Entropy increases due to irreversibility and decreases due to heat loss. If these two changes are equal in magnitude, the process would be isentropic without being adiabatic.

**7. Why the second law is called directional law of nature? [AU Nov 2009]**

It specifies the direction in which heat transfer takes place in a process.

**8. What is a cyclic heat engine? [AU Dec 2011]**

A cyclic heat engine is a thermodynamic system working on a cycle having net heat input and net work output.

**9. What is the difference between a heat pump and a refrigerator? [AU May 2012]**

**Heat pump** is a device which operating in cyclic process, maintains the temperature of a hot body at a temperature higher than the temperature of surroundings.

**A refrigerator** is a device which operating in a cyclic process, maintains the temperature of a cold body at a temperature lower than the temperature of the surroundings.

**10. State Carnot theorem. [AU May 2014] [AU Nov 2010]**

No heat engine operating in a cyclic process between two fixed temperature, can be more efficient than a reversible engine operating between the same temperature limits.

**11. What are the Corollaries of Carnot theorem [AU May 2014] [AU May 2011]**

- In all the reversible engine operating between the two given thermal reservoirs with fixed temperature have the same efficiency.
- The efficiency of any reversible heat engine operating between two reservoirs is independent of the nature of the working fluid and depends only on the temperature of the reservoirs.

**12. A heat engine is supplied with 2512 kJ/min of heat at 650°C. Heat rejection takes place at 100°C. Specify which of the following heat rejection represents a reversible, irreversible or impossible result. (a) 867 kJ/min (b) 1015 kJ/min [AU April 2015]**

$\text{max} = \text{rev} = 1 - 373/923 = 54.68\%$

(a)  $1 - Q_2/Q_1 = 1 - 867/2512 = 65.48\%$  max, it is impossible

(b)  $1 - Q_2/Q_1 = 1 - 1015/2512 = 59.59\%$  max, it is impossible

**13. Draw a schematic of a heat pump. [AU Dec 2013]**

**14. Carnot refrigerator requires 1.25 kW per ton of refrigeration to maintain the temperature of 243K. Find the COP of Carnot refrigerator. [AU Apr 2015]**

$\text{COP} = Q_2/W$ , here  $Q_2 = 1$  Ton of Refrigeration, which is equal to 3.5 kW

Therefore  $\text{COP} = 3.5/1.25 = 2.8$

**15. Ice is formed at 0°C from water at 20°C. The temperature of the brine is -10°C. Find the ice formed per kW hour. Assume that the refrigeration cycle used is perfect reversed Carnot cycle. Latent heat of ice = 80 kcal/kg. [AU Apr 2015]**

The heat removed from one kg of water at 20°C to convert it into ice at 0°C,

$Q = 1 \times 4.186 \times (20-0) + 1 \times 80 \times 4.186 = 418.6 \text{ kJ/kg}$   
Mass of ice formed per kW hour =  $3600 / 418.6 = 8.6 \text{ kg}$ .

**16. Define availability. [AU May 2009]**

The maximum useful work obtained during a process in which the final condition of the system is the same as that of the surrounding is called availability of the system.

**17. Define available energy and unavailable energy. [AU May 2012]**

Available energy is the maximum thermal useful work under ideal condition. The remaining part which cannot be converted into work, is known as unavailable energy.

**18. Why the performance of refrigerator and heat pump are given in terms of C.O.P and not in terms of efficiency? [AU May 2009]**

The performance of any device is expressed in terms of efficiency for work developing machines. But heat pump and refrigerator are work absorbing machines. So, the performance of those devices based on C.O.P. only.

**19. What do you mean by "Clausius inequality"? [AU April 2015]**

It is impossible for a self-acting machine working in a cycle process unaided by any external agency to convey heat from a body at a lower temperature to a body at a higher temperature.

**20. Can entropy of universe ever decrease? Why? [AU Dec 2013]**

Entropy of universe cannot ever decrease. It will remain constant or will increase due to irreversibility.

**21. Why Carnot cycle cannot be realized in practical?**

- (i) In a Carnot cycle all the four processes are reversible but in actual practice there is no process is reversible. (ii) There are two processes to be carried out during compression and expansion. For isothermal process the piston moves very slowly and for adiabatic process the piston moves as fast as possible. This speed variation during the same stroke of the piston is not possible. (iii) It is not possible to avoid friction moving parts completely.

**22. Why a heat engine cannot have 100% efficiency?**

For all the heat engines there will be a heat loss between system and surroundings. Therefore we can't convert all the heat input into useful work.

**23. What are the processes involved in Carnot cycle.**

Carnot cycle consist of

- i) Reversible isothermal compression ii) Isentropic compression



iii) Reversible isothermal expansion iv) Isentropic expansion

**24. List out the general types of irreversibilities.[April / May 2011]**

- (a) Internal irreversibility
- (b) External irreversibility

**25. What do you understand by dissipative effects? When work is said to be dissipated? [April / May 2010]**

A dissipative process is a process in which energy (internal, bulk flow kinetic or system potential) is transformed from some initial form to some final form; the capacity of the final form to do mechanical work is less than that of the initial form. For example, transfer of energy as heat is dissipative because it is a transfer of internal energy from a hotter body to a colder one.

**Part-B**

1) Explain the various processes in the Carnot cycle and derive its efficiency. (13)

2) A heat engine develops 10 kW power, when receiving heat at the rate of 2250 kJ/min. Evaluate the corresponding rate of heat rejection from the engine and its thermal efficiency. (13)

3) A machine operating as a heat pump extracts heat from the surrounding atmosphere, is driven by a 7.5 kW motor and supplies  $2 \times 10^5$  kJ/hr heat to a house needed for its heating in winter. Find the coefficient of performance for the heat pump. How this COP will be affected if the objective of the same machine is to cool the house in summer requiring  $2 \times 10^5$  kJ/hr of heat rejection. Comment on the result. (13)

4) An inventor claims a new engine that will develop 2.5 kW for a heat addition of 300 kJ/min. The highest temperature of the cycle is 1800 K and the lowest temperature is 600K. Examine the feasibility of the engine. (13)

5) A Carnot heat engine which operates between temperature level of 927°C and 330°C. It rejects 30 kJ of heat to the low temperature sink. The heat pump receives 270 kJ of heat from low temperature reservoir and rejects it to the surrounding at 330°C. Determine the temperature in °C of the low temperature reservoir of the heat pump. (13)

6). A Carnot heat engine with efficiency 0.4 drives a refrigerator with (COP)<sub>Ref</sub> = 4. Both the engine and refrigerator rejects energy to the atmosphere. Determine the amount of energy rejected into the

atmosphere by both the devices for each kJ of energy from the cold space by the refrigerator. (13)

7) A heat engine working on a Carnot cycle absorbs heat from three thermal reservoirs at 1000 K, 800 K and 600 K. The engine does 10 kW of net work and rejects 400 kJ/min of heat to a heat sink at 300 K. If the heat supplied by the reservoir at 1000 K is 60% of the heat supplied by the reservoir at 600 K. Make calculations for the quantity of heat absorbed by the engine from each reservoir. (13)

8) A reversible heat engine is supplied with 900 kJ of heat from a heat source at 500 K. The engine develops 300 kJ of net work and rejects heat to two heat sinks at 400 K and 300 K. Determine the engine thermal efficiency and magnitude of heat interaction with each of the sink. (13)

9) 1 kg of air initially at 7 bar pressure and 360 K temperature expands according to the polytropic law,  $pV^{1.2} = \text{constant}$ , until the pressure is reduced to 1/5th pressure. Determine (i) final specific volume and temperature. (ii) change in internal energy, work done and heat interaction, and (iii) change in entropy. Take  $R = 0.287$  kJ/kgK,  $\gamma = 1.4$ . (13)

10) 1 kg of air is contained in a piston cylinder assembly at 10 bar pressure and 500 K temperature. The piston moves outwards and the air expands to 2 bar and 350 K temperature. Determine the maximum work attainable. Assume the environmental conditions to be 1 bar and 290 K. Also make calculations for the availability in the initial and final states. For air,  $R = 0.287$  kJ/kgK,  $C_p = 1.005$  kJ/kgK and  $C_v = 0.718$  kJ/kgK (13)

### PART-C

1. one kg of ice at -10 degrees Celsius is allowed to melt in atmosphere at 30 degrees Celsius. The ice melt and the water so formed rises in temperature to that to the atmosphere. Determine the entropy change of ice, the entropy change of surrounding, the entropy change of universe. the specific heat of ice is 2 kJ/Kg-k and its latent heat is 335 kJ/Kg. (15)

2. (i) Prove that the efficiency of the Carnot cycle is  $[(T_1 - T_2)/T_1]$ , Where  $T_1 > T_2$  and draw the p-v and T-s diagram of Carnot cycle. (ii) Derive the cop of the heat pump. (15)

## **UNIT III**

### **PROPERTIES OF PURE SUBSTANCE AND STEAMPOWER CYCLE**

#### **PART - A**

**1. Define a pure substance. Give examples. [AU Dec 2012] [AU Dec2011][AU Dec2013]**

A pure substance is a substance of constant chemical composition throughout its mass. e.g. H<sub>2</sub>O

**2. What is the triple point of water? Give the values of properties at that point.**

**[AU Nov 2009]**

The state at which all the 3phases of water co-exist in equilibrium is called triple point of water. Temperature = 273.16K and pressure = 4.587 mmHg

**3. What is critical state? Define the term critical pressure, critical temperature, and critical volume of water. [AU May 2010]**

The state at which the transition from liquid to vapour phase suddenly takes place. The pressure, temperature and volume at critical state are known as critical pressure, critical temperature, and critical volume. The corresponding values for water are: 221.2bar, 374.15°C and 0.00317m<sup>3</sup>/kg.

**4. Why is Carnot cycle not a realistic model for steam power plants? [AU Nov 2009, May2010,Dec2011]**

Heat addition in steam power plants does not take place at constant temperature in the boiler when superheated steam is used in turbine. Condensation process cannot be ended in such a way that at the end of pumping saturated liquid state is reached in case of Carnot cycle in two-phase region.

**5. What are the advantages of reheating? [AU Nov 2010] [AU Nov 2014]**

**6. What are the methods of improving the performance of Rankine cycle? [AUMay2011, APRIL2014]**

**7. What is the effect of regeneration of a steam power plant? [AUMay2009]**

Regeneration does not affect work output; however the efficiency of the plant increases as the temperature of heat addition is decreased.

**8. Define dryness fraction of steam or what is quality of steam? [AU Nov 2010]**

It is defined as the amount of the mass of the total steam actually present to the mass of the steam.

Dryness fraction = mass of dry steam / mass of total mixture.

**9. Explain the terms: degree of superheat, degree of sub-cooling. [AU May 2009]**

**Degree of superheat:**

It is the difference between superheated temperature and saturated temperature at the same pressure.

**Degree of sub-cooling:**

It is the amount by which the water is cooled beyond the saturated temperature at the same pressure.

**10. Define triple point and critical point for pure substance. [AU May 2010]**

**Triple point:**

Triple point is the state at where all the three phases i.e. Solid, liquid and vapour to exist in equilibrium.

**Critical point:**

It represents the highest pressure and temperature at which the liquid and vapour phases co-exist in equilibrium. At the critical point the liquid and vapour phases are indistinguishable i.e. Liquid directly converted into vapour.

**11. What is meant by steam power cycle?**

Thermodynamic cycles which use steam as the working fluid is called steam power cycles.

**12. Name the different processes of Rankine cycle on T-s diagram processes:**

Isentropic expansion. Constant pressure and temperature heat rejection.

Water is pumped to boiler pressure. Constant pressure heat addition in boiler upto saturation temperature. Constant pressure and temperature in boiler.

**13. Name the different components in steam power plant working on a Rankine cycle.**

Boiler, turbine, cooling tower of condenser, and pump.

**14. List the advantage of reheat cycle. [AU May 2010]**

1. Marginal increase in thermal efficiency.
2. Increase in workdone per kg of steam which results in reduced size of boiler and auxiliaries for the same output.
3. We can prevent the turbine from erosion.

**15. Draw the phase equilibrium diagram for a pure substance on T-S plot with relevant constant property line. [AU May 2009]**

**16. Draw the phase equilibrium diagram on p-v co ordinates with relevant constant property lines for water. How is Triple point represented in the P-v diagram? [AU DEC2011]** **17. Draw the p-T diagram for a pure substance. [AUMay2014]**

**18. Show Rankine cycle on T-s diagram. [AUMay2011]**

**19. What are the major components in a steam power plant? [AU May 2009]**

**20. List the advantage of reheat cycle. [AU DEC2013]**

1. Marginal increase in thermal efficiency.
2. Increase in work done per kg of steam which results in reduced size of boiler and auxiliaries for the same output.
3. We can prevent the turbine from erosion.

**21. Define dryness fraction (or) What is the quality of steam?**

It is defined as their mass of the dry steam to the mass of the total steam.

**22. Define enthalpy of steam.**

It is the sum of heat added to water from freezing point to saturation temperature and the heat absorbed during evaporation.

**23. How do you determine the state of steam?**

If  $V > v_g$  then super heated steam,  $V = v_g$  then dry steam and  $V < v_g$  then wet steam.

**24. Define triple point.**

The triple point is merely the point of intersection of sublimation and vaporisation curves.

**25. Define heat of vaporisation.**

The amount of heat required to convert the liquid water completely into vapour under this condition is called the heat of vaporisation.

### **Part-B**

1. (a) Draw the p-T diagram of a pure substance and label all the phases and phase changes. (3)
- (b) What do you understand by dryness fraction? What is its importance? (2)
- (c) A rigid tank of 0.03 m<sup>3</sup> capacity contains liquid-vapour mixture at 80 kPa. If the total mass of the mixture is 12 kg, calculate the heat

added and the quality of the mixture when the pressure inside the tank reaches 7 MPa. (8)

2. (a) Briefly describe the process of super-heated steam formation with the help of a T-s diagram. (5)

(b) Find the saturation temperature, the changes in specific volume and entropy during evaporation, and the latent heat of vaporisation of steam at 1 MPa. (3)

(c) A vessel of volume  $0.04 \text{ m}^3$  contains a mixture of saturated water and saturated steam at a temperature of  $250^\circ\text{C}$ . The mass of the liquid present is 9 kg. Find the pressure, the mass, the specific volume, the enthalpy, the entropy, and the internal energy. (5)

3. (a) Explain the factors which affect the thermal efficiency and quality of turbine exhaust of Rankine cycle. Justify your discussion using T-s diagrams. (6)

(b) Sketch the Rankine regenerative cycle with one feedwater heater as schematic and T-s diagrams and deduce the expression for the thermal efficiency. (7)

4. In a Rankine steam power cycle, the steam supply is at 15 bar and dry and saturated. The condenser pressure is 0.4 bar. Calculate the efficiency of the cycle. Neglect pump work. Also, find the Carnot efficiency. (13)

5. A simple Rankine cycle works between pressures 28 bar and 0.06 bar, the initial condition of steam being dry saturated. Calculate the cycle efficiency, work ratio and specific steam consumption. (13)

6. (a) Describe Throttling Process. Steam at 10 bar pressure and 0.5 dry, is throttled to 1 bar. Find the condition of steam after throttling. (8)

(b) Steam initially at 1.5 MPa,  $300^\circ\text{C}$  expands reversibly and adiabatically in a steam turbine to  $40^\circ\text{C}$ . Determine the ideal work output of the turbine per kg of steam. (8)

7. Steam initially at 0.3 MPa,  $250^\circ\text{C}$  is cooled at constant volume. (13)

(a) At what temperature will the steam become saturated vapour?

(b) What is the quality at  $80^\circ\text{C}$ ? Why is the heat transferred per kg of steam in cooling from  $250^\circ\text{C}$  to  $80^\circ\text{C}$ ?

8. (a) Steam at 0.8 MPa,  $250^\circ\text{C}$  and owing at the rate of 1 kg/s passes into a pipe carrying wet steam at 0.8 MPa, 0.95 dry. After adiabatic mixing, the ow rate is 2.3 kg/s. Determine the condition of steam after mixing. Represent the process on a T-S diagram. (8)

(b) With the help of T-s diagram, explain the regenerative Rankine cycle. (5)

9. A steam power plant uses the following cycle: (13)

Steam at boiler outlet 150 bar, 550°C

Reheat at 40 bar to 550°C

Condenser at 0.1 bar

Using the Mollier chart and assuming ideal processes, find (a) quality at turbine exhaust, (b) cycle efficiency, (c) steam rate.

10. A steam power plant operates on a theoretical reheat cycle. Steam at boiler at 32 bar, 410°C expands through the high pressure turbine. It is reheated at a constant pressure of 5.5 bar to 395°C and expands through the low-pressure turbine to a condenser at 0.08 bar. Draw T - s and h - s diagrams. Find:

(a) Quality of steam at the turbine exhaust (b) cycle efficiency (13)

11. Steam at 20 bar, 360°C is expanded in a steam turbine to 0.08 bar. It then enters a condenser, where it is condensed to saturated liquid water. The pump feeds back the water back into the boiler.

(a) Assuming ideal processes, find per kg of steam, the net work and the cycle efficiency (6)

(b) If the turbine and the pump have each 80% efficiency, and the percentage reduction in the net work and cycle efficiency. (7)

12. In a reheat cycle, the initial steam pressure and the maximum temperature are 150 bar and 550°C respectively. If the condenser pressure is 0.1 bar and the moisture at the condenser inlet is 5%, and assuming ideal processes, determine:

(a) The reheat pressure

(b) cycle efficiency

(c) the steam rate (13)

13. A cyclic steam power plant is to be designed for a steam temperature at turbine inlet of 360°C and an exhaust pressure of 0.08 bar. After isentropic expansion of steam in the turbine, the moisture content at the turbine exhaust is not to exceed 15%. Determine the greatest allowable steam pressure at the turbine inlet and calculate the Rankine cycle efficiency for these steam conditions. Estimate the mean temperature of heat addition. (13)

14. In a regenerative cycle, steam pressure is 30 bar at the entry to the turbine, and 0.04 bar at the turbine exhaust. Steam is initially dry saturated. Steam is bled at 3 bar to heat the feed water. Determine the cycle efficiency. (13)

## UNIT IV

### IDEAL AND REAL GASES, THERMODYNAMIC RELATIONS

#### PART-A

- 1. State any one application of Clapeyron equation. [May/June 2006]**

Clapeyron equation can be applied in chemistry and chemical engineering for transitions between a gas and a condensed phase. This equation also has its application in climatology and meteorology.

- 2. What are reduced properties? Give their significance? [Nov/Dec 2009]**

Reduced properties of a fluid are a set of variables normalized to the fluid's state properties at its critical point. These dimensionless thermodynamic coordinates, taken together with a substance's compressibility factor, correspond to corresponding states.

- 3. Write down the Berthelot equation of state for a real gas. [May/June 2006]**

- 4. What does the Joule-Thomson coefficient represent? [Nov/Dec 2009]**

Joule-Thomson coefficient is a measure of the rate of change of temperature with respect to pressure as a gas is expanded through a valve or orifice without any heat transfer to or from the surroundings.

- 5. Sketch a skeleton compressibility chart with constant reduced temperature characteristics and indicate uses of this chart. [May/June 2006]**

- 6. State the laws of perfect gas. [May/June 2007]**

**Boyle's Law** states that, at constant temperature, the product of the pressure and volume of a given mass of an ideal gas is always constant.

**Charles's Law**, or the law of volumes, states that, for a given mass of an ideal gas at constant pressure, the volume is directly proportional to its absolute temperature.

- 7. What does the Joule-Thomson coefficient represent? [AU Nov/Dec 2009]**

Joule-Thomson coefficient is a measure of the rate of change of temperature with respect to pressure as a gas is expanded through a valve or orifice without any heat transfer to or from the surroundings.



**8. Define Joule-Thomson co-efficient. [May/June 2007]**

Joule-Thomson co-efficient is defined as the change in temperature with change in pressure, keeping the enthalpy remains constant. It is denoted by the  $\mu = (\partial T / \partial p)_h$ .

**9. What is compressibility factor? [AU Nov/Dec2009]**

We know that, the perfect gas equation is  $pV=RT$ . But for real gas, a correction factor has to be introduced in the perfect gas equation to take into account the derivation of real gas equation. This factor is known as compressibility factor ( $Z$ ) and is defined by  $Z = pV/RT$ .

**10. What is Clapeyron equation? [May/June 2006]**

Clapeyron equation which involves relationship between the saturation pressure, saturation temperature, the enthalpy of evaporation and the specific volume of the two phases involved.  $dp/dT = h_{fg} / T v_{fg}$

**11. State Tds equations. [May/June2007]**

Tds equations are  $Tds = C_p dT - T (\partial v / \partial T) dp$  and  $Tds = C_v dT + T (\partial v / \partial T) dp$

**12. State Helmholtz function. [AUMay2011]**

Helmholtz function is property of a system and is given by subtracting the product of absolute temperature ( $T$ ) and entropy ( $s$ ) from the internal energy  $u$ . I.e. Helmholtz function  $= u - Ts$

**13. State Gibbs function.**

Gibbs function is property of a system and is given by  $G = u - Ts + pv = h - Ts$  [  $h = u + pv$  ]

Where

$H$  – Enthalpy

$T$  – Temperature

$S$  – Entropy

**14. Define co-efficient of volume expansion and isothermal compressibility. [AUMay2014]**

**Co-efficient of volume expansions:** Coefficient of volume expansion is defined as the change in volume it changes in temperature per unit volume keeping the pressure constant. It is denoted by  $\beta$ .  $\beta = 1/V (\partial v / \partial T)_p$

**Isothermal compressibility:**

It is defined as the change in volume with change in pressure per unit volume by keeping the temperature constant. It is denoted by  $K$   
 $K = -1/v (\partial v / \partial T)_T$

**15. State the assumptions made in kinetic theory of gases. [AU Nov/Dec2009]**

1. There is no inter molecular force between particles.

2. The volume of the molecules is negligible in comparison with the gas

**16. What is compressibility factor? [AU May2011]**

We know that, the perfect gas equation is  $pV=RT$ . But for real gas, a correction factor has to be introduced in the perfect gas equation to take into account the derivation of real gas equation. This factor is known as compressibility factor ( $Z$ ) and is defined by  $Z = pV / RT$ .

**17. State Tds equations. [AU Dec2013]**

Tds equations are

$$Tds = C_p dT - T \left( \frac{\partial v}{\partial T} \right) dp$$

$$Tds = C_p dT + T \left( \frac{\partial v}{\partial T} \right) dp$$

**18. Explain law of corresponding states.**

If any two gases have equal values of reduced pressure and reduced temperature, then they have same values of reduced volume.

**19. How does the Vander Walls equation differ from the ideal gas equation of state?**

1. Inter molecular attractive study is made.

2. Shape factor is considered.

These assumptions are not made in ideal gas equation of state.

**20. State Dalton's law of partial pressure. [AU Dec2010]**

Dalton's law of partial pressure states—The total pressure of mixture of gases is equal to the sum of the partial pressures exerted by individual gases if each one of them occupied separately in the total volume of the mixture at mixture temperature.  $p = p_1 + p_2 + p_3 + \dots + p_k$ .

**22. What is Joule-Thomson coefficient?**

The temperature behaviours of a fluid during a throttling ( $h = \text{constant}$ ) process is described by the Joule-Thomson coefficient.

**23. What is compressibility factor?**

The gas equation for an ideal gas is given by  $(PV/RT) = 1$ , for real gas  $(PV/RT)$  is not equal to 1 ( $PV/RT = Z$ ) for real gas is called the compressibility factor.

**24. What is partial pressure?**

The partial pressure of each constituent is that pressure which the gas would exert if it occupied alone that volume occupied by the mixtures at the same temperature.

**25. What do you mean by reduced properties?**

The ratios of pressure, temperature and specific volume of a real gas to the corresponding critical values are called the reduced properties.

## **Part-B**

1. (a) Derive the Maxwell's equations. (7)  
(b) What is the use of the Clapeyron equation in thermodynamics? Derive the Clapeyron equation. (6)
2. (a) What is Joule Thomson coefficient? Prove that it is zero for an ideal gas. (7)  
(b) Prove that  $c_p$  of an ideal gas is a function of temperature only. (6)
3. Deduce the Maxwell's relations and from the third relation, deduce the Clausius-Clapeyron equation. Also, apply this equation to the vapourisation process for a pure substance. (13)
4. (a) What is the use of Clapeyron equation? Write down it for the liquid – vapour region. (5)  
(b) Explain the Joule-Thomson process of a real gas through a throttle valve. Derive the expression for Joule Thomson coefficient and reduce its value for an ideal gas. (8)
5. (a) What are the differences between ideal and real gases? (2)  
(b) Write down the Van der Waals' equation of state for real gases and explain how it is obtained from ideal gas equation by incorporating real gas corrections. (4)  
(c) Show that for an ideal gas  $c_p/c_v = \gamma$ . (4)  
(d) What is the compressibility factor? (3)
6. (a) Write down the first and second TdS equations, and derive the expression for the difference in heat capacities,  $c_p$  and  $c_v$ . (7)  
(b) Explain Joule-Kelvin effect. What is inversion temperature? (6)
7. (a) An ideal gas with  $c_p = 1 \text{ kJ/kg K}$  and  $c_v = 0.7 \text{ kJ/kg K}$  undergoes a polytropic expansion from 6 bar to 1 bar, following the law  $p v^{1.2} = \text{Constant}$ . Calculate the change in entropy per kg of the gas, during the process. (8)  
(b) Why is there no temperature change when an ideal gas is throttled? (5)
8. A mass of air is initially at  $260^\circ\text{C}$  and  $700 \text{ kPa}$ , occupies  $0.028 \text{ m}^3$ . The air is expanded at constant pressure to  $0.084 \text{ m}^3$ . A polytropic process with  $n = 1.5$  is carried out, followed by a constant temperature process which completes the cycle. All the processes are reversible. (a) Sketch the cycle in the  $p-v$  and  $T-s$  planes. (b) Find the heat received and the heat rejected in the cycle. (c) Find the efficiency of the cycle. (13)
9. (a) What is the Joule-Thomson coefficient? Derive an expression for the Joule-Thomson coefficient. (9)

(b) Compute the Joule-Thomson coefficient for a gas whose equation of state is  $p(v - b) = RT$  (4)

15. The values of specific volume of H<sub>2</sub>O at 100°C for saturated liquid and saturated vapour states are 0.001044 m<sup>3</sup>/kg and 1.673 m<sup>3</sup>/kg, respectively. The slope of the saturation pressure versus temperature curve, i.e.,  $(dp/dT)_{\text{sat}}$  is 3750 Pa/K. Calculate using the appropriate thermodynamic relation, the change in enthalpy between the two saturation states. (13)

## **UNIT – V - MIXTURES AND PSYCHROMETRY**

### **PART - A**

#### **1. What is the difference between air conditioning and refrigeration?[AU Dec2011]**

Refrigeration is the process of providing and maintaining the temperature in space below atmosphere temperature.

Air conditioning is the process of supplying sufficient volume of clean air containing a specific amount of water vapour and maintaining the predetermined atmospheric condition within a selected enclosure.

#### **2. Define psychrometry. [AU Nov/Dec2011]**

The science which deals with the study of behaviour of moist air (mixture of dry air and water vapour) is known as psychrometry.

#### **3. Define the dry bulb temperature (DBT).[AU Nov 2011]**

The temperature which is measured by an ordinary thermometer is known as dry bulb temperature. It is generally denoted by  $t_d$

#### **4. Define the wet bulb temperature. [AU May/June 2009]**

It is the temperature of air measured by a thermometer when its bulb is covered with wet cloth and exposed to a current rapidly moving air. It is denoted by  $t_w$

#### **5. Define the dew point temperature.[AU Nov/Dec2009]**

The temperature at which the water vapour present in air begins to condense when the air is cooled is known as dew point temperature. It is denoted by  $t_{dp}$ .

#### **6. Define relative humidity (RH) and specific humidity. [AU June 2007]**

RH is the ratio of the mass of water vapour ( $m_v$ ) in a certain volume of moist air at a given temperature to the mass of water vapour ( $m_{vs}$ ) in the same volume of saturated air at the same temperature. i.e.,  $RH \text{ (or) } \phi = m_v / m_{vs}$

**Specific humidity ( $\omega$ )** is the ratio of mass of water vapour ( $m_v$ ) to the mass of dry air in the given volume of mixture. i.e.,  $\omega = m_v / m_a$

**7. Difference between absolute and relative humidity.[AU Jan2013]** **Absolute humidity** is defined as the ratio of the mass of water vapour (mv) in a certain volume of moist air at a given temperature to the mass of water vapour(mvs)at atmospheric conditions.

**RH** is the ratio of the mass of water vapour (mv)in a certain volume of moist air at a given temperature to the mass of water vapour(mvs) in the same volume of saturated air at the same temperature. i.e.,  $RH(or)\phi = mv/mvs$

**8. Define DPT and degree of saturation. [AU Dec 2011]**

**DPT Dewpoint Temperature** is the temperature to which moist air is to be cooled before it starts condensing.

Degree of saturation is the ratio of specific humidity of moist air to the specific humidity of saturated air at temperature.

$\mu = \text{Specific humidity of moist air} / \text{Specific humidity of saturated air} = \omega / \omega_s$

**9. What is dew point temperature? How is it related to dry bulb and wet bulb temperature at the saturation condition? [AU May/Jun 2009]**

It is the temperature at which the water vapour presents in air being to condense when the air is cooled. For saturated air, the dry bulb, wet bulb and dew point temperature are all same.

**10.State Dalton's law of partial pressure.[AU Nov/Dec2011]**

The total pressure exerted by air and water vapour mixture is equal to the barometric pressure.  $p_h = p_a + p_v$

$p_h$  = Barometric pressure.

$p_a$  = Partial pressure of dry air.  $p_v$  = Partial pressure of water vapour.

**11. Define Apparatus Dew Point (ADP) of coolingcoil. [AUMay 2013]**

For dehumidification ,the cooling coil is to be kept at a mean temperature which is below the dew point temperature (DPT) of the entering. This temperature of the coil is called ADP temperature.

**12. List down the psychrometric process. [AU Dec2012]**

1. Sensible heating process
2. Sensible cooling process
3. Humidification process
4. Dehumidification process
5. Heating and humidification process
6. Cooling and dehumidification process
7. Adiabatic mixing air streams process
8. Evaporative cooling process.

**13. Define by pass factor (BPF) of a coil. [AUMay/June2011]**

The ratio of the amount of air which does not contact the cooling coil (amount of by passing air) to the amount of supply air is called BPF.  
i.e.,  $BPF = \frac{\text{Amount of air by passing the coil}}{\text{Total amount of air passed}}$ .

**14. Define the humidification process. [AU Dec2009]**

Humidification is defined as the process of adding moisture at constant dry bulb temperature. So  $\omega_1 > \omega_2$  but  $td_1 = td_2$ . So, the humidity ratio increases from  $\phi_1$  to  $\phi_2$ .

**15. Define „Mole fraction“. [May/June2009]**

Mole fraction describes the number of molecules (or moles) of one component divided by total the number of molecules (or moles) in the mixture.

**16. What is evaporative cooling? Will it work in humid climates? [April/May 2010]**

It is defined as the reduction in temperature resulting from the evaporation of a liquid, which removes latent heat from the surface from which evaporation takes place and it will not work in humid climates.

**17. When is humidification of air necessary? [May/June 2013]**

Humidification of air is the process of adding moisture to the air and it is necessary to provide human comfort.

**18. Deduce the expression for the molecular weight of the mixture of two non reacting ideal gases. [May 2007]**

**19. State the effects of very high and a very low by pass factor. [AU May 2009]**

**Very high by pass factor**

1. It requires lower ADP. Refrigerant plant should be of larger capacity.
2. It requires more air, larger fan and motor required.
3. It requires less heat transfer area.
4. It requires more chilling water. Larger piping required.

**Very low bypass factor:**

1. Higher ADP is to be employed.
2. It requires less air. Fan and motor size reduced.

**20. What is meant by wet bulb temperature(WBT)?**

It is the temperature recorded by a thermometer whose bulb is covered with cotton wick (wet) saturated with water. The wet bulb temperature maybe the measure of enthalpy of air. WBT is the lowest temperature recorded by moist end bulb.

**21. Define dew point depression.**

It is the difference between dry bulb temperature and dew point temperature of air vapour mixture.

**22. What is meant by adiabatic saturation temperature(or)thermodynamic wet bulb temperature?**

It is the temperature at which the outlet air can be brought into saturation state by passing through the water in the long insulated duct (adiabatic) by the evaporation of water due to latent heat of vapourisation.

**23. What is psychrometer?**

Psychrometer is an instrument which measures both dry bulb temperature and wet bulb temperature.

**24. What is Psychrometric chart?**

It is the graphical plot with specific humidity and partial pressure of water vapour in Y- axis and dry bulb temperature along X-axis. The specific volume of mixture, wet bulb temperature, relative humidity and enthalpy are the properties appeared in the psychrometric chart.

**PART B**

1. (a) State Dalton's law of partial pressures. (4)

1. (b) An air-water vapour mixture at 0.1 MPa, 30°C, 80% RH has a volume of 50 m<sup>3</sup>. Calculate the specific humidity, dew point, wet bulb temperature, mass of dry air and mass of water vapour. (9)

2. (a) Draw the psychrometric chart and show any two psychrometric processes on it. (3)

2 (b) A sample of moist air at 1 atm and 25°C has a moisture content of 0.01% by volume. Determine the humidity ratio, the partial pressure of water vapour, the degree of saturation, the relative humidity and the dew point temperature. (10)

3. A mole-basis analysis of a gaseous mixture yields the following results:

CO<sub>2</sub> 12.0%

O<sub>2</sub> 4.0%

N<sub>2</sub> 82.0%

CO 2.0%

Determine the analysis on a mass basis and the molecular mass for the mixture. (13)

4. A mixture of ideal gases consists of 3 kg of nitrogen and 5 kg of carbon-dioxide at a pressure of 300 kPa and a temperature of 20°C. Find:

(a) the mole fraction of each constituent.

(b) the equivalent molecular weight of the mixture

- (c) the equivalent gas constant of the mixture  
(d) the partial pressures of each constituent. (13)

5. (a) Describe the process of adiabatic mixing of two streams and deduce the ratio of masses of two streams in terms of humidity and/or enthalpy. (8)

(b) Air at 20°C, 40% RH is mixed adiabatically with air at 40°C, 40% RH in the ratio 1 kg of the former with 2 kg of the latter (on dry basis). Find the specific humidity and enthalpy of Final state of air.

6. A rigid container, 10 m<sup>3</sup> in volume, contains moist air at 45°C and 100 kPa with RH= 40%. The container is now cooled to 5°C. Neglect the volume of any liquid that might be present and find the final mass of water vapour, the final total pressure, and the heat transfer.(13)

7. A mixture of 2 kg oxygen (MW = 32 kg/kmol) and 2 kg argon (MW = 40 kg/kmol) is present in an insulated piston-cylinder arrangement at 100 kPa, 300 K. The piston now compresses the mixture to half its initial volume. Find the final pressure, temperature and the piston work. Assume CV for oxygen and argon as 0.6618 kJ/kgK and 0.3122 kJ/kgK respectively.(13)

9. An air-water vapour mixture at 0.1 MPa, 30°C, 80% RH has a volume of 50 m<sup>3</sup>. Calculate the specific humidity, dew point, wet bulb temperature, mass of dry air and mass of water vapour. If the mixture is cooled at constant pressure to 5°C, calculate the amount of water vapour condensed. (13)

11. (a) Atmospheric air is at 100 kPa and 25°C with a relative humidity of 75%. Find the absolute humidity and the dew point of the mixture. (5)

(b) Two streams of air at 25°C, 50% RH and 25°C, 60% RH are mixed adiabatically to obtain 0.3 kg/s of air (dry basis) at 30°C. Calculate the amounts of air drawn from both the streams and the humidity ratio of the mixed air. (8)

12. An air-conditioning unit is shown in the below Fig, with pressure, temperature, and relative humidity data. Calculate the heat transfer per kilogram of dry air, assuming that changes in kinetic energy are negligible

13) Atmospheric air at 1.01325 bar has a dbt of 32°C and a wbt 26°C.

Compute (a) the partial pressure of water vapour, (b) the specific humidity, (c) the dew point temperature, (d) the relative humidity, (e) the degree of saturation, (f) the enthalpy of the mixture. (13)



**CE8394 FLUID  
MECHANICS AND  
MACHINERY**

**OBJECTIVES**

- The properties of fluids and concept of control volume are studied
- The applications of the conservation laws to flow through pipes are studied.
- To understand the importance of dimensional analysis
- To understand the importance of various types of flow in pumps.
- To understand the importance of various types of flow in turbines.

**UNIT I FLUID PROPERTIES AND FLOW CHARACTERISTICS 12**

Units and dimensions- Properties of fluids- mass density, specific weight, specific volume, specific gravity, viscosity, compressibility, vapor pressure, surface tension and capillarity. Flow characteristics – concept of control volume - application of continuity equation, energy equation and momentum equation.

**UNIT II FLOW THROUGH CIRCULAR CONDUITS 12**

Hydraulic and energy gradient - Laminar flow through circular conduits and circular annuli-Boundary layer concepts – types of boundary layer thickness – Darcy Weisbach equation –friction factor-Moody diagram- commercial pipes- minor losses – Flow through pipes in series and parallel.

**UNIT III DIMENSIONAL ANALYSIS 12**

Need for dimensional analysis – methods of dimensional analysis – Similitude –types of similitude - Dimensionless parameters-application of dimensionless parameters – Model analysis.

**UNIT PUMPS 12**

Impact of jets - Euler's equation - Theory of roto-dynamic machines – various efficiencies– velocity components at entry and exit of the rotor- velocity triangles - Centrifugal pumps– working principle - work done by the impeller - performance curves - Reciprocating pump-working principle – Rotary pumps –classification.

**UNIT V TURBINES 12**

Classification of turbines – heads and efficiencies – velocity triangles. Axial, radial and mixed flow turbines. Pelton wheel, Francis turbine and Kaplan turbines- working principles - work done by water on the runner – draft tube. Specific speed - unit quantities – performance curves for turbines – governing of turbines.

**TOTAL: 60 PERIODS**

**COURSE OUTCOMES:**

**Upon completion of this course, the students will be able to**

<b>CO 1</b>	Apply mathematical knowledge to predict the properties and characteristics of a fluid.
<b>CO 2</b>	Can analyse and calculate major and minor losses associated with pipe flow in piping networks.
<b>CO 3</b>	Can mathematically predict the nature of physical quantities
<b>CO 4</b>	Can critically analyse the performance of pumps
<b>CO 5</b>	Can critically analyse the performance of turbines.

**TEXT BOOK:**

1. Modi P.N. and Seth, S.M. "Hydraulics and Fluid Mechanics", Standard Book House, New Delhi 2013.

**REFERENCES:**

1. Graebel. W.P, "Engineering Fluid Mechanics", Taylor & Francis, Indian Reprint, 2011
2. Kumar K. L., "Engineering Fluid Mechanics", Eurasia Publishing House(p) Ltd., New Delhi 2016
3. Robert W.Fox, Alan T. McDonald, Philip J.Pritchard, "Fluid Mechanics and Machinery", 2011.
4. Streeter, V. L. and Wylie E. B., "Fluid Mechanics", McGraw Hill Publishing Co. 2010

## CE 8394 FLUID MECHANICS AND MACHINERY

### UNIT I - FLUID PROPERTIES AND FLOW CHARACTERISTICS

#### PART A Questions

1) Define fluids. Classify the different types of fluids.(or) How are fluids classified?

**Ans:** Fluid may be defined as a substance which is capable of flowing. It has no definite shape of its own, but conforms to the shape of the containing vessel.

Fluids are classified as i) Ideal and Real Fluids ii) Newtonian and Non Newtonian fluids.

2) What are the properties of ideal fluids? Give examples.

**Ans: i)** It is incompressible ii) It has zero viscosity iii) Shear force is zero, when it is in motion.

Examples: low viscosity fluids such as air, water etc. can be considered as ideal fluids.

3) Distinguish between ideal and real fluids.

**Ans:**

S.No.	Ideal Fluids	Real Fluids
1.	It is incompressible	It is compressible
2.	It has zero viscosity	They are viscous in nature
3.	Shear force is zero, when it is in motion.	Shear stress always exists in such fluids

4) Distinguish between Newtonian fluids and Non Newtonian fluids.

**Ans:** Newtonian Fluids: Fluids which follow newton's law of viscosity are called Newtonian fluids. Ex: Water, Kerosene

Non – Newtonian Fluids: Fluids which do not follow newton's law of viscosity are called Non – Newtonian fluids. Ex: Thixotropic liquids ( Printers ink)

Newton's Law of viscosity states that magnitude of shear stress is proportional to the rate of deformation.  $\tau \propto du/dy$        $\tau = \mu du/dy$

5) What is the difference between compressible fluid and incompressible fluid? (or) Define Compressible and incompressible fluid.

**Ans:** A compressible fluid will reduce its volume in the presence of an external pressure.

An incompressible fluid is a fluid that does not change the volume of the fluid due to external pressure.

For compressible fluid, Density is not constant.

For incompressible fluids, Density is constant.

6) Distinguish between mass density and specific weight.(or) Define specific weight and density.

**Ans:** Density or mass density is defined as mass occupied per unit volume.  $\rho = m / V \text{ Kg/m}^3$

Where, m – mass in Kg, V – Volume in  $\text{m}^3$

Specific weight or weight density is defined as weight occupied per unit volume.  $w = W / V \text{ N/m}^3$  Where, W – weight in N or KN V – Volume in  $\text{m}^3$

Density does not vary from place to place but specific weight varies from place to place.

7) Define specific volume and express its unit.

**Ans:** Specific volume is defined as volume occupied per unit mass.  $v = V / m \text{ m}^3/\text{Kg}$  where V – volume in  $\text{m}^3$  and m – mass in Kg.

8) What is specific gravity? How is it related to density?(or) Define density and specific gravity of a fluid.

**Ans:** Specific gravity is defined as ratio of specific weight ( or)

Density of given fluid to the specific weight (or) density of the standard fluid.  $s = \text{Density of given fluid} / \text{Density of standard fluid}$ .

9) Calculate the mass density and specific volume of one litre of petrol of specific gravity 0.7?

**Ans:** Volume = 1 litre =  $0.001 \text{ m}^3$  Specific gravity  $s = 0.7$

Density of petrol  $\rho_{\text{petrol}} = s \times \rho_{\text{water}} = 0.7 \times 1000 = 700 \text{ Kg/m}^3$

Specific volume  $v = 1 / \rho_{\text{petrol}} = 1/700 = 0.00143 \text{ m}^3/\text{Kg}$

10) Define the term Pressure. What are its units?

**Ans:** Pressure is defined as the force exerted on the fluid per unit area.  $p = F/A$ , Units are  $\text{N/m}^2$ , Pa,  $\text{KN/m}^2$

11) What is the difference between gauge pressure and absolute pressure?

**Ans:** Gauge pressure is the pressure recorded by the pressure gauge. It actually measures the difference between fluid pressure and atmospheric pressure.

Pressure measured from absolute zero pressure is called as absolute pressure.

Absolute Pressure = Atmospheric Pressure + Gauge Pressure

Absolute Pressure = Atmospheric Pressure - Vacuum Pressure

12) Express 3m of water head in cm of mercury and pressure in Kpa.

**Ans:** 3m of water =  $(3 \times 1) / 13.6 = 0.2206 \text{ m}$  of mercury = 22.06cm of mercury

3m of water =  $3 \times 9.81 = 29.43 \text{ KN/m}^2$

13) Find the height of a mountain where the atmospheric pressure is 730 mm of Hg at normal conditions.

**Ans:**  $p_{\text{top}} = \rho g h_{\text{mercury}} = 13600 \times 9.81 \times 0.730 = 97.39 \text{ kN/m}^2$

$p_{\text{bottom}} = 101.325 \text{ kN/m}^2$

$\rho g h_{\text{air}} = p_{\text{bottom}} - p_{\text{top}} \quad 1.18 \times 9.81 \times \text{height of mountain} = 101.325 - 97.39$

Height of mountain = 339.9m

14) State Pascal's law.

**Ans:** Pascal's Law states that 'The normal stress acting at a point in a fluid is independent of the orientation of the surface on which it acts.'

15) What is meant by stagnation pressure?

**Ans:** The Pressure at which the velocity of fluid particles is zero is called stagnation pressure.

16) Define Compressibility and bulk modulus.

**Ans:** Bulk Modulus (K) is defined as the ratio of the change in pressure to the rate of change of volume due to the change in pressure.  $K = - dp/(dv/v)$

Compressibility is the property by which fluid undergoes a change in volume under the action of external pressure. It is the reciprocal of Bulk Modulus. Compressibility =  $1/K$

17) The volume of a liquid is decreased by 0.2% when the pressure of the liquid is increased from 8Mpa to 20Mpa. Determine the bulk modulus of elasticity and compressibility.

**Ans:**  $(dv/v) = 0.2/100 = 0.002$   $dp = 20 - 8 = 12 \text{ MPa} = 12 \times 10^6 \text{ N/m}^2$

$K = dp/(dv/v) = 12 \times 10^6 / 0.002 = 6 \times 10^9 \text{ N/m}^2$

Compressibility =  $1/K = 1/6 \times 10^9 = 1.67 \times 10^{-10} \text{ m}^2/\text{N}$

18) The volume of water in a rigid piston cylinder arrangement is  $0.3 \text{ m}^3$  and the pressure is 15 bar. The piston diameter is 120mm. Determine the distance through which the piston has to move so that the pressure will increase to 170 bar. Take **bulk modulus** as  $2360 \times 10^6 \text{ N/m}^2$ .

**Ans:**  $V = 0.3 \text{ m}^3$   $P_1 = 15 \text{ bar}$   $P_2 = 170 \text{ bar}$   $dP = P_2 - P_1 = 170 - 15 = 155 \text{ bar} = 155 \times 10^5 \text{ N/m}^2$

$D = 120 \text{ mm} = 0.12 \text{ m}$   $A = \pi D^2/4 = 0.0113 \text{ m}^2$

$K = dp/(dv/V) \quad 2360 \times 10^6 = 155 \times 10^5 / (dv/0.3) \quad dv = 1.97 \times 10^{-3} \text{ m}^3$

$dV = A \times dx$  Distance through which the piston has to move =  $dV/A$   
 $= 1.97 \times 10^{-3} / 0.0113 =$

0.174m

19) Density of sea water at the surface was measured as  $1025 \text{ Kg/m}^3$  at an atmospheric pressure of 1.01bar. At certain depth in water, the

density was found to be  $1043 \text{ Kg/m}^3$ . Determine the pressure at that point. The **Bulk Modulus** of sea water is  $2320 \text{ Mpa}$ .

$$\begin{aligned}\text{Ans: } K &= dP/(dp/\rho) \quad 2320 \times 10^6 = dP / ((1043 - 1025)/1025) \\ dP &= 40.74 \times 10^6 \text{ N/m}^2 \quad dP = P_2 - P_1 \\ P_2 &= dP + P_1 = 40.74 \times 10^6 + 1.01 \times 10^5 \\ P_2 &= 40.84 \times 10^6 \text{ N/m}^2\end{aligned}$$

20) A fluid having coefficient of **compressibility**  $0.8 \times 10^{-9} \text{ m}^2/\text{N}$  is completely filled in a reservoir of capacity  $0.015 \text{ m}^3$ . Calculate the amount of fluid that will spill over, if pressure in the reservoir is reduced by  $18 \text{ MPa}$ .

$$\begin{aligned}\text{Ans: } \text{Bulk Modulus } K &= 1/C = 1/0.8 \times 10^{-9} = 1.25 \times 10^9 \text{ N/m}^2 \quad V = 0.015 \text{ m}^3 \\ dP &= 18 \times 10^6 \text{ N/m}^2 \\ K &= dP/(dV/V) \quad 1.25 \times 10^9 = 18 \times 10^6 / (dV / 0.015) \\ dV &= 2.16 \times 10^{-4} \text{ m}^3\end{aligned}$$

21) What is Viscosity? What is the cause of it in liquids and in gases? (or) What is Viscosity and give its units? (or) Define dynamic viscosity.

**Ans:** Viscosity (or) Dynamic Viscosity is defined as the resistance offered to the flowing fluid.

Units for Viscosity:  $\text{N-sec/m}^2$  or  $\text{Kg/m-sec}$

Cause of Viscosity in liquids: When temperature increases, Viscosity decreases.

Cause of Viscosity in gases: When temperature increases, Viscosity increases.

22) Define the term Kinematic Viscosity and give its dimension.(or) Mention the significance of kinematic viscosity (or) What is the importance of Kinematic Viscosity?

**Ans:** Kinematic viscosity represents momentum diffusivity. Kinematic Viscosity increases with increase in temperature in case of gases whereas it decreases in case of liquids.

For liquids and gases, dynamic viscosity is not influenced significantly by pressure but kinematic viscosity of gases is influenced by pressure due to change in density.

23)) How does the dynamic viscosity of i) liquids and ii) gases vary with temperature? (or) What is the variation of viscosity with respect to temperature?

**Ans:** Cause of Viscosity in liquids: When temperature increases, Viscosity decreases.

Cause of Viscosity in gases: When temperature increases, Viscosity increases.

24) To what property does the term lighter refer?

**Ans:** The term lighter refers to specific gravity of oil which indirectly relates with density and viscosity. Hence the lighter oil has the specific gravity less than unity.

25) Distinguish between dynamic viscosity and Kinematic viscosity.

**Ans:**

S.No.	Dynamic Viscosity	Kinematic Viscosity
1.	Defined as resistance offered to the flowing fluid.	Defined as the ratio of dynamic viscosity to the density.
2.	Obtained with regard to the cause of motion	Obtained without regard to the cause of motion.
3.	Unit is N-s/m <sup>2</sup>	Unit is m <sup>2</sup> /sec
4.	Not concerned with length and time only	Concerned with length and time only.

26) Differentiate between Kinematic viscosity of liquids and gases with respect to pressure.

**Ans:** In liquids, as pressure increases, kinematic viscosity increases. In gases, change in viscosity due to change in pressure is negligible.

27) Define Newton's law of Viscosity.

**Ans:** Newtons law of viscosity states a linear relationship between shear stress and resulting rate of deformation. Shear stress  $\tau = \mu \frac{du}{dy}$

Where,  $\mu$  – dynamic viscosity  $\frac{du}{dy}$  – rate of deformation

28) Define relative or specific viscosity.

**Ans:** It is the ratio of dynamic viscosity of given fluid to dynamic viscosity of water at 20°C.

Relative or specific viscosity = dynamic viscosity of given fluid / dynamic viscosity of water at 20°C

29) If a liquid has a viscosity of 0.051poise and kinematic viscosity of 0.14 stokes. Calculate its specific gravity.

**Ans:** Viscosity  $\mu = 0.051\text{poise} = 0.0051\text{N-s/m}^2$

kinematic viscosity  $\nu = 0.14\text{stokes} = 0.14 \times 10^{-4}\text{m}^2/\text{s}$

$\nu = \mu/\rho$   $\rho = \mu/\nu = 0.0051/0.14 \times 10^{-4} = 364.29\text{Kg/m}^3$

specific gravity  $s = \rho_{\text{liquid}} / \rho_{\text{water}} = 364.29/1000 = 0.36429$

30) Calculate the viscosity of a liquid having kinematic viscosity 8.5stokes and specific gravity 1.4.

**Ans:** kinematic viscosity  $\nu = 8.5\text{stokes} = 8.5 \times 10^{-4}\text{m}^2/\text{s}$  specific gravity  $s = 1.4$

Density  $\rho_{\text{liquid}} = \rho_{\text{water}} \times s = 1000 \times 1.4 = 1400\text{Kg/m}^3$

Viscosity  $\mu = \nu \times \rho_{\text{liquid}} = 8.5 \times 10^{-4} \times 1400 = 1.19\text{N-s/m}^2$



31) If certain oil has density  $1370 \text{ Kg/m}^3$ , shear stress of a point in oil of  $0.354 \text{ N/m}^2$  and velocity gradient at that point is  $0.23 \text{ per second}$ . Calculate the kinematic viscosity.

**Ans:**  $\rho_{\text{liquid}} = 1370 \text{ Kg/m}^3$   $\tau = 0.354 \text{ N/m}^2$   $du/dy = 0.23/\text{sec}$

$\mu = \tau / du/dy = 0.354 / 0.23 = 1.539 \text{ N-s/m}^2$

kinematic viscosity  $\nu = \mu/\rho = 1.539/1370 = 1.12 \times 10^{-3} \text{ m}^2/\text{sec}$

32) Two horizontal plates are placed  $12.5 \text{ mm}$  apart, the space between them being filled with oil of viscosity  $14 \text{ poise}$ . Calculate the shear stress in the oil if the upper plate moved with a velocity of  $2.5 \text{ m/s}$ . Define specific weight.

**Ans:**  $dy = 12.5 \text{ mm} = 12.5 \times 10^{-3} \text{ m}$   $\mu = 14 \text{ poise} = 1.4 \text{ N-s/m}^2$   $du = 2.5 \text{ m/s}$

Shear stress  $\tau = \mu du/dy = 1.4 \times 2.5 / 12.5 \times 10^{-3} = 280 \text{ N/m}^2$

Specific weight  $w$  is defined as weight occupied per unit volume.  $w = W/V \text{ N/m}^3$

33) What is Cohesion and Adhesion in fluids?

**Ans:** Cohesion: Force of attraction between molecules of same liquid.

Adhesion: Force of attraction between molecules of different liquids or between the molecules of liquid and molecules of solid surface.

34)) Define Surface tension and express its unit.

**Ans:** Surface tension is due to the force of cohesion between the liquid particles at the free surface. It is the tensile force acting on the free surface under equilibrium.

Unit for surface Tension –  $\text{N/m}$

35) Calculate the pressure difference between the inside and outside of a **soap bubble** of  $28 \text{ mm}$  diameter, if the surface tension is  $0.021 \text{ N/m}$ .

**Ans:** pressure difference  $p_i - p_o = 8\sigma/d = 8 \times 0.021 / 0.028 = 6 \text{ N/m}^2$

36) Find the pressure inside a **soap bubble** of  $30 \text{ mm}$  diameter. The surface tension is  $0.074 \text{ N/m}$ .

**Ans:**  $p_i - p_o = 8\sigma/d$   $p_i = 8\sigma/d + p_o = (8 \times 0.074) / 0.03 + 101.325 \times 10^3 = 101.344 \times 10^3 \text{ N/m}^2$

37) Estimate pressure inside a **water droplet** of  $0.5 \text{ mm}$  diameter. Assume  $\sigma = 0.073 \text{ N/m}$ .

**Ans:**  $p = 4\sigma/d = 4 \times 0.073 / 0.5 \times 10^{-3} = 584 \text{ N/m}^2$

38) A soap bubble is formed when the inside pressure is  $5 \text{ N/m}^2$  above the atmospheric pressure. If surface tension in the soap bubble is  $0.0125 \text{ N/m}$ . Find the diameter of the bubble formed.

**Ans:**  $p_i - p_o = 8\sigma/d$   $5 = (8 \times 0.0125) / d$   $d = 0.02 \text{ m}$

39) The pressure inside a cylindrical **jet of water** is  $18\text{N/m}^2$  in excess of ambient pressure. If the diameter of jet is 30mm, calculate the **surface tension** of water in contact with air.

**Ans:**  $p = 2\sigma/d$        $\sigma = p \times d / 2 = 18 \times 0.03 / 2 = 0.27\text{N/m}$

40) Suppose the small air bubbles in a glass of tap water may be on the order of  $50\mu\text{m}$  in diameter. What is the pressure inside these bubbles?

**Ans:** Assume  $\sigma = 0.073\text{N/m}$  for water       $p = 4\sigma/d = 4 \times 0.073 / 50 \times 10^{-6} = 5840\text{N/m}^2$

41) Write the equation of surface tension of liquid jet, liquid droplet and soap bubble.

**Ans:** Surface tension in liquid droplet  $p = 4\sigma/d$ ,  $\sigma = p \times d / 4$

Surface tension in liquid jet  $p = 2\sigma/d$ ,  $\sigma = p \times d / 2$

Surface tension in soap bubble  $p = 8\sigma/d$ ,  $\sigma = p \times d / 8$

42) What is meant by capillarity? (or) What is capillarity?

**Ans:** Capillarity is a phenomenon of rise or fall of liquid surface relative to the adjacent normal level of liquid. This is due to the combined effect of cohesion and adhesion of liquid particles. Rise of liquid level is known as capillary rise and fall of liquid is known as capillary fall or capillary depression.

43) What are the parameters upon which the capillarity depends?

**Ans:** i) Diameter of capillary tube ii) Specific weight of liquid iii) Surface tension of liquid.

iv) Contact angle

44) Explain Capillary depression.

**Ans:** The fall of liquid surface relative to the adjacent normal level due to the combined effect of cohesion and adhesion of liquid particles is known as capillary depression.

45) Calculate the height of Capillary rise for water in a glass tube of diameter 1mm.

**Ans:** for water, contact angle  $\beta=0$  surface tension  $\sigma=0.073\text{N/m}$  specific weight  $w = 9810\text{N/m}^3$

Capillary rise  $h = 4\sigma \cos\beta / w \times d = (4 \times 0.073 \times \cos 0) / (9810 \times 0.001) = 0.0298\text{m}$

46) The gauge pressure measured in a manometer is  $120\text{N/m}^2$ . Find the smallest diameter of manometer tube such that error due to **capillary** action is less than 5%. The manometer liquid is water.

**Ans:** pressure  $p = 120\text{N/m}^2$  for water, contact angle  $\beta=0$  surface tension  $\sigma=0.073\text{N/m}$  specific weight  $w = 9810\text{N/m}^3$  Height of water column  $h = p/w = 120/9810 = 0.0122\text{m}$

Error in diameter due to capillary action = 5% = 0.95d

$$h = 4\sigma \cos\beta / w d \quad 0.95d = 4\sigma \cos\beta / w h$$

$$d = (4 \times 0.073 \times \cos 0) / (9810 \times 0.0122 \times 0.95) = 2.568 \times 10^{-3} \text{m}$$

47) Explain Concept of Continuum.

**Ans:** Molecules inside the substances are in constant motion and collide with each other. In liquids the molecules are closely spaced which create strong intermolecular cohesive forces. Thus the liquid behaves as a continuous mass. Continuum is a continuous medium in which there is a continuous distribution of matter with no empty space.

48) Distinguish between a control and differential control volume.(or)  
What is the use of control volume?

**Ans:** The region in which the mass crosses the system boundary is called control volume. The control volume in which the conservation of mass equation is applied is called as differential control volume.

49) Write the Bernoulli's equation in terms of head. Explain each terms.

**Ans:**  $z + p/w + v^2/2g$  ---- Bernoulli's Equation

Where,  $z$  – Datum head  $p/w$  – pressure head  $v^2/2g$  – velocity or kinetic head

50) What are the assumptions made in deriving Bernoulli's equation?

**Ans:** i) Flow is steady and continuous ii) Liquid is ideal and compressible iii) velocity is uniform over the cross section iv) Frictional forces are negligible v) only gravity force and pressure forces are considered.

51) List the types of flow measuring devices fitted in a pipe flow, which uses the principle of Bernoulli's theorem.(or) Mention any four applications of Bernoulli's theorem.

**Ans:** i) Venturimeter ii) Orificemeter iii) Nozzle meter iv) Pitots tube

52) State the limitations of Bernoulli's Theorem.

**Ans:** i) Velocity of flow is assumed to be constant but it is not so in actual practice.  
ii) No forces, other than gravity and pressure forces are considered.

53) What is Venturimeter ? Explain its basic principles.

**Ans:** Venturimeter is a device, which is used for measuring the rate of flow of fluid through pipes. Basic principle is, by reducing the cross sectional area along the pipe, a pressure difference is created from which we can calculate the discharge through the pipe.

54) What are the various parts in Venturimeter?

**Ans:** i) Inlet section followed by convergent portion ii) Throat part iii) Divergent cone followed by outlet section.

55) Why convergent portion is smaller than divergent portion?

**Ans:** Convergent portion is smaller than divergent portion to avoid flow separation and consequent energy loss. flow separation is occurring in divergent portion and therefore it is not used for flow measurement.

56) Why pressure difference is not measured between throat and exit?

**Ans:** In exit portion of the venturimeter, the flow separation takes place. So the pressure difference is measured between inlet and throat.

57) Write the expression for the rate of flow through a Venturimeter?

**Ans:** Actual Discharge  $Q_{act} = \{C_d a_1 a_2 (2gh)^{1/2} / \{a_1^2 - a_2^2\}^{1/2}$

Where,  $C_d$  – Coefficient of discharge  $a_1$  – area of inlet section  $a_2$  – area of throat section  $h$  = head causing flow.

58) What are the advantages and disadvantages of Venturimeter?

**Ans:** Advantages i) Loss of head is small ii) suitable for large rate of flow

Disadvantages – i) More space requirements ii) possibility of cavitation.

59) What is Orificemeter ? Explain its basic principles.

**Ans:** Orificemeter is a device, which is used for measuring the rate of flow of fluid through pipes. Basic principle is, by reducing the cross sectional area along the pipe, a pressure difference is created from which we can calculate the discharge through the pipe.

60) What are the advantages and disadvantages of Orifice meter?

**Ans:** Advantages: i) Cost is less ii) requires less space iii) Installation and replacement is easy

Disadvantages: i) Loss of head is high ii) coefficient of discharge is low

61) Compare Venturimeter and Orificemeter.

**Ans:**

S.No.	Venturimeter	Orificemeter
1.	Loss of head is less	Loss of head is high
2.	Coefficient of discharge is high	Coefficient of discharge is low
3.	Requires more space	Requires less space

4.                      Installation and replacement is difficult                      Installation and replacement is easy

62) Write the continuity equation.

**Ans:** Continuity equation –  $\rho_1 A_1 v_1 = \rho_2 A_2 v_2$

Where,  $\rho_1$  and  $\rho_2$  – Density of fluid at entry and exit

$A_1$  and  $A_2$  – Area of the pipe at entry and exit

$v_1$  and  $v_2$  – Velocity of flow at entry and exit

63) State the assumptions made in deriving continuity equation.

**Ans;** i) Flow is steady and continuous ii) Liquid is ideal and compressible

64) State the continuity equation in one dimensional flow.

**Ans:**  $(d(\rho u) / dx) = 0$

65) State the equation of continuity in three dimensional incompressible flow.

**Ans:**  $(d(\rho u) / dx) + (d(\rho v) / dy) + (d(\rho w) / dz) = 0$

66) The converging pipe with inlet and outlet diameters of 200mm and 150mm carries the oil whose specific gravity is 0.8. The velocity of oil at the entry is 2.5m/sec, find the velocity at the exit of the pipe and oil flow rate in Kg/sec.

**Ans:**  $d_1 = 0.2m$   $d_2 = 0.15m$   $s = 0.8$   $v_1 = 2.5m/s$

Continuity equation –  $\rho_1 A_1 v_1 = \rho_2 A_2 v_2$

$$\pi d_1^2 v_1 / 4 = \pi d_2^2 v_2 / 4 \quad \pi \times 0.2^2 \times 2.5 / 4 = \pi \times$$

$$0.15^2 \times v_2 / 4 \quad v_2 = 4.44m/sec$$

$$\text{mass } m = \text{density} \times \text{area} \times \text{velocity} = 0.8 \times 1000 \times \pi \times 0.2^2 \times 2.5 / 4 = 62.8Kg/sec$$

67) Define i) Coefficient of Velocity ii) Coefficient of Contraction.

**Ans:** i) Coefficient of Velocity : It is the ratio of actual velocity of fluid to theoretical velocity.

ii) Coefficient of Contraction : It is the ratio of area of Vena contracta to area of orifice.

68) Write the Euler's equation.

**Ans:** Euler's equation of motion -  $(dp/\rho) + v dv + g dz$

69) Write the impulse momentum equation. (or) What do you understand by Impulse momentum equation?

**Ans:** Impulse momentum equation states that" the impulse force F acting on a fluid mass m over a short interval of time dt is equal to the change of momentum d(mv) in the direction of force.

$$F \cdot dt = d(mv)$$

70) Why is it necessary in winter to use lighter oil for automobiles than in summer? To what property does the term lighter refer?

**Ans:** The term lighter refers to the property called viscosity. In winter, if heavy oil is used for automobiles, the oil becomes more viscous, and doesn't serve lubrication purpose. So lighter oil is used.

## **PART – B Questions**

### **I. PROBLEMS ON SPECIFIC WEIGHT, DENSITY, SPECIFIC VOLUME AND SPECIFIC GRAVITY**

- 1) What are the various **classifications of fluids**? Discuss. ( 8 marks)
- 2) Explain the term **specific gravity, density, compressibility and vapour pressure**. (8marks)
- 3) Distinguish between **mass density and specific weight**. ( 6 marks)
- 4) Explain the **Properties of a hydraulic fluid**. ( 16 marks)
- 5) Explain the terms **specific weight, density, absolute pressure and gauge pressure**. ( 8 marks)
- 6) Calculate the **specific weight, mass density, specific gravity and specific volume** of oil having a volume of  $4.5\text{m}^3$  and weight of 40KN.
- 7) What depth of oil of **specific gravity** 0.8 will produce a pressure of  $120\text{KN/m}^2$ ? What would be the corresponding **depth of water**? (8marks)
- 8) The barometric pressure at sea level is 760mm of mercury while that on a mountain top is 735mm. If the **density** of air is assumed constant as  $1.2\text{Kg/m}^3$ , What is the elevation of the mountain top? ( 8 marks)
- 9) Explain in detail the **Newton's law of viscosity**. Briefly **classify the fluids** based on the density and viscosity. Give the limitations of applicability of Newton's law of viscosity. ( 16 marks)
- 10) At a depth of 8 Km from the surface of ocean, the pressure is stated to be  $82\text{MN/m}^2$ . Determine the **mass density, weight density and specific volume** of water at this depth. Take density at the surface  $\rho = 1025\text{Kg/m}^3$  and Bulk modulus of elasticity  $K = 2350\text{ MPa}$  for the indicated pressure range. (8marks)
- 11) **Convert** intensity of pressure of 2MPa into equivalent **pressure head** of oil of specific gravity 0.8 (4marks)

### **2. PROBLEMS ON BULK MODULUS AND COMPRESSIBILITY**

- 12) A liquid is compressed in a cylinder having a volume of  $0.012\text{m}^3$  at a pressure of  $690\text{N/cm}^2$ . What should be the new pressure in order to make its volume  $0.0119\text{m}^3$ ? Assume **Bulk modulus** of elasticity for the liquid =  $6.9 \times 10^4\text{ N/cm}^2$ . ( 8 marks )

13) Explain the following: i) **Capillarity** ii) **Surface Tension** iii) **Compressibility** iv) **Kinematic viscosity**. ( 16 marks)

14) Explain the phenomenon of **surface tension and capillarity**. ( 8 marks)

15) Write short notes on i) **Viscosity** ii) **Surface Tension** iii) **Newtonian and Non-Newtonian fluids** ( 8 marks)

16) When a pressure of  $20.7 \text{ MN/m}^2$  is applied to 100 litres of a liquid and its volume decreases by one litre. Find the **Bulk modulus** of the liquid. ( 6marks)

17) Assuming the **Bulk modulus of elasticity** of water is  $2.07 \times 10^6 \text{ KN/m}^2$  at standard atmospheric conditions, determine the increase of pressure necessary to produce one percent reduction in the volume at the same temperature.

### **3. PROBLEMS ON CAPILLARITY AND SURFACE TENSION**

18) Derive the expressions for capillary rise and depression. ( 8 marks)

19) Find the height through which water rises by **capillary action** in a 2mm bore, if surface tension at the prevailing temperature is  $0.075 \text{ g/cm}$ .

20) Calculate the **capillary rise** in glass tube of 3mm diameter when immersed in mercury. Take the surface tension and angle of contact of mercury as  $0.52 \text{ N/m}$  and  $130^\circ$  respectively. Also determine the minimum size of the glass tube, if it is immersed in water, given that the surface tension of water is  $0.0725 \text{ N/m}$  and capillary rise in the tube is not to exceed  $0.5 \text{ mm}$ .

21) A **capillary tube** having inside diameter 6mm is dipped in  $\text{CCl}_4$  at  $20^\circ\text{C}$ . Find the rise of  $\text{CCl}_4$  in the tube, if surface tension is  $2.67 \text{ N/m}$  and specific gravity is 1.594 and contact angle  $\theta$  is  $60^\circ$  and specific weight of water at  $20^\circ\text{C}$  is  $9981 \text{ N/m}^3$ .

22) Calculate the **capillary effect** in mm in a glass tube of 4mm diameter, when immersed in i) water and ii) mercury the temperature of the liquid is  $20^\circ\text{C}$  and the value of surface tension of water and mercury at  $20^\circ\text{C}$  in contact with air are  $0.0735 \text{ N/m}$  and  $0.51 \text{ N/m}$  respectively. The contact angle for water  $\theta = 0$  and for mercury  $\theta = 130^\circ$ . Take specific weight of water at  $20^\circ\text{C}$  as equal to  $9790 \text{ N/m}^3$  and specific gravity of mercury is 13.6. ( 12 marks)

23) A U tube is made of **two capillaries** of diameter 1.0mm and 1.5mm respectively. The tube is kept vertically and partially filled with water of surface tension  $0.0736 \text{ N/m}$  and zero contact angle. Calculate the difference in the levels of the menisci caused by the capillary. ( 6 marks)

24) A U tube manometer is made up of **two capillaries** of bore 1.2mm and 1.8 mm respectively. The tube is held vertically and is partially filled with liquid of surface tension  $0.06 \text{ N/m}$  and zero contact angle. Calculate the density of the liquid, if the estimated pressure difference in the level of two menisci is  $12 \text{ mm}$ .

- 25) Derive an **expression for the capillary rise** of a liquid in a capillary tube of radius  $r$  having surface tension  $s$  and contact angle  $\theta$ . If the plates are of glass, what will be the capillary rise of water having  $\sigma = 0.073\text{N/m}$ ,  $\theta = 0^\circ$ ? Take  $r$  as  $1\text{mm}$ . ( 8 marks)
- 26) The **capillary rise** in a glass tube is not to exceed  $0.2\text{mm}$  of water. Determine its minimum size, given that the surface tension for water in contact with air  $= 0.0725\text{N/m}$ . ( 6marks)
- 27) In a differential U tube manometer, the glass tubing of various diameters is used to measure pressure. The diameters of the glass tubes are **3mm and 4mm**. The manometer is used to measure readings in the range of  $30\text{mm}$  to  $120\text{mm}$ . Calculate the **percentage of error** that can creep in the highest and lowest readings. Take  $\sigma = 0.072\text{N/m}$  and  $\theta = 0$ .
- 28) What do you mean by **surface tension**? If the pressure difference between the inside and outside of the air bubble of diameter  $0.01\text{mm}$  is  $29.2\text{Kpa}$ , what will be the surface tension at air water interface? Derive an expression for the surface tension in the **air bubble** and from it, deduce the result for the given conditions. ( 8 marks)
- 29) A  $1.9\text{mm}$  diameter tube is inserted into an unknown liquid whose density is  $960\text{Kg/m}^3$  and it is observed that the liquid rises  $5\text{cm}$  in the tube making a contact angle of  $15^\circ$ . Determine the **surface tension** of the liquid.
- 30) Prove that the relationship between **surface tension** and pressure inside a **droplet of liquid** in excess of outside pressure is given by  $P = 4\sigma / d$ . ( 8 marks)
- 31) In a closed end single tube manometer, the height of mercury column above the mercury well shows  $757\text{ mm}$  against the atmospheric pressure. The inlet diameter of the tube is  $2\text{mm}$ . The contact angle is  $135^\circ$ . Determine the actual height representing the atmospheric pressure if **surface tension** is  $0.48\text{N/m}$ . The space above the column may be considered as vacuum. ( 8 marks)
- 32) A spherical **water droplet** of  $5\text{mm}$  in diameter splits up in air into 16 smaller droplets of equal size. Find the work involved in splitting up the droplet. The surface tension of water may be assumed as  $0.072\text{N/m}$ . ( 8 marks)
- 33) Air is introduced through a nozzle into a tank of water to form a stream of bubble. If the bubbles are intended to have a diameter of  $5\text{mm}$ , calculate by how much the pressure of the air at the nozzle must exceed that of surrounding water. Take  $\sigma = 0.092\text{N/m}$ . Also calculate the absolute **pressure inside the bubble**, if the surrounding water is at  $1.01\text{bar}$ .
- 34) A hollow cylinder of  $150\text{mm}$  OD with its weight equal to the buoyant force is to be kept floating vertically in a liquid with a surface tension of  $0.45\text{N/m}$ . The contact angle is  $60^\circ$ . Determine the additional force required due to **surface tension**. ( 6 marks )
- 35) During measurement of the **unit surface energy** of a mineral oil ( sp. Gr.  $0.8$ ) by the bubble method, a tube having an internal



diameter of 1.2 mm is immersed to a depth of 15mm in the oil. Air is forced through the tube forming a bubble at the lower end. What magnitude of unit surface energy will be indicated by a maximum bubble pressure intensity of  $200\text{N/m}^2$ .

#### **4. PROBLEMS ON DYNAMIC VISCOSITY AND KINEMATIC VISCOSITY**

- 36) State the **effect of temperature** and pressure on **Viscosity**. ( 4 marks)
- 37) The velocity gradient at a certain point of stream of glycerin is  $0.32\text{S}^{-1}$  The density of the glycerin is  $1270\text{Kg/m}^3$  and the **kinematic viscosity** is  $6.5 \times 10^{-6} \text{ m}^2/\text{sec}$ . Calculate the **shear stress** at the point.
- 38) A 200mm diameter shaft slides through a sleeve 200.5mm in diameter and 400mm long at a velocity of 30cm/s. The **viscosity** of the oil filling the annular space is  $\mu = 0.1125\text{Ns/m}^2$ . Find the **resistance to the motion**.
- 39) A hydraulic lift shaft of 225mm diameter moves in a cylinder of 227mm diameter with the length of engagement of 1.2m. The interface is filled with oil of **kinematic viscosity** of  $3.4 \times 10^{-4} \text{ m}^2/\text{sec}$  and density  $950\text{Kg/m}^3$ . Determine the **uniform velocity of movement** of the shaft, if the drag resistance was 480N.
- 40) A shaft of 69mm diameter **rotates concentrically inside a cylinder** of diameter 70mm. Both the cylinder and piston are 80mm long. Find the **tangential velocity** and rpm of the shaft if the space between the cylinder and shaft is filled with an oil of **viscosity** 2.35 poise and a torque of 1.37N-m is applied.
- 41) A liquid with **Kinematic viscosity** of 4 centistokes and specific weight  $8000\text{N/m}^3$  fills the **space between a large stationary plate** and a parallel plate of  $550\text{mm}^2$ , which are kept 2mm apart. If a parallel plate is to be pulled with uniform velocity of 3m/sec, determine the force and power required to maintain this speed.
- 42) An oil of **viscosity** 0.25 poise and relative density 0.86 is flowing through a circular pipe of diameter 80mm and of length 150m. The rate of flow is 3.1 lit/sec. Find the **shear stress** at the pipe wall.
- 43) **Two large inclined parallel planes** are kept 8mm apart and are filled with oil. The parallel planes are inclined at  $30^\circ$  to the horizontal. A small thin square plate of 75mm side slides freely down parallel and midway between the inclined planes with a constant velocity .5m/s due to its own weight of 1.5N Determine the **viscosity** of the oil.
- poise at 200 rpm. Calculate the power lost for a length of 100mm if the thickness of the oil is 1mm. ( 16 marks)
- 45) Lateral stability of a **long shaft 150mm** in diameter is obtained by means of a 250mm stationary bearing having an internal diameter of 150.25mm. If the space between the bearing and shaft is filled with a lubricant having a **viscosity**  $0.245\text{Ns/m}^2$ , What **power** will be

required to overcome the viscous resistance when the shaft is rotated at a constant rate of 180rpm? (10 marks)

46) The **space between two square flat parallel plate** is filled with oil. Each side of the plate is 600mm. The thickness of oil film is 12.5mm. The upper plate which moves at 2.5m/s requires a force of 98.1N to maintain the speed. Determine i) The **dynamic viscosity** of oil in poise ii) The **Kinematic viscosity** of oil in stokes if the specific gravity of the oil is 0.95. ( 16 marks)

47) Determine the **power required** to run a 300mm shaft at 400 rpm in journals with uniform oil thickness of 1mm. Two bearings of 300mm width are used to support the shaft. The dynamic **viscosity** of oil is  $0.03 \text{Ns/m}^2$ . ( 8 marks)

48) The **space between two parallel plates** 0.5mm apart is filled with mineral oil. A force of 15N is required to pull the upper plate at a constant velocity of 0.6m/sec. Calculate the viscosity of oil, if the area of the upper plate is  $2 \times 10^6 \text{ mm}^2$ . Find also the **kinematic viscosity** of the oil if its relative density is 0.8.

49) The space between **two large flat and parallel walls** 25mm apart is filled with a liquid of **absolute viscosity** 0.7 Pa.sec. within this space a thin flat plate , 250mm x 250mm is towed at a velocity of 150mm/sec at a distance of 6mm from one wall, the plate and its movement being parallel to the walls. Assuming linear variations of velocity between the plate and the walls, determine the **force exerted** by the liquid on the plate. ( 8 marks)

50) State **Newton's law** of viscosity. The velocity distribution over a plate is given by the relation,  $u = y ((2/3)-y)$  where y is the vertical distance above the plate in metres. Assuming a **viscosity** of 0.9 Pa-s, Find the shear stress at  $y = 0$  and  $y = 0.15\text{m}$ . ( 8marks)

51) The velocity distribution over a plate is given by  $u = (3/4) * y - y^2$  where u is velocity in m/sec and at a depth y in m above the plate. Determine the shear stress at a distance of 0.3m from the top of plate. Assume dynamic **viscosity** of the fluid is taken as  $0.95 \text{N-s /m}^2$ .

52) If the velocity distribution of a fluid over a plate is given by  $u = ay^2 + by + c$  with the vertex 0.2m from the plate, where the velocity is 1.2m/s. Calculate the **velocity gradients** and **shear stresses** at a distance of 0m, 0.1m and 0.2m from the plate, if the **viscosity** of the fluid is  $0.85 \text{N-s/m}^2$ . ( 16 marks )

53) A fluid of specific gravity 0.9 flows along a surface with a velocity profile given by  $u = 4y - 8y^3 \text{ m/s}$ . where y is in m. What is the velocity gradient at the boundary? If the kinematic **viscosity** is  $0.36 \text{ s}^{-1}$  What is the **shear stress** at the boundary?

54) A 15 cm diameter vertical **cylinder rotates concentrically** inside another cylinder of diameter 15.10cm. Both cylinders are 25cm high. The space between the cylinders is filled with a liquid whose viscosity is unknown. If a torque of 12.0Nm is required to rotate the inner cylinder at 100rpm. Determine the **viscosity** of the fluid. ( 8 marks)

55) In a **rotating cylinder** viscometer, the cylinder supported by a torsion spring is 150mm diameter and 200mm long. The outer

cylinder of 152mm diameter rotates at 800 rpm and the torque measured on inner cylinder is 0.01N-m. If the oil of **viscosity**  $\mu$  between inner and outer cylinder is filled, find the value of  $\mu$ .

56) Consider flow of oil through a pipe of 0.3m diameter. The velocity distribution is parabolic with the maximum velocity of 3m/sec at the pipe center. Estimate the **shear stresses** at the pipe wall and within the fluid 50mm from the pipe wall. The **viscosity** of the oil is 1.7Pa.s. ( 8 marks)

### **5. PROBLEMS ON TORQUE, POWER, DRAG FORCE**

57) Determine the **torque and power** required to turn a 12cm long 6cm diameter shaft at 500rpm in a 62cm concentric bearing flooded with a lubricating oil of **viscosity** 100cp.

58) A flat plate of  $0.5\text{m}^2$  in area moves through the oil between **large fixed parallel planes** 150mm apart. The **gap between two parallel planes** is filled with oil of kinematic viscosity  $0.4 \times 10^{-4} \text{m}^2/\text{s}$ , specific gravity 0.75 and the plate moving with the velocity of 0.8m/s. Calculate the drag force when i) the plate is **30mm from one of the planes** and ii) the plate is **placed midway** between the plates.

59) A 30mm wide **gap between two vertical plane surface** is filled with an oil of specific gravity 0.8 and **dynamic viscosity**  $2 \text{N}\cdot\text{s}/\text{m}^2$ . A metal plate  $1\text{m} \times 1\text{m} \times 2.5\text{mm}$  thick and weighing 35N is placed midway in the gap. Find the **force required**, if plate is to be lifted with a constant velocity of 0.15m/s.

60) A **hollow cylinder** of 150 mm inner diameter filled with fluid of **viscosity**  $0.018\text{N}\cdot\text{s}/\text{m}^2$  rotates at 700 rpm. A shaft of diameter 60mm placed centrally inside the hollow cylinder. Determine the **shear stress** on the shaft wall.

61) A 180mm diameter **disc rotates over a flat surface** separated by an oil film of 1.2mm thickness. Calculate the **viscosity** of oil, if the torque required to rotate the disc at 450rpm was 0.8N-m.

62) The clearance between the shaft of 75mm diameter and the **bearing varies from 2mm to 1mm** over a length of 0.5m. The **viscosity** of oil filling the clearance is  $0.056\text{N}\cdot\text{s}/\text{m}^2$ . The axial velocity of the shaft is 0.8m/s. Determine the force required for **axial movement** of the shaft.

63) Determine the **viscous drag torque and power** absorbed on one surface of a **collar bearing** of 0.2m ID and 0.3m OD with an oil film thickness of 1mm and a viscosity of 30 centipoise if it rotates at 500rpm. ( 6 marks )

### **6. PROBLEMS ON CONTINUITY EQUATION, DISCHARGE AND VELOCITY**

64) **Derive** the general form of **continuity equation** in Cartesian coordinates.( 8 marks)

- 65) Obtain an expression for continuity equation in Cartesian coordinates. ( 8 marks)
- 66) State the law of conservation of mass and **derive the equation of continuity** in Cartesian coordinates for an incompressible fluid. Would it alter if the flow were steady, highly viscous and compressible? ( 16 marks)
- 67) A pipeline 60cm in diameter bifurcates at a Y junction into two branches 40cm and 30cm in diameter. If the rate of flow in the main pipe is  $1.5\text{m}^3/\text{sec}$  and the mean velocity of flow in the 30cm pipe is  $7.5\text{m/s}$ , determine the **rate of flow** in the 40cm pipe. (4marks)
- 68) A swimming pool of  $8\text{m} \times 15\text{m}$  is to be filled to a depth of  $2.5\text{m}$ . Determine the inflow required in  $\text{m}^3/\text{s}$  for a filling time of 90 minutes. If 40mm pipes are available and the water velocity in each hose is limited to  $2\text{m/s}$ , determine the **number of hoses required**.
- 69) A 400mm diameter pipe branches into two pipes of diameters of 200mm and 250mm respectively. If the average velocity in the 400mm diameter pipe is  $2.2\text{m/s}$ . Find the **discharge** in this pipe. Also determine the velocity in 250mm pipe, if the average velocity in 200mm diameter pipe is  $2.6\text{m/s}$ .
- 70) A pipe line of 175mm diameter branches into two pipes which delivers the water at atmospheric pressure. The diameter of the branch 1 which is at  **$35^\circ$  counter clockwise** to the pipe axis is 75mm and the velocity at outlet is  $15\text{m/s}$ . The branch 2 is at  **$15^\circ$  with the pipe centre line in the clockwise direction** has a diameter 100mm. The outlet velocity is  $15\text{m/s}$ . The pipes lie in a horizontal plane. Determine the **magnitude and direction** of the forces on the pipes. (8 marks)
- 71) A  $0.3\text{m}$  diameter pipe carrying oil at  $1.5\text{m/s}$  velocity suddenly expands to  $0.60\text{m}$  diameter pipe. Determine the **discharge and velocity** in  $0.6\text{m}$  diameter pipe. ( 4marks)

## **7. PROBLEMS ON BERNOULLIES EQUATION, EULERS EQUATION**

- 72) **Derive Euler's equation** of motion for a flow along a streamline. ( 16 marks)
- 73) **Derive the Euler's equation** of motion and **deduce** the expression to **Bernoulli's equation**. ( 16 marks)
- 74) State **Bernoulli's theorem** for steady flow of an incompressible fluid. **Derive** an expression for Bernoulli equation and state the **assumptions** made. ( 10 marks)
- 75) **Develop the Euler equation** of motion and then **derive** the one dimensional form of Bernoulli's equation. ( 8 marks)
- 76) **Derive the energy equation** and state the **assumptions** made while deriving the equation. (12marks)
- 77) **Derive the Bernoulli's equation** from the first principles? State the **assumptions** made while deriving Bernoulli's equation. ( 8 marks)

78) A pipe containing water at  $180\text{KN/m}^2$  pressure is connected by a differential gauge to another pipe 1.6m lower than the first pipe and containing water at high pressure. If the difference in heights of 2 mercury columns of the gauge is equal to 90mm, what is the **pressure in the lower pipe?**

79) A 15 cm diameter **vertical pipe** is connected to 10 cm diameter vertical pipe with a reducing socket. The pipe carries a flow of 100 lit/sec. At point 1 in 15cm pipe gauge pressure is 250Kpa. At point 2 in the 10cm pipe located 1.0m below point 1 the gauge pressure is 175Kpa. i) find **whether the flow is upwards/downwards** ii) **head loss** between the two points.

80) A drainage **pipe is tapered** in a section running with full of water. The pipe diameters at inlet and exit are 1000mm and 500mm respectively. The water surface is 2m above the centre of inlet and exit is 3m above the free surface of the water. The pressure at the exit is 250mm of Hg vacuum. The friction loss between the inlet and exit of the pipe is  **$1/10$  of the velocity head** at the exit. Determine the **discharge** through the pipe. ( 16 marks)

81) The water is flowing through a **taper pipe** of length 100m having diameters 600mm at the upper end and 300mm at the lower end at the rate of 50litres/sec. The pipe has a **slope of 1 in 30**. Find the **pressure at the lower end** if the pressure at the higher level is  $19.62\text{N/cm}^2$ . ( 12marks)

82) Water is flowing through a tapering diameters pipe having 300mm and 150mm at sections 1 and 2 respectively. The discharge through the pipe is 40lit/s. The section 1 is 10m above datum and section2 is 6m above datum. Find the pressure at section2, if that at section 1 is  $400\text{KN/m}^2$ .

83) The water is flowing through a taper pipe having diameter 400mm at the bottom end and 250mm at the upper end. The intensity of pressure at the bottom and upper end are  $250\text{KN/m}^2$  and  $100\text{KN/m}^2$  respectively. Calculate the **difference in datum head**, if the rate of flow through pipe is 30lit/s.

84) Water flows at the rate of 200 litres per second upward through a tapered vertical pipe. The diameter at the bottom is 240mm and at the top 200mm and the length is 5m. The pressure at the bottom is 8 bar and the pressure at the topside is 7.3 bar. Determine the **head loss** through the pipe. Express it as a **function of exit velocity head**. (10 marks )

85) A pipe 200m long slopes down at **1 in 100** and **tapers** from 600mm diameter at the higher end to 300mm diameter at the lower end, and carries 100 litres/sec of oil having specific gravity 0.8 If the pressure gauge at the higher end reads  $60\text{KN/m}^2$ , determine the **velocities at the two ends and also the pressure at the lower end**. Neglect all losses. ( 16 marks )

## 8. PROBLEMS ON VENTURIMETER, ORIFICEMETER

- 86) Explain the principle of **orifice meter** with a neat diagram. Derive the expression for the rate of flow of liquid through it. ( 8 marks)
- 87) A 300 mm x 150 mm **Venturimeter** is provided in a vertical pipe line carrying oil of relative density 0.9, the flow being upwards. The differential U tube mercury manometer shows a gauge deflection of 250mm Calculate the **discharge** of oil if the coefficient of meter as 0.98. ( 8 marks)
- 88) An **Orificemeter** with orifice diameter 15cm is inserted in a pipe of 30cm diameter. The pressure of the upstream and downstream of orificemeter is  $14.7\text{N/cm}^2$  and  $9.81\text{N/cm}^2$ . Find the **discharge**, if  $C_d = 0.6$
- 89) A horizontal **venturimeter** with inlet diameter 200 mm and throat diameter 100 mm is employed to measure the flow of water. The readings of differential manometer connected to the inlet is 180mm of mercury. If  $C_d = 0.98$ , determine the **rate of flow**. (10 marks)
- 90) A pipe of 300mm diameter **inclined at  $30^\circ$**  to the horizontal is carrying gasoline ( specific gravity = 0.82) A **venturimeter** is fitted in the pipe to find out the flow rate whose throat diameter is 150mm. The throat is 1.2m from the entrance along its length. The pressure gauges fitted to the venturimeter read  $140\text{KN/m}^2$  and  $80\text{KN/m}^2$  respectively. Find out the **coefficient of discharge** of venturimeter if the flow is  $0.20\text{m}^3/\text{sec}$ . ( 16 marks )
- 91) A **venturimeter** having inlet and throat diameters 30cm and 15cm is fitted in a horizontal diesel pipe line ( Sp. Gr. = 0.92) to measure the discharge through the pipe. The venturimeter is connected to a mercury manometer. It was found that the discharge is 8 litres/sec. Find the **reading of mercury manometer** head in cm. Take  $C_d = 0.96$  . ( 8 marks)
- 92) A **Venturimeter** of 150mm X 75mm size is used to measure the flow rate of oil having specific gravity of 0.9. The reading shown by U tube manometer connected to the venturimeter is 150mm of mercury column. Calculate the **coefficient of discharge** for the venturimeter if the flow rate is  $1.7\text{m}^3/\text{min}$ . ( 16 marks)

## 9. PROBLEMS ON MOMENTUM EQUATION

- 93) Derive the **momentum equation** for steady flow. ( 12 marks)
- 94) Derive the linear **momentum equation** using the control volume approach and determine the force exerted by the fluid flowing through a pipe bend. ( 8 marks)
- 95) A pipe of 30cm diameter carrying  $0.25\text{m}^3/\text{s}$  water. The **pipe is bent by  $135^\circ$**  from the horizontal anti clockwise. The pressure of water flowing through the pipe is  $400\text{KN}$ . Find the magnitude and direction of the resultant force on the bend. ( 8 marks)

## UNIT II - FLOW THROUGH CIRCULAR CONDUITS

### PART A Questions

1) What is T.E.L?

**Ans:** If at different sections of the pipe, the total energy  $\{ z + p/w + v^2/2g \}$  is plotted to scale and joined by a line, the line is called as Total energy line.

2) What are Total Energy lines and Hydraulic gradient lines?

**Ans:** Total Energy line: If at different sections of the pipe, the total energy  $\{ z + p/w + v^2/2g \}$  is plotted to scale and joined by a line, the line is called as Total energy line.

Hydraulic gradient line: If at different sections of the pipe, the pressure heads  $\{ z + p/w \}$  is plotted to scale and joined by a line, the line is called as Hydraulic gradient line.

3) Distinguish between hydraulic and energy gradients.

**Ans:** Hydraulic gradient is the piezometric head which is the sum of potential head and datum head. Hydraulic gradient =  $\{ z + p/w \}$

Energy gradient is the sum of hydraulic gradient and kinetic head.

Energy gradient =  $\{ z + p/w + v^2/2g \}$

4) Mention the general characteristics of laminar flow.

**Ans:** i) There is shear stress between fluid layers ii) No slip at the boundary iii) Loss of energy is proportional to first power of velocity. iv) flow is rotational.

5) Write down the examples of laminar flow.

**Ans:** i) blood flow through capillaries ii) flow through pipes iii) flow of oil through thin tube.

6) What factors account for energy loss in laminar flow?

**Ans:** Frictional losses occur because of fluid viscosity and shear.

7) Define critical velocity.

**Ans:** Velocity at which the flow changes from laminar to turbulent for a given fluid at a given temperature and given pipe is known as critical velocity.

8) What is meant by transition state?

**Ans:** The state at which the flow changes from laminar to turbulent is known as transition state.

9) Define Reynolds number.

**Ans:** Reynolds number is a dimensionless number defined as the ratio of inertia force to the viscous force. Reynolds number  $R_e =$

Inertia force / Viscous force =  $\rho v d / \mu$  where,  $\rho$  – density,  $\mu$  – dynamic viscosity,  $d$  – diameter of pipe,  $v$  – velocity of flow.

10) Write down the values of Reynolds number for laminar, transition and turbulent flow.

**Ans:** Reynolds number  $Re$  less than 2000 - Laminar flow  
Reynolds number  $Re$  between 2000 and 4000 – Transition flow  
Reynolds number  $Re$  greater than 4000 – Turbulent flow

11) A circular and a square pipe of equal sectional area. For the same flow rate, determine which section will lead to a higher value of Reynolds number.

**Ans:** Area of square section = Area of circular section  
 $L^2 = \pi d^2 / 4$  where,  $L$  – side of square  $d$  – diameter of circular section  $L = 0.886d$   
For circular section,  $Re = \rho v d / \mu$  For square section  $Re = \rho v L / \mu$   
 $Re_{\text{square}} = 0.886 Re_{\text{circular}}$  hence circular section will lead to higher value of Reynolds number

12) Write down two examples of laminar flow.

**Ans:** i) blood flow through capillaries ii) flow through pipes iii) flow of oil through thin tube.

13) Write down Hagen – Poiseuille equation for laminar flow.

**Ans:**  $P_1 - P_2 = 32\mu UL / D^4 = 128\mu QL / \pi D^4$  where,  $P_1$  and  $P_2$  – pressure at inlet and outlet of pipe  
 $\mu$  – dynamic viscosity  $U$  – velocity,  $L$  – length of pipe,  $D$  - diameter of pipe  $Q$  – Discharge

14) What is boundary layer? Give a sketch of a boundary layer region over a flat plate.

**Ans:** The fluid layer in the vicinity of the solid boundary where the variation of velocity are predominant is known as the boundary layer.  
(For Sketch – Refer book)

15) What is Laminar boundary layer?

**Ans:** Near the surface of the leading edge of the plate, the thickness of boundary layer is small and the flow in the boundary layer is laminar though the main stream flow is turbulent. So the layer of the fluid is said to be laminar boundary layer.

16) What is turbulent boundary layer?

**Ans:** In boundary layer growth on flat plate, after Transition zone, the turbulent boundary layer starts and continues to grow in thickness along the downstream.



17) Define displacement thickness.

**Ans:** It is defined as the distance measured perpendicular to the boundary by which the mainstream is displaced to an account of formation of boundary layer. (write the formula also)

18) Define momentum thickness.

**Ans:** It is defined as the distance measured perpendicular to the boundary by which the boundary should be displaced to compensate for the reduction in momentum of flowing fluid on account of formation of boundary layer. (write the formula also)

19) Define Energy thickness.

**Ans:** It is defined as the distance measured perpendicular to the boundary by which the boundary should be displaced to compensate for the reduction of kinetic energy of flowing fluid on account of formation of boundary layer. (write the formula also)

20) Write the equation for displacement thickness and momentum thickness.

**Ans:** Displacement thickness  $\delta^* = \int_0^\delta (1 - \frac{u}{U}) dy$

Momentum thickness  $\theta = \int_0^\delta \frac{u}{U} (1 - \frac{u}{U}) dy$

21) Define the terms Drag and Lift.

**Ans:** The component of the total force in the direction of flow of fluid is known as drag.

The component of the total force in the direction perpendicular to the of flow of fluid is known as lift.

22) Give the expression for Drag coefficient and Lift coefficient.

**Ans:** Drag Coefficient  $C_D = F_D \times 2g / w a U^2$

Lift Coefficient  $C_L = F_L \times 2g / w a U^2$  where,  $w$  – specific weight,  $a$  – projected area perpendicular to the direction of flow,  $U$  – velocity of fluid,  $F_D$  – Drag force,  $F_L$  – lift force

23) What is meant by boundary layer separation?

**Ans:** The boundary layer is formed on the flat plate when it is held immersed in a flowing fluid. If the immersed plate is curved or angular, the boundary layer does not stick to the whole surface of the body. The boundary layer leaves the surface and gets separated from it. This phenomenon is known as boundary layer separation.

24) State the effect of boundary layer separation.

**Ans:** Separation of the boundary layer results in formation of eddies and wake zone of disturbed flow on the downstream which in turn causes continuous loss of energy.

25) Mention any two methods to prevent boundary layer separation.

**Ans:** i) Providing slots near the leading edge ii) streamlining of body shape

26) How does surface roughness affect the pressure drop in a pipe if the flow is turbulent?

**Ans:** The surface roughness affects the pressure drop in a pipe during turbulent in two ways such as i) constricting the flow area and ii) increasing the wall shear stress.

27) Differentiate between laminar and turbulent flow.

**Ans:**

S.No.	Laminar flow	Turbulent flow
1.	Fluid particles move in layers with one layer sliding smoothly over adjacent layer.	Fluid particles move in random and erratic manner.
2.	There is no eddies or vortices present	Eddies or vortices present
3.	Laminar flow occurs in liquids having high viscosity	Turbulent flow occurs in liquids having low viscosity
4.	This type of flow occurs in smooth pipes.	This type of flow occurs in rivers, canals, streams etc.

28) What is the use of Moody's diagram?

**Ans:** i) Moody's diagram for commercial pipe become a convenient and more reliable tool for solving practical problems in pipe. ii) This chart gives values of friction factor of any pipe provided its relative roughness and Reynolds number of flow are known.

29) What is Darcy's equation? Identify various terms in the equation.(or) Write the Darcy – Weishbach equation for loss of head in pipes due to friction.

**Ans:** Darcy – Weishbach equation for loss of head due to friction is  $h_f = 4fLv^2/2gd$

Where,  $f$  – friction coefficient,  $L$  – length of pipe,  $v$  – velocity of flow,  $d$  – diameter of pipe

30) Define i) Wetted perimeter ii) Hydraulic mean depth (or) Hydraulic Radius.

**Ans:** Wetted perimeter is the surface which is in contact with water. Hydraulic mean depth is defined as ratio of area of flow to the wetted perimeter.

31) What is the relation between Darcy friction factor, Fanning friction factor and friction coefficient?

**Ans:** Darcy friction factor is 4 times larger than the Fanning friction factor.

32) What do you understand by hydraulic diameter.

**Ans:** Hydraulic diameter is the effective diameter of the wetted surface. It is the ratio of wetted area to the wetted perimeter.

33) Classify the losses in pipes.

**Ans:** Loss of energy mainly classified into 2 types. i) Major losses ii) Minor losses.

34) What are the losses experienced by a fluid when it is passing through a pipe? ( or) List the causes of minor energy losses in flow through pipes.

**Ans:** i) Major loss due to friction and viscosity ii) Minor losses due to a) sudden enlargement b) sudden contraction c) exit from the pipe d) entrance to a pipe e) bend f) various pipe fittings g) gradual contraction or enlargement h) obstruction in a pipe.

35) What are minor losses? Under what circumstances will they be negligible?

**Ans:** Loss of energy caused on account of the change in velocity of flowing fluid is called minor losses. In case of long pipes, these losses are very small as compared to major loss and hence they can be neglected.

36) Write the equation of loss of energy due to sudden enlargement.

**Ans:** loss of head due to sudden enlargement  $h_e = (V_1 - V_2)^2 / 2g$   
Where,  $V_1$ -velocity at section 1-1,  
 $V_2$ -velocity at section 2-2

37) Find the loss of head when a pipe of diameter 200mm is suddenly enlarged to a diameter of 400mm. Rate of flow of water through the pipe is 250lit/s.

**Ans:**  $Q = 250 \text{ lit/s} = 0.25 \text{ m}^3/\text{s}$   $d_1 = 0.2 \text{ m}$ ,  $d_2 = 0.4 \text{ m}$   $V_1 = (0.25 \times 4) / \pi \times 0.2^2 = 7.958 \text{ m/s}$   
 $V_2 = (0.25 \times 4) / \pi \times 0.4^2 = 1.989 \text{ m/s}$   $h_e = (V_1 - V_2)^2 / 2g = (7.958 - 1.989)^2 / 2 \times 9.81 = 1.816 \text{ m}$

38) Write the expression of loss of energy due to sudden contraction.

**Ans:**  $h_c = 0.5V_2^2/2g$  Where,  $V_2$ -velocity at section 2-2

39) Write the expression for energy loss due to entrance and exit of the pipe.

**Ans:** Loss of head at the entrance  $h_i = 0.5V^2/2g$   
Loss of head at the exit  $h_o = V^2/2g$

40) Write the formula for loss of energy due to gradual enlargement and also bend in pipe.

**Ans:** Loss of energy due to gradual enlargement  $h_L = K(V_1^2 - V_2^2) / 2g$

Loss of energy due to bend in pipe  $h_b = K(V^2) / 2g$  where, K – coefficient which depends on angle of convergence.

41) What are pipes in series?

**Ans:** It is defined as the pipes of different diameters and lengths are connected with one another to form a single pipe line.

42) What are pipes in parallel? ( or) What do you mean by flow through parallel pipes?

**Ans:** When a main pipe line divides into two or more parallel pipes, which again join together to form a single pipe and continue as a main line, the pipes are said to be in parallel.

43) What is equivalent pipe?

**Ans:** A compound pipe consisting of several pipes of varying diameters and length may be replaced by a pipe of uniform diameter, which is known as equivalent pipe.

44) A piping system involves two pipes of different diameters ( but of identical length, material and roughness) connected in parallel. How would you compare the flow rates and pressure drops in these two pipes?

**Ans:** The purpose of using parallel pipes is to increase the discharge. For this arrangement, flow rate  $Q = Q_1 + Q_2$  Head loss through each branch is same.  $h_f = h_{f1} = h_{f2}$ .

45) What are the factors influencing the frictional loss in pipe flow?

- Ans:** Frictional resistance for the turbulent flow is
- Proportional to  $vn$  where  $v$  varies from 1.5 to 2.0.
  - Proportional to the density of fluid.
  - Proportional to the area of surface in contact.
  - Independent of pressure.
  - Depend on the nature of the surface in contact.

46) What is Dupit's equation?

**Ans:**  $L_1/d_1^5 + L_2/d_2^5 + L_3/d_3^5 = L / d^5$

Where

$L_1, d_1$  = Length and diameter of the pipe 1,  $L_2, d_2$  = Length and diameter of the pipe 2

$L_3, d_3$  = Length and diameter of the pipe 3

## **PART – B**

1) A pipe 30cm in diameter and 3200m long is used to pump up 50Kg per second of an oil whose density is 950Kg/m<sup>3</sup> and whose kinematic viscosity is 2.1 stokes. The centre of the pipe line at the

upper end is 40m above than at the lower end. The discharge at the upper end is atmospheric. Find the **pressure at the lower end** and draw **Hydraulic gradient and Total energy line**. ( 8 marks )

2) A horizontal pipe of 250mm diameter and 60m long is connected to a water tank at one end and discharges freely to atmosphere through the other end. If the height of water in the tank is 4.5m above the centre of the pipe, **calculate the rate of flow** of water. Consider all losses and take  $f = 0.008$ . Also draw the **HGL and TEL**.

3) Two reservoirs are connected by a horizontal pipe of diameter 200mm and length 270m. If the rate of flow of water through the pipe is  $0.5\text{m}^3/\text{sec}$ . **Calculate the difference in elevation** between the water surfaces of the reservoirs. Consider all losses. Take  $f = 0.007$ . Also draw **HGL and TEL**.

4) A horizontal pipeline 50m is connected to a water tank at one end and discharge freely to atmosphere through the other end. For the first 30m length from tank, the diameter of pipe is 15cm and for rest it is 30cm in diameter. The water level in the tank is 8m above the centre of the pipe. Take  $f = 0.01$ . By considering all losses, **determine the discharge through the pipe**. Also draw the **HGL and TEL**.

5) Derive an **expression for the velocity distribution for viscous flow** through a circular pipe. ( 8 marks )

(or)

Derive an **expression for the velocity distribution for viscous flow through a circular pipe**. Also sketch the distribution of velocity across a section of the pipe. ( 8 marks )

(or)

Considering **laminar flow through a circular pipe**, obtain an expression for the velocity distribution. ( 8 marks )

(or)

For a flow of viscous fluid flowing through a circular pipe under **laminar flow** conditions show that the **velocity distribution is a parabola**. And also show that the **average velocity is half of the maximum velocity**. ( 16 marks )

6) Derive **Hagen – Poiseuille equation** and state the **assumptions** made. ( 16 marks )

( or )

Obtain an **expression for Hagen poiseuille flow**. Deduce the **condition for maximum velocity**. ( 16 marks )

(or)

Give a proof of **Hagen – Poiseuille's equation** for fully developed laminar flow in a pipe and hence show that **Darcy friction coefficient is equal to  $16/Re$** , Where  $Re$  is Reynolds number. ( 16 marks )

7) A **laminar flow** is taking place in a **pipe of diameter 20cm**. The maximum velocity is  $1.5\text{m/s}$ . Find the **mean velocity** and radius at which it occurs. Also **calculate the velocity** at 4cm from the wall of pipe. ( 7 marks )

8) Determine the a) pressure gradient b) the shear stress at the **two horizontal parallel plates** and c) discharge per meter width for the **laminar flow** of oil with maximum velocity 2m/s between two horizontal parallel fixed plates which are 10cm apart. Given  $\mu = 2.4525\text{N-s/m}^2$ .

( 8 marks )

9) A lubricating oil flows in a 10cm diameter pipe at 1m/s. Determine whether the flow is **laminar or turbulent**. For the lubricating oil,  $\mu = 0.1\text{N-s/m}^2$  and  $\rho = 930\text{Kg/m}^3$ . Calculate also **transition and turbulent velocities**. ( 16 marks )

10) Lubricating oil at a velocity of 1m/s ( average ) flows through a pipe of 100mm inlet diameter. Determine whether the flow is **laminar or turbulent**. Also determine the **friction factor and the pressure drop** over 10m length. What should be the velocity for the flow to turn turbulent? Density =  $930\text{Kg/m}^3$ , dynamic viscosity  $\mu = 0.1\text{N-s/m}^2$ . ( 16 marks )

11) i) A pipe line 20cm in diameter, 70m long, conveys oil of specific gravity 0.95 and viscosity  $0.23\text{N-s/m}^2$ . If the velocity of oil is  $1.38\text{m/s}$ , find the **difference in pressure between the two ends** of the pipe. ( 8 marks )

ii) Oil of mass density  $800\text{Kg/m}^3$  and dynamic viscosity 0.02 poise flows through 50mm diameter pipe of length 500m at the rate of 0.19lit/s. Determine 1) **Reynolds number** of flow 2) centre line velocity 3) **Pressure gradient** 4) **Loss of pressure** in 500m length 5) Wall shear stress and 6) **Power required** to maintain the flow. ( 8 marks )

12) Consider flow of oil through a pipe of 0.3m diameter. The **velocity of distribution is parabolic** with a maximum velocity of 3m/s at the pipe centre. Estimate the **shear stresses at the pipe wall** and within the fluid 50mm from the pipe wall. The viscosity of the oil is  $1.7\text{Pa-s}$ . ( 8 marks )

13) Two tanks of fluid (  $\rho = 998\text{Kg/m}^3$  and  $\mu = 0.001\text{Kg/m-s}$  ) at  $20^\circ\text{C}$  are connected by a capillary tube 4mm in diameter and 3.5m long. The surface of tank 1 is 30cm higher than the surface of tank 2. Estimate the flow rate in  $\text{m}^3/\text{hr}$ . Is the flow **laminar** ? For what tube diameter will **Reynolds number** be 500? ( 10 marks )

14) Oil with a density of  $900\text{Kg/m}^3$  and kinematic viscosity of  $6.2 \times 10^{-4}\text{m}^2/\text{s}$  is being discharged by a 6mm diameter, 40m long horizontal pipe from a storage tank open to the atmosphere. The height of the liquid level above the centre of the pipe is 3m. Neglecting the minor losses, determine the **rate of flow** through the pipe.

15) Lubricating oil of specific gravity 0.84 and dynamic viscosity  $0.137\text{N-s/m}^2$  is pumped at a rate of  $0.024\text{m}^3/\text{s}$  through a 0.17m **diameter** 400mm long **horizontal pipe**. Calculate the **pressure drop**, average shear stress at the wall of the pipe and the **power required** to maintain flow.

16) Oil of viscosity  $0.183\text{N-s/m}^2$  and specific gravity 0.87 flows through a horizontal pipe of 35mm diameter. If the **pressure drop**

per metre length of the pipe is  $14\text{KN/m}^2$ , determine a) The rate of flow b) The shear stress at the pipe wall c) shear stress within the fluid 10mm from pipe centre d) The **Reynolds number** of flow and e) The **power required** per 50m length of pipe to maintain the flow.

17) A horizontal pipe of 55mm diameter was used in the laboratory to measure the viscosity of the oil of specific weight  $8750\text{N/m}^3$ . During the test run, a **pressure difference** of 17KPa was recorded from two pressure gauges located 8m apart on the pipe. The oil was allowed to discharge into a weighing tank and **6000N of oil was collected in 3.5minutes** duration. Find the **dynamic viscosity** of the oil.

18) Oil of specific gravity 0.85 pumped under a steady mean flow rate of 750lit/min through 80mm diameter and 500m long pipe. The **pump requires 4.25KW to maintain** the oil flow with the **efficiency of 72%**. Calculate the **dynamic viscosity** of the oil.

19) Oil is pumped along a horizontal 10cm diameter pipe, 220m long. The specific gravity of the oil is 0.88 and its kinematic viscosity is 1.4 stokes. **Flow is laminar** so that the friction factor for the pipe will be  $64 \text{ Re}^{-1}$  in which Re is the Reynolds number. It takes 17KW to drive the pump which has an **overall efficiency of 65%**. Find the **quantity of oil** flowing through the pump in litres per minute.

20) An oil of specific weight  $8750\text{N/m}^3$  flows under a head of 4.6m through 2500m pipe of 260mm diameter. Due to cooling, the viscosity changes along the length and may be taken as 0.17 poise over the first 1500m and 0.32 poise over the second 1000m. Determine the **flow rate of oil** in  $\text{m}^3/\text{sec}$ .

21) A pipe line of 250mm diameter and 3000m long is used to pump 65lit/s of an oil whose specific gravity is 0.92 and kinematic viscosity is 2 stokes. The centre of the pipe line at the upper end is 35m above that at the lower end. Find **difference of pressure** at the ends.

22) Discuss in detail about the **boundary layer thickness and separation of boundary layer**. ( 8 marks )

23) Explain the **concept of boundary layer** in pipes for both **laminar and turbulent** flow with neat sketches. ( 6 marks )

24) A smooth flat plate with a sharp **leading edge** is placed along a free stream of water flowing at 3m/s. Calculate the distance from the leading edge and the **boundary thickness** where the **transition from laminar to turbulent** flow may commence. Assume the density of water as  $1000\text{Kg/m}^3$  and viscosity as 1 centipoise. ( 16 marks )

25) A smooth **rectangular plate** of 6m long, 4m wide is kept immersed in water which moves with a velocity of 0.6m/s. Calculate the **thickness of boundary layer** at a distance of 2.0m from the leading edge. Take kinematic viscosity of water as  $1.1 \times 10^{-6}\text{m}^2/\text{s}$ .

26) A stream lined train has 200m long 3m wide on the top surface. Find the **thickness of boundary layer** at a distance of 30m from the **leading edge** when the train is running at 75Km/hr. Take kinematic

viscosity of air as  $1.6 \times 10^{-5} \text{ m}^2/\text{s}$ . Also find the **thickness of boundary layer at the trailing edge**.

27) The **velocity distribution in the boundary layer** is given by  $(u/U) = (y/\delta)$  where  $u$  = velocity at a distance  $y$  from the flat plate and  $u = U$  at  $y = \delta$ ,  $\delta$  = boundary layer thickness.

Determine the value of i) The **displacement thickness** ii) The **momentum thickness** iii) The **energy thickness** and iv)  $\delta^*/\theta$ .

28) The velocity distribution in laminar boundary layer is given by  $(u/U) = 3(y/\delta) - 2(y/\delta)^2$  where  $u$  = velocity at distance  $y$  from the boundary  $U$  = velocity at a distance  $\delta$ , the thickness of the boundary layer, Calculate i) The **ratio of displacement thickness to boundary layer thickness** ( $\delta^*/\delta$ ) ii) The **ratio of momentum thickness to boundary layer thickness** ( $\theta/\delta$ ).

29) Determine the **displacement thickness, momentum thickness and energy thickness** in terms of boundary layer thickness  $\delta$  for the velocity profile in the boundary layer on a flat plate given by  $(u/U) = 2(y/\delta) - (y/\delta)^2$  where  $u$  is the velocity at a height  $y$  above the surface and  $U$  is the main stream velocity.

30) For the velocity profile in **laminar boundary layer** as  $(u/U) = 2(y/\delta) - 2(y/\delta)^3 + (y/\delta)^4$ . Find the thickness of the boundary layer and shear stress **at the trailing edge of a plate**. The plate is 2m long and 1.8m wide is placed in water which is moving with a velocity of 20cm/sec. Find the drag on one side of the plate if the viscosity of water is  $0.001 \text{ N-s/m}^2$ .

31) A plate of 600mm length and 400mm wide is immersed in a fluid of specific gravity 0.9 and kinematic viscosity of  $10^{-4} \text{ m}^2/\text{s}$ . the fluid is moving with the velocity of 6m/s. Determine i) **Boundary layer thickness** 2) Shear stress at the end of the plate 3) **Drag force** on one of the sides of the plate. ( 10 marks )

32) Compare the rate of growth of the **laminar boundary layer** over a smooth **flat plate** in the following cases : i) Flat plate placed in a **water stream** flowing at 2m/s ii) Flat plate in an **air stream** flowing at 2m/s iii) Flat plate placed in an air stream flowing at 8m/s. Given that the densities of water and air are 1000 and  $1.2 \text{ Kg/m}^3$  respectively and the viscosities of water and air are 0.001 and  $0.000019 \text{ N-s/m}^2$  respectively.

33) For the velocity profile in **laminar boundary layer** as  $(u/U) = (3/2)(y/\delta) - (1/2)(y/\delta)^3$  Find the thickness of the boundary layer and shear stress, **1.8 m from the leading edge of a plate**. The plate is 2.5m long and 1.5m wide is placed in water, which is moving with a velocity of 15cm/s. Find the **drag on one side** of the plate, if the viscosity of water is 0.01 poise.

34) A flat plate 1.5m x 1.5m moves at 50Km/hr in a stationary air of density  $1.15 \text{ Kg/m}^3$ . If the **coefficient of drag and lift** are 0.15 and 0.75 respectively, determine i) Lift force ii) Drag force iii) the resultant force iv) power required to set the plane in motion. ( 16 marks )



35) A jet plane which weighs 29430N and has a wing area of 20m<sup>2</sup> flies at a velocity of 250Km/hr. when the engine delivers 7357.5KW. 65% of power is used to overcome the drag resistance of the wing. Calculate the **coefficient of lift and coefficient of drag** for the wing. Take density of air equal to 1.21Kg/m<sup>3</sup>. ( 8marks )

36) Derive the **Darcy – Weishbach equation** for calculating **loss of head due to friction** in a pipe. ( 8 marks )

(or)

Obtain expression for **Darcy – Weishbach friction factor** for flow in a pipe. ( 6 marks )

37) Derive **Chezy's formula for loss of head due to friction** in pipes. ( 6 marks )

38) An oil of specific gravity 0.7 is flowing through a pipe of diameter 30cm at the rate of 500 litres/s. Find the **head lost due to friction and power required to maintain the flow** for a length of 1000m. Take kinematic viscosity as 0.29 stokes. ( 8 marks )

39) A **pipe of 12 cm diameter** is carrying an oil (  $\mu = 2.2 \text{ Pa-s}$  and  $\rho = 1250 \text{ Kg/m}^3$  ) with a velocity of 4.5m/s. Determine the shear stress at the wall surface of the pipe, **head loss** if the length of the pipe is 25m and the **power lost**. ( 8 marks )

40) A smooth pipe carries 0.30m<sup>3</sup>/s of water discharge with a **head loss** of 3.0m per 100m length of pipe. If the water temperature is 20°C, determine the **diameter of the pipe**. ( 10 marks )

41) Calculate the power required to maintain 0.06m<sup>3</sup>/s of oil ( specific gravity 0.75) through a steel pipe 300mm diameter and 1350m long. Take coefficient of friction  $f = 0.04$  in the **darcy relation**.

42) A pipeline 0.4m in diameter and 1250m long has a **slope of 1 in 150** for the first 750m and **1 in 100** for the next 500m. The pressure at the upper end of the pipeline is 120KPa and at the lower end is 70 KPa. Taking  $f = 0.025$ , determine the **discharge** through the pipe.

43) A smooth pipe conveys 7.5lit/sec of water with a **head loss** of 80mm per 10m length. Viscosity of water is  $10^{-6} \text{ m}^2/\text{s}$ . friction factor in the **Darcy's equation** is given by  $f = 0.316 / \text{Re}^{0.25}$  Determine the diameter of the pipe.

44) Water is flowing through a pipe of diameter 200mm with a velocity of 3.5m/s. If the **coefficient of friction** is given by  $f = 0.008 + ( 0.07 / \text{Re}^{0.25} )$  where Re – Reynolds number, Find the **head loss due to friction** for a length of 6.5m, Take  $\gamma = 0.014 \times 10^{-4} \text{ m}^2/\text{s}$ .

45) An Engineering college having 1200 students is to be supplied with water from a reservoir 12Km away. Water is to be supplied at the rate of 50litres per head per day and half of the daily supply is pumped in 8 hrs. If the **head loss due to friction** is 55m, find the diameter of the pipe. Take  $f = 0.004$ .

46) i) A pipe line 10km, long delivers a power of 50KW at its outlet ends. The pressure at inlet is 5000KN/m<sup>2</sup> and pressure drop per Km

of pipeline is  $50 \text{ kN/m}^2$ . Find the **size of the pipe and efficiency of transmission**. Take  $4f = 0.02$ . ( 12 marks )

ii) The velocity of water in a pipe 200mm diameter is  $5 \text{ m/s}$ . The length of the pipe is 500m. Find the **loss of head due to friction**, if  $f = 0.008$ . ( 4 marks )

47) A 2500m long pipeline is used for transmission of power. 120KW power is to be transmitted through the pipe in which water having a pressure of  $4000 \text{ kN/m}^2$  at inlet is flowing. If the pressure drop over the length of pipe is  $800 \text{ kN/m}^2$  and  $f = 0.006$ . Find i) **diameter of pipe** ii) **Efficiency of transmission**. ( 10 marks )

48) A **power transmission pipe** 10cm diameter and 500m long is fitted with a nozzle at the exit, the inlet is from a river with water level 60m above the discharge nozzle. Assume  $f = 0.02$ , calculate the **maximum power** which can be transmitted and the **diameter of nozzle** required.

49) Find the **maximum power** available at the end of a pipeline of 3Km long and 250mm diameter, When the total **head at the inlet** of the pipe is 200m. Assume  $f = 0.007$ .

50) Derive the **expression for the loss of head at sudden expansion** in pipe flow. ( 16 marks )

51) Explain the losses of energy in flow through pipes. ( 16 marks )

52) A pipeline carrying oil of specific gravity 0.85, **changes in diameter from 350mm at position 1 to 550mm diameter to a position 2**, which is at 6m at a higher level. If the pressure at position 1 and 2 are taken as  $20 \text{ N/cm}^2$  and  $15 \text{ N/cm}^2$  respectively and discharge through the pipe is  $0.2 \text{ m}^3/\text{sec}$ . Determine **the loss of head and direction of flow**. ( 8 marks )

53) The rate of flow of water through a horizontal pipe is  $0.25 \text{ m}^3/\text{s}$ . The **diameter of the pipe which is 20cm is enlarged to 40cm**. The pressure intensity in the smallest pipe is  $11.772 \text{ N/cm}^2$ . Determine a) loss of head due to sudden enlargement b) pressure intensity in larger pipe c) power loss due to enlargement. ( 9 marks )

54) A horizontal pipeline 40m long is connected to a water tank at one end and discharges freely into the atmosphere at the other end. For the first 25m of its length from the tank, the pipe is 150mm diameter and its diameter is **suddenly enlarged** to 300mm. The height of water level in the tank is 8m above the centre of the pipe. Considering all **losses of head** which occur, determine the rate of flow. Take  $f = 0.01$  for both sections of the pipe. ( 16 marks )

55) The rate of flow of water through a horizontal pipe is  $0.3 \text{ m}^3/\text{s}$ . The diameter of the pipe is **suddenly enlarged** from 25cm to 50 cm. The pressure intensity in the smaller pipe is  $14 \text{ N/m}^2$ . Determine the i) **loss of head due to sudden enlargement** ii) pressure intensity in the large pipe and iii) **power loss due to enlargement**.

56) Oil of specific gravity 0.85 flows in a 120mm diameter pipeline. A **sudden expansion** takes place into a second pipeline of such a diameter that maximum pressure rise is obtained. If the oil flow rate

in the pipeline is 22lit/s, find the **loss of head due to sudden expansion**.

57) A horizontal pipe of 400mm diameter is **suddenly contracted** to a diameter of 200mm. The pressure intensities in the large and small pipe are given as  $15\text{N/cm}^2$  and  $10\text{N/cm}^2$  respectively. Find the **loss of head due to contraction**, if  $C_c = 0.62$ , determine also the **rate of flow** of water.

58) A 150 mm diameter **pipe reduces in diameter** abruptly to 100mm diameter. If the pipe carries water at 30 litres /second, calculate the **pressure loss across the contraction**. Take coefficient of contraction as 0.6. ( 16 marks )

59) A horizontal pipe of diameter 300mm is attached to another pipe of diameter 250mm by means of flange. The rate of flow of water is  $0.5\text{m}^3/\text{s}$ . The pressure intensities in the large and small pipe are given as  $14.32\text{N/cm}^2$  and  $11.58\text{N/cm}^2$ .respectively. Calculate the **loss of head and the coefficient of contraction**.

60) A pipe of **18cm diameter is attached to a 12cm** diameter pipe by means of a coupling. The pressure loss at the transition is indicated by differential gauge length on a water – mercury manometer connected between the two pipes equals 75mm. Find the loss of head due to contraction, if  $C_c = 0.73$ , Also calculate the **rate of flow**.

61) A **sudden enlargement** is introduced in a horizontal pipeline from **175mm diameter to 300mm diameter**. Measurements indicate that when flow is from smaller to larger cross- section, the **head loss** is 0.6m in excess of that when the flow takes place from **larger to smaller** section. Determine the flow rate. Take the **coefficient of contraction** as 0.62.

62) When a sudden contraction is introduced in a horizontal pipeline from 400mm to 220mm, the pressure changes from 101KPa to 65KPa. Calculate the **rate of flow**. Take coefficient of contraction as 0.63. Following this there is a **sudden enlargement** from 220 mm to 400mm . If the pressure at the 220mm section is 65KPa, Calculate the **pressure at the 400mm enlarged section**.

63) Water is flowing **vertically downwards** through a 120mm diameter pipe at the rate of 70lit/s. At a certain location, another pipe of 200mm diameter is connected by a flange. A point X is located 650mm above the section of enlargement and another point Y is located 700mm below it in the **enlarged portion**. A pressure gauge connected at X gives a reading of 2.5 bar. Calculate the pressure at the location Y neglecting friction losses but considering the loss due to **sudden enlargement**. What will be the pressure at Y when the **flow is reversed** and pressure at X remains same? Take  $C_c = 0.61$ .

64) A pipe 75mm diameter is 5m long and the velocity of flow of water in the pipe is 2.8m/s. If the central 2m length of pipe was replaced by 100mm diameter pipe and the **change of section being sudden**, calculate the **loss of head** and the corresponding power saving. Take  $f = 0.038$  for the pipes of both diameters.

- 65) Water is flowing through a horizontal pipe of diameter 300mm at a velocity of 3.2m/s. Find the **loss of head** in the pipe, when a circular solid plate of diameter 200mm is placed in the pipe to **obstruct** the flow. assume  $C_c = 0.63$ .
- 66) Determine the **equivalent pipe corresponding to 3 pipes in series** with lengths and diameters  $l_1, l_2, l_3, d_1, d_2$  and  $d_3$  respectively. ( 16 marks )
- 67) What is meant by **equivalent pipe**? ( 6 marks )
- 68) Two reservoirs are connected by a pipe line consisting of **two pipes in series**. One pipe is of 15cm diameter and 6m long and another pipe of 22.5cm diameter and 15m long. If the difference in water level of the two reservoirs is 6m, calculate the **discharge by considering all losses**. Assume  $f = 0.02$  for both pipes.
- 69) Two reservoirs with a difference in water surface elevation of 15m are connected by a pipeline ABCD that consists of three **pipes** AB, BC and CD joined **in series**. Pipe AB is 10cm in diameter, 20m long and has a value of  $f = 0.02$  Pipe BC is of 16cm diameter, 25m long and has  $f = 0.018$ . Pipe CD is of 12cm diameter, 15m long and has  $f = 0.02$ . The junctions with the reservoirs and between the pipes are abrupt. a) Calculate the discharge b) What difference in reservoir elevation is necessary to have a discharge of 20lit/s **including all minor losses**.
- 70) **Three pipes** of diameters 300mm, 200mm and 400mm and lengths 450m, 255m and 315m respectively are **connected in series**. The difference in water surface levels in two tanks is 18m. determine the rate of flow of water if coefficients of frictions are 0.0075, 0.0078 and 0.0072 respectively **considering the minor losses and by neglecting minor losses**. ( 8 marks )
- 71) A 30cm diameter **pipe** of length 30cm is **connected in series** to a 20cm diameter pipe of length 20cm to convey discharge. Determine the **equivalent length of pipe** of diameter 25cm, assuming that the friction factor remains the same and the minor losses are negligible.
- 72) A main **pipe divides into two parallel pipes, which again forms one pipe**. The length and diameter for the first parallel pipe are 2000m and 1m respectively, while the length and diameter of second parallel pipe are 2000m and 0.8m respectively. Find the rate of flow in each parallel pipe, if total flow in the main is  $3\text{m}^3/\text{s}$ . The coefficient of friction for each parallel pipe is same and equal to 0.005.( 8 marks )
- 73) For a town water supply, a main pipe line of diameter 0.4m is required. As pipes more than 0.35m diameter are not readily available, **two parallel pipes** of same diameter are used for water supply. If the total discharge in the parallel pipes is same as in the **single main pipe**, find the diameter of parallel pipe, Assume coefficient of discharge to be same for all the pipes. ( 16 marks )

74) A pipe line of 0.6m diameter is 1.5Km long. To increase the discharge, another line of same diameter is introduced **parallel** to the first in second half of the length. Neglecting minor losses, find the **increase in discharge**, if  $4f = 0.04$ . The head at inlet is 30cm. ( 8 marks )

75) A pipeline conveys 10lit/s of water from an overhead tank to a building. The pipe is 2 Km long and 15 cm diameter, the friction factor is 0.03. It is planned to increase the discharge by 30% by installing another **pipeline in parallel** with this over half the length. Find the **suitable diameter of the pipe** to be installed. Is there any upper limit on **discharge augmentation** by this arrangement? ( 16 marks )

76) A pipe of 60cm diameter is carrying water for a length of 1.5Km. In order to **augment the discharge**, another line of same diameter is introduced **parallel to the first** in the second half of the length. **Neglecting minor losses**, find the increase in discharge. The head at inlet is 30m. Take  $f = 0.02$ .

77) **Three pipes** of same length  $L$ , diameter  $D$  and friction factor  $f$  are **connected in parallel**. Determine the diameter of the pipe of length  $L$  and friction factor  $f$  that will carry the same discharge for the same head loss. Use  $h_f = fL v^2 / 2gD$ .

78) Two sharp ended pipes of diameter 50mm and 100mm respectively, each of length 100m are **connected in parallel** between two reservoirs, which have a difference of level of 10m. If the coefficient of friction for each pipe is  $4f = 0.32$ , calculate the **rate of flow for each pipe** and also the diameter of a single pipe 100m long which would give the same discharge, if were substituted for the original two pipes.

79) Two reservoirs 10Km apart have a difference of water level of 25m and are connected together by a pipeline. The pipeline consists of a 6Km single pipe feeding a junction from which **two pipes run in parallel** for the remaining length of 4Km to the lower reservoir. If the same size of pipe be used throughout, what should be the diameter of the pipe so that the velocity may not exceed 1.25m/s? Take friction factor  $4f = 0.03$ .

80) A pumping plant forces water through a 500mm diameter main, the friction head being 32m. In order to **reduce the power consumption**, it is prepared to lay another main of appropriate diameter along the side of the existing one, so that the **two pipes may work in parallel** for the entire length and reduce the friction head to 12m only. Find the diameter of the new main, if with the exception of diameter; it is similar to the existing one in every respect.

81) **Two pipes** of 15cm and 30cm diameters are laid **in parallel** to pass a total discharge of 100litres/second. Each pipe is 250m long. Determine discharge through each pipe. Now these **pipes are connected in series** to connect two tanks 500m apart, to carry same total discharge. Determine water level difference between the tanks.

Neglect minor losses in both cases.  $f = 0.02f_n$  for both pipes. ( 8 marks )

82) **Two pipes** of diameter 400mm and 200mm are each 300m long, where the pipes are **connected in series**. The discharge through the pipe line is  $0.10\text{m}^3/\text{s}$ . find the loss of head. What would be the **loss of head** in the system to pass the same total discharge when the **pipes are connected in parallel**? Assume Darcy's friction factor as 0.0075. ( 16 marks )

83) Two pipes each 400m long are available for connecting to a reservoir from which a flow of  $0.1\text{m}^3/\text{s}$  is required. If the diameters of the two pipes are 0.4m and 0.25m respectively. Determine the ratio of the **head lost** when the pipes are **connected in series** to the **head lost** when they are **connected in parallel**. Neglect minor losses.

### UNIT III - DIMENSIONAL ANALYSIS

#### PART A Questions

1) What do you understand by fundamental units and derived units?(or) Differentiate between fundamental units and derived units. Give examples.

**Ans:** The physical parameters which are independent of each other are called fundamental or primary parameters. Ex: Mass, Length, Time. The parameters which are expressed in terms of primary quantities are called secondary quantities. Ex: force, velocity, power etc.

2) Give the dimension of the following physical quantities: a) pressure b) surface tension c) dynamic viscosity d) kinematic viscosity.

**Ans:** Pressure :  $\text{ML}^{-1}\text{T}^{-2}$  Surface tension:  $\text{MT}^{-2}$  Dynamic Viscosity:  $\text{ML}^{-1}\text{T}^{-1}$  Kinematic Viscosity:  $\text{L}^2\text{T}^{-1}$

3) The excess pressure  $\Delta p$  inside a bubble is known to be a function of the surface tension and the radius. By dimensional reasoning determine how the excess pressure will vary if we double the surface tension and radius.

**Ans:** Excess pressure  $\Delta p$  is a function of surface tension  $\sigma$  and radius  $r$ .

Mathematically  $\Delta p = f(\sigma, r)$   $\Delta p = C(\sigma^a, r^b)$  where, C is a constant. Using MLT system, corresponding dimensions are  $\text{ML}^{-1}\text{T}^{-2} = \text{C}[(\text{MT}^{-2})^a (\text{L})^b]$

For dimensional homogeneity, the exponents of each dimensions on both sides of the equation must be identical. For M:  $1 = a$ , for L:  $-1 = b$ , for T:  $-2 = -2a$

$a = 1, b = -1$   $\Delta p = C(\sigma^1, r^{-1}) = C \sigma / r$  Therefore if the surface tension and radius of bubble are doubled, the excess pressure will remain the same.

4) Determine the dynamic pressure exerted by a flowing incompressible fluid on an immersed object, assuming the pressure is a function of the density and velocity.

**Ans:** pressure  $p$  is a function of density  $\rho$  and velocity  $v$ .

Mathematically  $p = f(\rho, v)$   $p = C(\rho^a, v^b)$  where,  $C$  is a constant.

Using MLT system, corresponding dimensions are  $ML^{-1}T^{-2} = C[(ML^{-3})^a (LT^{-1})^b]$

For dimensional homogeneity, the exponents of each dimensions on both sides of the equation must be identical. For  $M$ :  $1 = a$ , for  $L$ :  $-1 = -3a + b$ , for  $T$ :  $-2 = -b$

$a = 1, b = 2$   $p = C(\rho^1, v^2) = C \rho v^2$

5) State the fourier law of dimensional homogeneity.

**Ans:** Fourier law of dimensional homogeneity states that "an equation which expresses a physical phenomenon of fluid flow should be algebraically correct and dimensionally homogenous".

6) What is a dimensionally homogeneous equation? Give example.(or) Explain the term dimensional homogeneity.

**Ans:** Dimensionally homogenous equation means, the dimensions of the terms on left hand side should be same as the dimensions of the terms on right hand side. Ex:  $Q = C_d a (2gh)^{1/2}$

7) Check whether the following equation is dimensionally homogeneous.  $Q = C_d a (2gh)^{1/2}$

**Ans:** Dimensions for discharge  $Q$ :  $L^3T^{-1}$ , area  $a$ :  $L^2$ , gravity  $g$ :  $LT^{-2}$ , head  $h$ :  $L$

$L.H.S = L^3T^{-1}$   $R.H.S = C_d L^2 (2LT^{-2} L)^{1/2} = 2 C_d L^{2+1/2 + 1/2} T^{-2 \times 1/2} = L^3T^{-1}$  hence  $L.H.S = R.H.S$

8) What are the uses of dimensional homogeneity?

**Ans:** i) To check the dimensional homogeneity of the given equation. ii) To determine the dimension of a physical variable. iii) To convert units from one system to another through dimensional homogeneity. iv) it is a step towards dimensional analysis.

9) Give examples for dimensionally homogeneous and non homogeneous equation.

**Ans:** Examples for dimensionally homogeneous equations: i) Head loss in flow through pipes ii) Reynolds number iii) Bernoulli's equation Examples for non homogeneous equations: i) Equation of simple harmonic motion ii) second order linear equation of both free damped oscillation and forced damped oscillation.

10) What are the points to be remembered while deriving expressions using dimensional analysis?

**Ans:** i) Any mathematical equation should be dimensionally homogeneous. ii) First, the variables controlling the phenomenon should be identified and expressed in terms of primary dimensions. iii) In typical cases, a suitable mathematical model is constructed to simplify the problem with suitable assumptions.

11) Give the Rayleigh method to determine dimensionless groups.

**Ans:** i) First, the functional relationship is written with the given data ii) Then the equation is expressed in terms of a constant with exponents like powers of  $a, b, c, \dots$  iii) values of  $a, b, c, \dots, z$  are determined with the help of dimensional homogeneity iv) Finally, these exponent values are substituted in the functional equation and simplified to obtain the suitable form.

12) State the methods of dimensional analysis.

**Ans:** i) Rayleigh's method ii) Buckingham  $\pi$  method.

13) How are equations derived in Rayleigh's method?

**Ans:** The expression is determined for a variable depending upon maximum three or four variables only. If the number of independent variables becomes more than four, it is very difficult to find the expression for the dependent variable. So, a functional relationship between variables is expressed in exponential form of equations.

14) State the buckingham  $\pi$  theorem.

**Ans:** It states that if there are  $n$  variables in a dimensionally homogenous equation and if these variables contain  $m$  fundamental dimensions ( $M, L, T$ ), then they are grouped into  $n-m$  dimensionless independent  $\pi$  terms.

15) Distinguish between Rayleigh's method and Buckingham's  $\pi$  theorem.

**Ans:**

S.No.	Rayleigh's method	Buckingham's $\pi$ method
1.	The expression is determined for a variable depending upon maximum three or four variables only.	It is easy for the number of independent variables being more than three or four.
2.	This method is difficult for more than four variables	In this method, $n$ variables can be used without any difficulties.



16) Describe briefly the selection of repeating variables in Buckingham's  $\pi$  theorem.(or) State any two choices of selecting repeating variables in Buckingham's  $\pi$  theorem.

**Ans:** The number of repeating variables is equal to the fundamental dimensions of the problem. Generally  $\rho, v, l$  or  $\rho, v, D$  are chosen as repeating variables. It means one refer to fluid property( $\rho$ ) one refers to flow property( $v$ ) and the other one refers to geometric property( $l$  or  $D$ )

The following points should be kept in mind while selecting repeating variables.

i) No one variables should be dimensionless. ii) The selected two repeating variables should not have the same dimensions iii) The selected repeating variables should be independent as far as possible.

17) Under what circumstances, will the Buckingham's Pi theorem yield incorrect number of dimensionless group.

**Ans:** If, when defining the problem, extra - unimportant - variables are introduced then extra pi groups will be formed. They will play very little role influencing the physical behaviour of the problem concerned and should be identified during experimental work. If an important / influential variable was missed then a pi group would be missing. Experimental analysis based on these results may miss significant behavioural changes. It is therefore, very important that the initial choice of variables is carried out with great care.

18) Define Weber number.

**Ans:** It is the ratio of the square root of the inertia force to the surface tension force.

Weber number  $W_e = (\text{Inertia force/ surface tension force})^{1/2} = (\rho L v^2 / \sigma)^{1/2}$

19) Define Reynolds number.

**Ans:** It is the ratio of the square root of the inertia force to the viscous force of the flowing fluid.

Reynolds number  $R_e = (\text{Inertia force/ viscous force}) = (\rho L v / \mu)$

20) What is the physical significance of Reynolds number?

**Ans:** i) Motion of Air planes ii) Motion of Submarines iii) Flow around structures immersed in moving fluids. iv) Flow of incompressible fluid in closed pipes.

21) Define Mach number and state its application.

**Ans:** It is the ratio of the square root of the inertia force of the flowing fluid to the Elastic force.

Mach number  $M = (\text{Inertia force/ Elastic force})^{1/2} = (\rho v^2 / k)^{1/2}$

22) Define Froude's number.

**Ans:** It is the ratio of the square root of the inertia force of the flowing fluid to the gravity force.

$$\text{Froude's number } Fr = (\text{Inertia force} / \text{gravity force})^{1/2} = (v^2/gL)^{1/2}$$

23) State the limitations of dimensional analysis.

**Ans:** i) Dimensional analysis does not give any clue regarding selection of variables. ii) The complete information is not provided by dimensional analysis. It only indicates that there is some relationship between parameters. iii) The values of coefficient and the nature of function can be obtained only by experiments from mathematical analysis.

24) What are the advantages of model testing?

**Ans:** i) With the help of models the performance of hydraulic structures/ hydraulic machines can be predicted in advance. ii) Model testing can be used to detect and rectify the defects of an existing structure, which is not functioning properly. iii) The model tests are quite economical and convenient.

25) Mention the applications of model testing.

**Ans:** i) Civil engineering structures such as dams, weirs, canals etc. ii) Design of harbor, ships and submarines. iii) Airplanes, rockets and machines, missiles.

26) Define similitude.

**Ans:** Similitude is the complete similarity between the model and the prototype. Complete similarity is attained, if the following three types of similarities exist: i) Geometric similarity ii) Kinematic similarity iii) Dynamic similarity.

27) What are the similarities between model and prototype?

**Ans:** Complete similarity is attained, if the following three types of similarities exist: i) Geometric similarity ii) Kinematic similarity iii) Dynamic similarity.

28) What is meant by kinematic similarity?

**Ans:** Kinematic similarity is the similarity of motion. If the acceleration ratios and acceleration vector points are same in the same direction, then two flows are said to be kinematically similar.

29) In fluid flow, What does dynamic similarity mean? What are the non dimensional numbers associated with dynamic similarity?

**Ans:** Dynamic similarity is similarity of forces. The flows in the model and prototype are of dynamic similar. Dimensional numbers are weight, force, dynamic viscosity, surface tension, capillarity etc. Non dimensional numbers associated with dynamic similarity are i) Reynolds number ii) Froude's number iii) Weber Number iv) Euler number v) Mach number.

30) What is meant by dynamic similarity?

**Ans:** Dynamic similarity is similarity of forces. The flows in the model and prototype are of dynamic similar. In dynamic similarity, the force polygon of the two forces can be superimposed by change in force scale.

31) If two systems ( model and prototype) are dynamically similar, is it implied that they are also kinematically and geometrically similar?

**Ans:** YES, If model and prototype are dynamically similar, it implies that they are also kinematically and geometrically similar.

32) Mention the significance of Reynolds model law.

**Ans:** Reynolds model laws are more significant in the following phenomena: i) Motion of Air planes ii) Motion of Submarines iii) Flow around structures immersed in moving fluids. iv) Flow of incompressible fluid in closed pipes.

33) State froude's model law.

**Ans:** Only gravitational force is more predetermining force. The law states, "The Froude number is same for both model and prototype".

34) Write down the scale ratio for discharge, energy and momentum.

**Ans:** i) Discharge scale ratio  $Q_r = L_r^{5/2}$  ii) Energy scale ratio  $E_r = L_r^4$   
iii) Momentum scale ratio  $M_r = L_r^{7/2}$

35) State the Euler model law and give its significance.

**Ans:** Only pressure force is the more predominant force in addition to the inertia force. Euler model law states " The Euler number is same for both model and prototype.

Significance of Euler model law: Euler number itself is sufficient criterion in the following phenomena: i) where the gravity and surface tension forces are fully absent and the turbulence is fully developed with negligible viscous force. ii) It is applied in cavitation phenomena.

36) Submarine is tested in the air tunnel. Identify the model law applicable.

**Ans:** Reynolds model law

37) Mention the types of models.

**Ans:** i) Undistorted models ii) Distorted models

38) What is meant by undistorted models?

**Ans:** The model which is geometrically similar to its prototype is known as undistorted models. In such models, the conditions of similitude are fully satisfied. So the results obtained from the models are used to predict the performance of the prototype easily.

39) Mention the circumstances which necessitate the use of distorted models.

**Ans:** i) To maintain accuracy. ii) To maintain turbulent flow. iii) To accommodate available facilities. iv) To obtain suitable bed materials v) To obtain required roughness condition.

40) Define the term scale effect.(or) What is scale effect in physical model study?

**Ans:** It is impossible to predict the exact behavior of the prototype by model testing alone. The two models of same prototype behavior with different scale ratios will not be same. So, discrepancy between models and prototype will always occur. It is known as scale effect.

41) State three demerits of a distorted model.

**Ans:** i) Exit pressure and velocity distributions are not true. ii) A model wave may differ from that of prototype iii) Both extrapolation and interpolation of results are difficult.

42) Obtain scale ratio of discharge for distorted models.

**Ans:**  $Q_r = Q_p/Q_m = A_p V_p/A_m V_m = B_p h_p (2gh_p)^{1/2}/B_m h_m (2gh_m)^{1/2}$   
 $Q_r = (L_r)_H \times (L_r)_V \times ((L_r)_V)^{1/2} = (L_r)_H \times ((L_r)_V)^{3/2}$

43) State the advantages of dimensional and model analysis.

**Ans:** Advantages of dimensional analysis: i) Dimensional equations are used to validate the correctness of a physical equation. ii) Dimensional equations are used to derive correct relationship between different physical quantities. iii) Dimensional equations are used to convert one system of units to another. iv) Dimensional equations are used to find the dimension of a physical constant.

Advantages of Model Analysis: i) Cost can be reduced by doing experiments with the models of full size operations. ii) Performance of the prototype can be determined from the test models. iii) Models can be used for the design of ships, Airplanes, pumps, turbines, dams, river channels, rockets and missiles etc.

44) Write the dimension of surface tension and vapour pressure in MLT system.

**Ans:** Dimension for surface tension –  $MT^{-2}$   
Dimension for vapour pressure -  $ML^{-1}T^{-2}$

45) What are the similitudes that should exist between the model and its prototype?

**Ans:** Complete similarity is attained, if the following three types of similarities exist: i) Geometric similarity ii) Kinematic similarity iii) Dynamic similarity.

46) Brief on Intuitive method. Give some examples.

**Ans:** It states that if there are  $n$  variables in a dimensionally homogenous equation and if these variables contain  $m$  fundamental dimensions (M,L,T), then they are grouped into  $n-m$  dimensionless independent  $\pi$  terms. Examples: resistance to flow, thrust, pump flow rate etc.

### **PART- B Questions**

1) An agitator of diameter  $D$  rotates at a speed  $N$  in a liquid of density  $\rho$  and viscosity  $\mu$ . Show that the power required to mix the liquid is expressed by a functional form

$$(P / \rho N^3 D^5) = f \{ \rho N D^2 / \mu, N^2 D / g \} \quad (16 \text{ marks})$$

2) Derive on the basis of dimensional analysis suitable parameters to present the **thrust developed by a propeller**. Assume that the thrust  $P$  depends upon the angular velocity  $\omega$ , speed of advance  $V$ , diameter  $D$ , dynamic viscosity  $\mu$ , mass density  $\rho$ , elasticity of the fluid medium which can be denoted by the speed of sound in the medium  $C$ . (16 marks)

3) Show that the **power  $P$**  developed in a water turbine can be expressed as:

$P = \rho N^3 D^5 \phi \{ D/B, \rho D^2 N / \mu, ND / (gH)^{1/2} \}$  where,  $\rho$  = mass density of the liquid,  $N$  = speed in rpm,  $D$  = diameter of the runner,  $B$  = width of the runner and  $\mu$  = dynamic viscosity. (16 marks)

4) The **power** required by the pump is a function of discharge  $Q$ , head  $H$ , acceleration due to gravity  $g$ , viscosity  $\mu$ , mass density of the fluid  $\rho$ , speed of rotation  $N$  and impeller diameter  $D$ . Obtain the relevant dimensionless parameters. (16 marks)

5) i) Derive an **expression for the shear stress** at the pipe wall when an incompressible fluid flows through a pipe under pressure. Use **dimensional analysis** with the following significant parameters: pipe diameter  $D$ , flow velocity  $V$ , and viscosity  $\mu$  and density  $\rho$  of the fluid. (10 marks)

ii) Use **dimensional analysis and the MLT system** to arrange the following into a **dimensionless number:  $L, \rho, \mu$  and  $\sigma$** . (6 marks)

6) The resisting force ( $R$ ) of a supersonic flight can be considered as dependent upon the length of the air craft ' $L$ ', velocity ' $v$ ', air viscosity ' $\mu$ ' air density ' $\rho$ ' and bulk modulus of air is ' $k$ '. Express the **functional relationship** between these variables and the resisting force. (11 marks)

(OR)

The resisting force  $F$  of a plane during flight can be considered as dependent upon the length of the air craft ' $L$ ', velocity ' $v$ ', air viscosity ' $\mu$ ' air density ' $\rho$ ' and bulk modulus of air is ' $k$ '. Express the **functional relationship** between these variables using **dimensional analysis**. Explain the physical significance of the dimensionless groups arrived. (8 marks)

7) The drag force ( $F$ ) on a partially submerged body depends on the relative velocity ( $v$ ) between the body and the fluid, characteristics

linear dimension (**l**), height of surface roughness(**k**), fluid density(**ρ**), the viscosity(**μ**) and the acceleration due to gravity(**g**). Obtain an **expression for the drag force**, using the method of **dimensional analysis**. ( 16 marks )

( or )

Using Buckingham  $\pi$  method of dimensional analysis obtain an expression for the drag force on a partially submerged body moving with a relative velocity **V** in a fluid; the other variables being the linear dimension **L**, height of surface roughness **K**, fluid density **ρ** and gravitational acceleration **g**. ( 16 marks )

8) i) The **power** developed by hydraulic machines is found to depend on the **head h**, **flow rate Q**, **density ρ**, **speed N**, **runner diameter D**, and **acceleration due to gravity g**. Obtain suitable **dimensionless parameters** to correlate experimental results. ( 8 marks )

ii) The **capillary rise h** is found to be influenced by the tube **diameter D**, **density ρ**, **gravitational acceleration g** and **surface tension σ**. Determine the **dimensionless parameters** for the correlation of experimental results. ( 8 marks )

9) i) Using **dimensional analysis**, obtain a correlation for the **frictional torque** due to rotation of a disc in a viscous fluid. The parameters influencing the torque can be identified as the **diameter**, **rotational speed**, **viscosity and density of fluid**. ( 8 marks )

ii) The **drag force** on a smooth sphere is found to be affected by the velocity of flow **u**, the diameter **D** of the sphere and the fluid properties density **ρ** and viscosity **μ**. Find the **dimensionless groups** to correlate the parameters. ( 8 marks )

10) Consider force **F** acting on the propeller of an air craft which depends upon the variable **U**, **ρ**, **μ**, **D** and **N**. Derive the non dimensional functional form,  $F / \rho U^2 D^2 = f \{ U D \rho / \mu \}$ ,  $N D / U$ .

11) A **partially submerged body** is towed in water. The resistance **R** to its motion depends on the density **ρ**, the viscosity **μ** of water, length **L** of the body, velocity **v** of the body and acceleration due to gravity. Show that the resistance to motion can be expressed in the form

$$R = \rho L^2 v^2 \phi \left( \frac{\mu}{\rho L v}, \left( \frac{L g}{v} \right) \right)$$

12) The **pressure drop Δp** in a pipe of diameter **D** and length **l** depends on the density **ρ** and viscosity **μ** of fluid flowing, mean velocity **v** of flow and average height of protuberance **t**. Show that the pressure drop can be expressed in the form  $\Delta p = \rho v^2 \phi \{ l/D, \mu/vD\rho, t/D \}$

13) It is judged that the performance of a lubricating oil ring depends upon the following variables quantity of oil delivered per unit time **Q**, inside diameter of the ring **D**, shaft speed in revolution per unit time **N**, oil viscosity **μ**, oil density **ρ**, specific weight of the oil **w** ( $w = \rho g$ ) and surface tension in air **σ**. Show that  $Q / D^3 N = \phi \{ \rho N D^2 / \mu, \mu N D / \sigma, w D / \mu N \}$

14) i) Define **dimensional homogeneity**. (6 marks)

ii) Using Buckingham's  $\pi$  theorem, show that velocity through a circular pipe orifice is given by  $v = (2gH)^{1/2} \phi \{ D/H, \mu/\rho v H \}$   $H$  – head causing flow;  $D$  – dia of orifice,  $\mu$  - coefficient of viscosity,  $\rho$  – mass density;  $g$  – acceleration due to gravity. (10 marks)

15) State Buckingham  $\pi$  theorem. What are the criteria for selecting repeating variable in this dimensional analysis. (6 marks)

16) Express efficiency in terms of dimensionless parameters using density, viscosity, angular velocity, diameter of the rotor and discharge using Buckingham  $\pi$  theorem. (16 marks)

(or)

The efficiency ( $\eta$ ) of a fan depends on  $\rho$  (density),  $\mu$  (viscosity) of the fluid,  $\omega$  (angular velocity),  $d$  (diameter of rotor) and  $Q$  (discharge). Express  $\eta$  in terms of non-dimensional parameters. Use Buckingham's  $\pi$  theorem. (16 marks)

17) The efficiency  $\eta$  of a fan depends on the density  $\rho$ , the dynamic viscosity  $\mu$ , the angular velocity  $\omega$ , diameter  $D$  of the rotor and the discharge  $Q$ . Express  $\eta$  in terms of dimensionless parameters. Use Rayleigh's method. (16 marks)

18) State Buckingham's  $\pi$  theorem. The discharge of a centrifugal pump ( $Q$ ) is dependent on  $N$  (speed of pump),  $d$  (diameter of impeller),  $g$  (acceleration due to gravity),  $H$  (manometric head developed by pump) and  $\rho$  and  $\mu$  (density and dynamic viscosity of the fluid). Using the dimensional analysis and Buckingham's  $\pi$  theorem, prove that it is given by

$$Q = Nd^3 f\{gH/N^2 d^2, \mu/Nd^2 \rho\} \quad (16 \text{ marks})$$

19) The frictional torque  $T$  of a disc diameter  $D$  rotating at a speed  $N$  in a fluid of viscosity  $\mu$  and density  $\rho$  in a turbulent flow is given by  $T = D^5 N^2 \rho \phi\{\mu/D^2 N \rho\}$  Prove it by Buckingham's  $\pi$  theorem.

20) The pressure difference  $\Delta p$  in a pipe of diameter  $D$  and length  $l$  due to viscous flow depends on the velocity  $v$ , viscosity  $\mu$  and density  $\rho$ . Using Buckingham  $\pi$  theorem obtain an expression for  $\Delta p$ .

21) Derive an expression showing the relationship between the torque and the variables diameter, rotational speed, viscosity and density by Buckingham  $\pi$  method.

22) Obtain a relation using dimensional analysis, for the resistance to uniform motion of a partially submerged body in a viscous compressible fluid. (16 marks)

23) Classify models with scale ratios. (16 marks)

24) What is the significance and the role of the following parameters? i) Reynolds number ii) Froude number iii) Mach number iv) Weber number. (16 marks)

25) i) Consider viscous flow over a very small object. Analysis of the equations of motion shows that the inertial terms are much smaller than viscous and pressure terms. Fluid density drops out, and these are called creeping flows. The only important parameters are velocity  $U$ , viscosity  $\mu$  and body length scale  $d$ . For three dimensional

bodies, like spheres, creeping flow analysis yields very good results. It is uncertain, however, if creeping flow applies to two dimensional bodies such as cylinders, since even though the diameter may be very small, the length of the cylinder is infinite. Let us see if dimensional analysis can help. (i) Apply the **Pi theorem** to two dimensional drag force  $F_{2-D}$  as a function of the other parameters. Be careful: two dimensional drag has dimensions of force per unit length, not simple force. (ii) Is your analysis in part (i) **physically plausible**? If not explain why not. (iii) It turns out that fluid density  $\rho$  cannot be neglected in analysis of creeping flow over two dimensional bodies. Repeat the **dimensional analysis**, this time including  $\rho$  as a variable and find the resulting **non-dimensional relation** between the parameters in this problem. ( 10 marks )

ii) When fluid in a pipe is accelerated linearly from rest, it begins as laminar flow and then undergoes transition to **turbulence at a time  $t_{tr}$**  which depends upon the pipe diameter  $D$ , fluid acceleration  $a$ , density  $\rho$  and viscosity  $\mu$ . Arrange this into a **dimensionless relation** between  $t_{tr}$  and  $D$ .

( 6 marks )

26) Using **Buckingham's  $\pi$  theorem**, show that the drag force  $F_D$  of a supersonic aircraft is given by  $F_D = \rho L V^2 \phi(R_e, M)$  where  $R_e = \rho L V / \mu =$  Reynolds number,  $M = V / c =$  mach number,  $\rho =$  fluid density,  $V =$  velocity of aircraft,  $c =$  sonic velocity  $= (K/\rho)^{1/2}$ ,  $K =$  bulk modulus of fluid,  $L =$  chord length,  $L^2 =$  wings area = chord x span,  $\phi =$  a functional notation.

27) i) What are the **similarities between model and prototype**. Mention the **applications of model testing**.

( 4 marks )

ii) A **spillway model** is to be built to a geometrically similar scale of 1/50 across a flume of 600mm width. The **prototype is 15m high** and maximum head on it is expected to be 1.5m.

i) What **height of model** and what **head on the model** should be used?

ii) If the flow over the **model** at a particular head is 12 litres per second, what **flow per metre length of the prototype** is expected?

iii) If the negative pressure in the **model** is 200mm, what is the **negative pressure** in the **prototype**? Is it **practicable**? (12 marks )

28) **Model tests** have been conducted to study the **energy loss** in a pipeline of 1m diameter required to transport of specific gravity 0.80 and dynamic viscosity 0.02 poise at the rate of 2000litres/sec. Tests were conducted on a 10cm diameter pipe using water at 20°C. What is the **flow rate in the model**? If the energy head loss in 30m length of the model is measured as 44.0 cm of water, what will be the corresponding **head loss in the prototype**? What will be the **friction factor** for the **prototype** pipe? (16 marks )

29) A pipe of diameter 1.5m is required to transport an oil of specific gravity 0.90 and viscosity  $3 \times 10^{-2}$  poise at the rate of 3000litre/sec.



Tests were conducted on a 15cm diameter pipe using water at 20°C. Find the **velocity and the rate of flow in the model**. Viscosity of water at 20°C = 0.01 poise. ( 16 marks )

30) A **geometrically similar model** of an air duct is built to 1/25 scale and tested with water which is 50 times more viscous and 800 times denser than air. When tested under dynamically similar conditions, the pressure drop is 200kN/m<sup>2</sup> in the **model**. Find the corresponding pressure drop in the **full scale prototype** and express in cm of water. ( 8 marks )

31) A **model** of a hydro electric power station tailrace is proposed to built by selecting vertical scale 1 in 50 and horizontal scale 1 in 100. If the design pipe has flow rate of 600m<sup>3</sup>/sec and the allowable discharge of 800m<sup>3</sup>/sec. Calculate the corresponding flow rate for the **model testing**.

32) The ratio of lengths of a **sub-marine and its model** is 25:1. The speed of sub-marine ( **prototype** ) is 15m/s. The model is to be tested in a wind tunnel. Find the speed of air in wind tunnel. Also determine the ratio of the **drag** (resistance) between the **model and its prototype**. Assume the value of kinematic viscosities for water and air are 0.012 stokes and 0.016 stokes respectively. The density of seawater and air are given as 1030Kg/m<sup>3</sup> and 1.24Kg/m<sup>3</sup> respectively.

33) A ship 500m long moves in sea water whose density is 1030Kg/m<sup>3</sup>. A 1:75 **model** of this ship is to be tested in a wind tunnel. The velocity of air in The wind tunnel around the **model** is 40m/s and the **resistance of the model** is 50N. Determine the velocity of ship in sea water and also the **resistance of ship** in sea water. The density of air is given as 1.24Kg/m<sup>3</sup>. Assume the kinematic viscosity of sea water and air as 0.012 stokes and 0.018 stokes respectively.

34) The performance of a **spillway** of an irrigation project is to be studied by means of a model constructed to a scale of 1:20, neglecting viscous and surface tension effects, determine: i) rate of flow of model for a **prototype discharge** of 1000m<sup>3</sup>/s. ii) The dissipation of energy in the **prototype** hydraulic jump, if the jump in the **model** dissipates 0.3KW.

35) A **model of a torpedo** is tested in a towing tank at a velocity of 32m/s whilst the **prototype** is to turn 8m/s. a) What **model scale** has been used? For water  $\nu = 1.13 \times 10^{-6} \text{m}^2/\text{s}$ . b) What would be the **model speed** if it is tested in wind tunnel under a pressure of 1750Kpa and a constant temperature of 27°C, Absolute velocity of air under these conditions is  $1.85 \times 10^{-4}$  poise, and gas constant,  $R = 287 \text{J/KgK}$ .

36) A 1:25 **model of a naval ship** having a submerged area of 10m<sup>2</sup> and length 15m has a total drag of 25N when towed through water as a velocity of 2m/s. Calculate the total drag on the **prototype** when moving at the corresponding speed. Use the relation  $F_r = \frac{1}{2} C_p \rho A V^2$  for calculating the skin resistance. The value of  $C_p$  is given by  $C_p =$

$0.0735/\text{Re}^{1/5}$  Assume kinematic viscosity of water (sea water) as 0.01 stoke and the density of water (sea water) as  $1030\text{Kg/m}^3$ .

37) A rectangular pier, 1.5m wide 5m long is to be built in a river whose average depth is 4m. To determine the discharge characteristics, a **small scale model** made to a scale of 1:25 was tested in a laboratory and the following data were recorded. Velocity of flow = 0.5m/s, Height of standing wave = 3.0cm and force acting on the **model** = 5N Make calculations of speed, force, height of standing wave and the coefficient of drag resistance for the **prototype**. Assume that the flow in the **model and prototype** are insensitive to change in Reynolds number.

38) A **Spillway model** built-up to a scale of 1/10 is discharging water with a velocity of 2m/s under a head of 75mm. Find the velocity of water of the **prototype**, if the head of water over the **prototype** is 5.4m

39) In a laboratory, a **spillway** 1:40 model was found to be discharged water with a velocity of 0.3m/s under a head of 600mm. Find the head under which the velocity of water in the **prototype** will be 1m/s.

40) A **dam model**, made to 1:75 scale is discharging water with a velocity of 0.75m/s under a head of 350mm. Find the velocity with which the water will flow in the dam under a head of 8m. Also find the discharge of the dam, when the **discharge of the model** is 2.5litres/s.

41) A quarter scale **turbine model** is tested under a head of 12m. The full scale turbine is to work under a head of 30m and to run at 428rpm. Find N for **model**, if **model** develops 100KW and uses 1100lit/s at this speed, what **power** will be obtained from full scale turbine assuming its  $\eta$  as 3% better than that of **model**.

42) It is desired to obtain the **dynamic similarity** between a 30cm diameter pipe carrying linseed oil at  $0.5\text{m}^3/\text{s}$  and a 5m diameter pipe carrying water. What should be the rate of flow of water in lps? If the pressure loss in the **model** is  $196\text{N/m}^2$ , what is the pressure loss in the **prototype pipe**? Kinematic viscosities of linseed oil and water are 0.457 and 0.0113 stokes respectively. Specific gravity of linseed oil = 0.82.

43) A 7.2m high and 15m long **spillway** discharges  $94\text{m}^3/\text{s}$  under a head of 2m. If a 1:9 scale **model** of this spillway is to be constructed, determine the **model law** to be used, **model dimensions**, head at spillway and discharge in the **model**. If **model** experiences a force of 764N, determine force on **prototype**.

## UNIT IV - PUMPS

### PART A Questions

1) Define fluid Machines.

**Ans:** The device in which the fluid is in continuous motion and imparts energy conversion is known as fluid machines.

2) Write the Eulers equation for turbo machines.

**Ans:** Force exerted by the water  $F = \rho a V_1 (V_{w1} + V_{w2})$

Workdone by the fluid  $= F \times u = \rho a V_1 (V_{w1} + V_{w2}) \times u$

3) State the principle on which turbo machines are based.

**Ans:** Impulse Momentum principle which states that "The impulse force  $F$  acting on a fluid mass ' $m$ ' over a short interval of time  $dt$  is equal to the change in momentum  $d(mv)$  in the direction of force".  
 $F \cdot dt = d(mv)$

4) How Pumps are classified?

**Ans:** Pumps are in general classified as Centrifugal Pumps (or Roto-dynamic pumps) and Positive Displacement Pumps.

Centrifugal pumps include radial, axial and mixed flow units.

Centrifugal pumps can be classified further as i) vertical multistage pumps ii) horizontal multistage pumps iii) submersible pumps iv) self-priming pumps v) axial-flow pumps

The positive displacement pump can be classified as: i) Reciprocating pumps - piston, plunger and diaphragm ii) Rotary pumps - gear, lobe, screw, vane pumps.

5) What are the components of energy transfer in turbo machines?

**Ans:** i) Whirl velocity ii) Flow velocity iii) Relative velocity

6) Mention the main parts of the Centrifugal pump.

**Ans:** i) Impeller ii) Casing iii) Suction pipe with strainer and foot valve  
iv) Delivery pipe with delivery valve.

7) What is the role of a volute chamber of a centrifugal pump?

**Ans:** i) To guide water to and from the impeller and ii) to partially convert the kinetic energy into pressure energy.

8) Where the suction pipe is placed? For what?

**Ans:** Suction pipe connects the sump and the pump inlet It is provide with i) a strainer at its lower end to prevent the entry of solid particles, debris etc into the pump ii) Foot valve to fill the pump before it is started and to prevent back flow, when the pump is stopped.

9) What are the various types of casing?

**Ans:** i) Volute casing ii) Vortex casing iii) Volute casing with guide blades

10) Define Manometric head.

**Ans:** Manometric head is the head against which the pump has to work.

11) What is the maximum theoretical suction head possible for a centrifugal pump?

**Ans:** Maximum theoretical suction head possible for a centrifugal pump = Atmospheric pressure = 10.33m

12) What do you mean by manometric efficiency and mechanical efficiency of a centrifugal pump?

**Ans:** Manometric efficiency is the ratio between manometric head and the head imparted by the impeller to the liquid.  $\eta_{\text{mano}} = \text{Manometric head} / \text{head imparted by the impeller to the liquid}$

Mechanical efficiency of a pump is defined as the ratio of output of the pump to the power imparted by the impeller.  $\eta_{\text{mech}} = \text{output of the pump} / \text{power imparted by the impeller}$ .

13) Write down the relationship between overall efficiency, manometric efficiency, volumetric efficiency and mechanical efficiency.

**Ans:** Overall efficiency = Manometric efficiency x Volumetric efficiency x Mechanical efficiency  $\eta_o = \eta_{\text{mano}} \times \eta_{\text{vol}} \times \eta_{\text{mech}}$

14) Define volumetric efficiency.

**Ans:** Volumetric efficiency is defined as the ratio of volume of water actually striking the buckets to the total water supplied by the jet.

15) Define overall efficiency.

**Ans:** Overall efficiency of the pump is defined as the ratio of output of the pump to the shaft power.  $\eta_{\text{overall}} = wQH / P$  where, P – shaft power, Q- Discharge, w – specific weight, H – head.

16) What precautions are to be taken while starting and closing the centrifugal pump?

**Ans:** i) The pump casing is to be vented of air and primed full of liquid before starting the pump. ii) Before stopping the pump the load on the pump is to be reduced by closing in on the discharge valve. iii) centrifugal pumps which run at constant speed, are to be started against a closed discharge valve. iv) Once the pump is running the discharge valve is to be opened until the desired discharge pressure is achieved.

17) What are backward curved vanes?

**Ans:** If the outlet tip of blade bends in a direction opposite to that of motion then the vane is called Backward curved vane.

18) Define Radial Vane.

**Ans:** Radial vanes are vanes placed at  $90^\circ$  so that the liquid leaves the vane with relative velocity in the radial direction.

19) What is a forward curved vane?

**Ans:** If the outlet tip of blade bends in the direction of motion then the vane is called forward curved vane.

20) The following data refer to a centrifugal pump which is designed to run at 1500 rpm.  $D_1 = 100\text{mm}$   $D_2 = 300\text{mm}$   $B_1 = 50\text{mm}$   $B_2 = 20\text{mm}$   $V_{f1} = 3\text{m/s}$   $\beta_2 = 60^\circ$ . Find the velocity of flow at outlet.

**Ans:**  $Q = \pi D_1 B_1 V_{f1} = \pi D_2 B_2 V_{f2}$

$$\pi \times 0.1 \times 0.05 \times 3 = \pi \times 0.3 \times 0.02 \times V_{f2}$$
$$V_{f2} = 2.5\text{m/s}$$

21) Write down the formula for specific speed of a pump.

**Ans:**  $N_s = N(Q)^{1/2} / H_m^{3/4}$  or  $N_s = N(P)^{1/2} / H_m^{5/4}$

22) What is the range of specific speed for high speed radial flow?

**Ans:** Range of specific speed for high speed radial flow = 50 to 80

23) A pump is to discharge  $0.82\text{m}^3/\text{s}$  at a head of 42m when running at

300rpm. What type of pump will be required?

**Ans:**  $N_s = N(Q)^{1/2} / H_m^{3/4}$

$N_s = 300(0.82)^{1/2} / (42)^{3/4} = 16.47$  Since, the specific speed lies between 10 and 30. So, slow speed radial flow pump is recommended.

24) Define speed ratio.

**Ans:** Speed ratio is the ratio of peripheral velocity at outlet to the theoretical velocity of jet corresponding to manometric head. Speed ratio  $K_u = u_2 / (2gH_m)^{1/2}$

25) Define flow ratio.

**Ans:** Flow ratio is the ratio of the velocity of flow at outlet to the theoretical velocity of jet corresponding to manometric head. Speed ratio  $K_f = V_{f2} / (2gH_m)^{1/2}$

26) How can we obtain a high head in a pump network?

**Ans:** A number of impellers on the same shaft are connected in series to obtain a high head.

$H_{\text{total}} = n \times H$  where  $n$  – no of impellers,  $H$  – head developed by one impeller.

27) What will be the effect of arranging the pumps in parallel?

**Ans:** A number of impellers are connected in parallel to obtain a high discharge.

$Q_{\text{total}} = n \times Q$  where  $n$  – no of impellers,  $Q$  – discharge from one impeller.

28) What are types of characteristic curves?

**Ans:** i) Main characteristic curves ii) Operating characteristic curves  
iii) Constant efficiency or Muschel curves iv) constant head and constant discharge curves.

29) What is meant by cavitation?

**Ans:** Cavitation is defined as the phenomenon of formation of vapour bubbles of a flowing fluid in a region where the pressure of the fluid falls below its vapour pressure and the sudden collapse of these vapour bubbles in a region of higher pressure.

30) What are the effects of cavitation? Give the necessary precautions against cavitation.

**Ans:** The major effects of cavitation are break down of machine itself due to severe pitting and erosion of blade surface. Other effects are:  
i) Sudden drop in efficiency ii) Head falls suddenly iii) More power requirement iv) Noise and vibration.

Necessary precautions against cavitation: i) The pressure should not be allowed to fall below its vapour pressure ii) Special material coatings can be given to the surfaces where the cavitation occurs.

31) State any two precautions against cavitation.

**Ans:** i) The pressure should not be allowed to fall below its vapour pressure ii) Special material coatings can be given to the surfaces where the cavitation occurs.

32) What is priming? Why is it necessary?

**Ans:** The operation of filling liquid in the suction pipe, casing of the pump and a portion of the delivery pipe up to delivery valve before starting the pump is called priming of a centrifugal pump. Priming is necessary to remove the air present in the pump.

33) What are the advantages of centrifugal pumps over reciprocating pumps?

**Ans:** i) Because of fewer parts, Centrifugal pumps have less cost than reciprocating pump. ii) Installation and maintenance are easier and cheaper as compared to reciprocating pump iii) Much greater discharging capacity than reciprocating pump iv) Compact and small in size and have less weight for the same capacity and energy transfer as compared to reciprocating pump.

34) How are reciprocating pumps classified?

**Ans:** Classification according to the fluid being in contact with piston or plunger i) single acting pump ii) double acting pump. Classification according to the number of cylinders i) single cylinder pump ii) double cylinder pump iii) Triple cylinder pump etc.

35) What is the principle of reciprocating pumps?

**Ans:** It operates on a principle of actual displacement of liquid by a piston or plunger, which reciprocates in a closely fitting cylinder.

36) Why is the reciprocating pump called a positive displacement pump?

**Ans:** It discharges a definite quantity of liquid during the displacement of its piston, therefore it is called as a positive displacement pump.

37) How are reciprocating pumps classified?

**Ans:** Classification according to the fluid being in contact with piston – i) single acting pump ii) Double acting pump  
Classification according to the number of cylinders i) Single cylinder pump ii) Double cylinder pump iii) Triple cylinder pump

38) Mention the main components of reciprocating pump.

**Ans:** i) piston or plunger fitted into the cylinder ii) suction and delivery pipes iii) crank and connecting rod.

39) What is the main difference between single acting and double acting reciprocating pump?

**Ans:** In a single acting reciprocating pump, the liquid acts on one side of the piston whereas in double acting reciprocating pump, the liquid acts on both sides of the piston.

40) Write down the formula for discharge, work done and power required for double acting pump.

**Ans:** Discharge  $Q = 2ALN/60$  Work done  $= 2wALN(h_s + h_d)/60$   
Power  $= 2wALN(h_s + h_d)/60$  Where, A – area of piston, L – stroke length of piston, N – speed of crank w – specific weight,  $h_s$  – suction head,  $h_d$  – delivery head.

41) Which factor determines the maximum speed of a reciprocating pump?

**Ans:** Atmospheric head, acceleration head, suction head and delivery head.

42) Define Slip of reciprocating pump and percentage slip. When does the negative slip occur?

**Ans:** Slip is the difference between theoretical discharge and actual discharge.  $\text{Slip} = Q_{th} - Q_{act}$

Percentage slip =  $((Q_{th} - Q_{act}) / Q_{th}) \times 100 = (1 - C_d) \times 100$

Sometimes,  $Q_{act}$  may be higher than  $Q_{th}$ , in such cases,  $C_d$  is greater than unity and the slip will be negative called as negative slip.

43) What are the reasons for negative slip in a reciprocating pump?

**Ans:** i) very long suction pipe ii) very low delivery head iii) high speed of the pump

44) Can actual discharge be greater than theoretical discharge in a reciprocating pump?

**Ans:** In most of the cases, actual discharge is less than theoretical discharge. But sometimes,  $Q_{act}$  may be higher than  $Q_{th}$ , in such cases,  $C_d$  is greater than unity and the slip will be negative called as negative slip.

45) What is Indicator diagram?

**Ans:** Indicator diagram is a graph plotted between the pressure head in the cylinder and the distance travelled by piston from inner dead centre for one complete revolution of the crank.

46) A single acting reciprocating pump, running at 50 rpm. The diameter of piston = 20cm and length = 40cm. Find the theoretical discharge of the pump.

**Ans:**  $Q = ALN/60 = \pi \times 0.2^2 \times 0.4 \times 50 / (4 \times 60) = 0.010472\text{m}^3/\text{sec}$

47) Draw the relationship between discharge and crank angle for a single acting pump.

**Ans:** Refer book

48) Mention the significance of "back leakage".

**Ans:** i) In a Gear pump, dynamic seals need sufficient clearances which permit the fluid to pump back from outlet of the high pressure side to the low pressure side of inlet called back leakage. ii) Trapping of recycled fluid within the pump which reduces the pumps total flow rate.

49) Write down the formula for work done by the pump in an indicator diagram.

**Ans:** Workdone =  $K \times \text{area of indicator diagram}$

=  $wAN/60 \times \text{Area of indicator diagram for single acting pump}$

=  $2 wAN/60 \times \text{Area of indicator diagram for double acting pump}$

50) Define suction head.

**Ans:** It is the vertical height of the centre line of the pump shaft above the liquid surface in the sump from which the liquid is being raised.



51) Define and explain briefly about acceleration head.

**Ans:** When the piston moves outward at the beginning of the suction stroke, a negative pressure is created which is equal to the suction head ( $h_s$ ). In addition to this, the liquid will also be accelerated in the suction stroke, due to which the total negative pressure head is equal to  $h_s + h_{as}$ . At the same time, the absolute pressure should not fall below the vapour pressure and separation should not take place.

52) Under what conditions would you suggest the use of double suction pump and a multistage pump?

**Ans:** i) If the need of continuous delivery of fluid ii) if uniform discharge of fluid is needed iii) If the required discharge of fluid is more iv) If the space is limited to use single suction pump for the higher capacity.

53) What is an Air Vessel? List the objectives that would be fulfilled by the use of air vessels?

**Ans:** An air vessel is a closed chamber made of cast iron, which contains compressed air.

Objectives: i) to provide uniform discharge ii) friction work is reduced iii) acceleration head is reduced iv) pump can be operated at high speed.

54) What are the functions of an air vessel?

**Ans:** i) to provide uniform discharge ii) friction work is reduced iii) acceleration head is reduced iv) pump can be operated at high speed.

55) What is the function of non return valve in a reciprocating pump?

**Ans:** Non return valve permits the fluid to flow in one direction only i.e. From sump to the inlet of the pump, from the outlet of the pump to the delivery.

56) Mention the working principle of an air vessel.

**Ans:** An air vessel is a closed chamber made of cast iron, which contains compressed air. Air vessels are connected with both suction and delivery pipes separately. During pumping of water, the piston moves inside the cylinder with acceleration in the first half of delivery stroke. Due to this, large quantity of water flows through the delivery pipe. But the quantity of water flow is reduced due to piston retardation during second half of the delivery stroke. So, the discharge is not uniform. These difficulties are overcome by fitting an air vessel in delivery pipe.

The air vessel stores excess quantity of water during the first half of delivery stroke and supplies excess quantity of water to the delivery pipe during the second half of the stroke. Water is stored in

the air vessel by compressing the air. Similarly the water is allowed to flow from the air vessel to the delivery pipe by expanding the compressed air. Thus, the uniform discharge is ensured in the delivery pipe.

57) Write down the formula for saving in work by fitting air vessels.

**Ans:** Work saved =  $(W_1 - W_2 / W_1) \times 100$  where,  $W_1$  – work done against friction without air vessel,  $W_2$  – work done against friction with air vessel.

58) What are the advantages of fitting an air vessel?

**Ans:** i) Amount of work to be supplied to the pump is reduced. ii) speed of the pump is maintained without separation.

59) When will you select a reciprocating pump?

**Ans:** For obtaining high pressure or head and low discharge, a reciprocating pump is selected.

60) What are rotary pumps? Give examples.

**Ans:** Rotary pumps resemble like a centrifugal pumps in appearance, but the working method differs. Uniform discharge and positive displacement can be obtained by using rotary pumps. It has the combined advantages of both reciprocating pump and centrifugal pump. Types of Rotary pumps are i) External Gear pump ii) Internal Gear pump iii) Vane pump iv) Lobe pump v) Screw pump.

61) What is the work saved by fitting an air vessel in a single acting, double acting pump?

**Ans:** Work saved by fitting air vessels in a single acting pump is 84.87%,

In a double acting pump the work saved is 39.2%.

## **PART – B Questions**

### **ROTARY PUMPS**

1) Explain the working of the following pumps with the help of neat sketches and mention two applications of each i) **External gear pump** ii) **lobe pump** iii) **vane pump** iv) **screw pump**. ( 16 marks )

(or) Discuss the working of Lobe and vane pumps. (6 marks)

2) Explain the working principle of **screw pump and gear pump** with neat diagram in detail. ( 6 marks )

3) Explain in detail with neat sketch the working of **Gear pump**. ( 8 marks )

4) Discuss in detail about **rotary positive displacement** pumps. ( 8 marks )

5) Explain the working of a **vane pump** with a neat diagram. ( 16 marks )

- 6) Explain with neat sketch the working of i) **gear pump** ii) **Vane pump**. ( 16 marks )
- 7) How **rotary pumps are classified**? Explain the working principles of **any one type of rotary pump** with the aid of a neat sketch. ( 16 marks )
- 8) Explain in detail the working principle and construction of **rotary pumps** with neat sketch. ( 8 marks )
- 9) With neat sketches, discuss about the **rotary positive displacement pump**. ( 16 marks )
- 10) Discuss briefly the working principle of a **vane pump** with a schematic diagram. ( 8 marks )

### **RECIPROCATING PUMP**

- 11) A **single acting** reciprocating pump has a plunger of diameter 30cm and stroke of 20cm. If the speed of the pump is 30 rpm and it delivers 6.5 lit/sec of water, find the **coefficient of discharge** and the **percentage slip** of the pump. ( 16 marks )
- 12) The piston area of a **single acting** reciprocating pump 0.15m<sup>2</sup> and stroke is 30cm. The water is lifted through a total head of 15m. The area of delivery pipe is 0.03m<sup>3</sup>. If the pump is running at 50rpm, find the **percentage slip, coefficient of discharge and the power required** to drive the pump. The actual discharge is 35 litre/s. Take mechanical efficiency as 0.85.( 8 marks )
- 13) The cylinder bore diameter and stroke of a **single acting** reciprocating pump are 150mm and 300mm respectively. The pump runs at 50 rpm and lifts the water to a height of 25m. The delivery pipe is 22m long, 100mm in diameter. Find the **theoretical discharge and theoretical power required** to run the pump. If the actual discharge is 4.2lit/s. Find the **percentage of slip**. ( 16 marks )
- 14) A **double acting** reciprocating pump, running at 50 rpm is discharging 900 litres of water per minute. The pump has a stroke of 400mm. The diameter of piston is 250mm. The delivery and suction heads are 25m and 4m respectively. Find the **slip** of the pump and **power required** to drive the pump.
- 15) A **three throw pump** has cylinders of 350mm diameter and stroke of 600mm each. The pump is required to deliver 0.12m<sup>3</sup>/sec at a head of 100m. Frictional losses are estimated to be 2m in suction pipe and 22m in delivery pipe. **Velocity of water in delivery pipe is 1m/s**. Overall efficiency is 80% and the slip is 4.25%. Determine i) **speed of the pump** and ii) power required to run the pump.
- 16) i) A **single acting** reciprocating pump running at 50 rpm delivers 0.01m<sup>3</sup>/sec of water. The diameter of the piston is 20cm and stroke length 40cm. Determine i) **theoretical discharge** of the pump ii) **coefficient of discharge** iii) **slip** of the pump. ( 12 marks )
- ii) Describe the function of **air vessel**. ( 4 marks )
- 17) For a **single acting** reciprocating pump, piston diameter is 150mm, stroke length is 300mm, rotational speed is 50 rpm. The

pump is required to lift water to a height of 18m. Determine the theoretical discharge, if the actual discharge is 4 lit/sec and the mechanical efficiency is 80% determine the **volumetric efficiency, slip, theoretical power and the actual power required.**

( 10 marks )

18) A single cylinder **double acting** reciprocating pump has a piston diameter of 300mm and stroke length of 400mm. When the pump runs at 45 rpm, it discharges  $0.039\text{m}^3/\text{sec}$  under a total head of 15m. What will be the **volumetric efficiency, work done per second and power required** if the mechanical efficiency of the pump is 75%. ( 10 marks )

19) Explain with a neat sketch the working of a single acting reciprocating pump. Also obtain the expression for weight of water delivered by the pump per second. ( 16 marks)

20) Derive an **expression for acceleration head** developed in a reciprocating pump. ( 8 marks )

21) Explain the **working principle of reciprocating pump** with neat diagram in detail and state its **advantages and disadvantages over centrifugal pump.** ( 8 marks )

22) Describe the **principle and working of a reciprocating pump** with a neat sketch.( 8 marks )

23) With a neat sketch explain the **working of double acting reciprocating pump** with its **performance characteristics.** ( 10 marks )

24) Write briefly on the **Indicator diagram for reciprocating pump.** (8 marks )

25) Explain the **working principle of single and double acting reciprocating pumps** with neat diagram in detail. Also explain the **effects of inertia pressure and friction** on the performance of the pump **using indicator diagram with and without air vessel.** ( 8 + 8 marks )

26) Discuss on the following: **Working of double acting pump, indicator diagram, acceleration head , friction head.** ( 16 marks )

27) The diameter and stroke of a **single acting** reciprocating pump are 120mm and 300mm respectively. The water is lifted by a pump through a total head of 25m. The diameter and length of delivery pipe are 100mm and 20m respectively. Find out: i) **Theoretical discharge** and **theoretical power** required to run the pump if the speed is 60 rpm. ii) **Percentage slip**, if the actual discharge is 2.95litres/s and iii) **The acceleration head at the beginning and middle** of the delivery stroke. ( 16 marks )

28) The cylinder of a **single acting** reciprocating pump is 15cm in diameter and 30cm in stroke. The pump is running at 30 rpm and discharges water to a height of 12m. The diameter and length of the delivery pipe are 10cm and 30m respectively. If a large air vessel is fitted in the delivery pipe at a distance of 2m from the centre of the pump, find the **pressure head** in the cylinder i) at the **beginning of**

**delivery stroke and ii) in the middle of the delivery stroke.** Take  $f = 0.01$  (16marks )

29) A **double acting** reciprocating pump has a cylinder of 250mm diameter and stroke of 450mm. The pump delivers water to a height of 15m through a pipe 40m long and 125mm diameter at 35rpm. Find the **pressure** in the cylinder **at the beginning** of delivery stroke, if a **large air vessel is fitted** in the delivery pipe at the same level of the pump, but 4m from the cylinder. Take  $f = 0.008$ .

30) A **single acting** reciprocating pump has a diameter of 150mm and stroke length 350mm. The centre of the pump is 3.5m above the water surface in the sump and 22m below the delivery water level. Both the suction and delivery pipes have the same diameter of 100mm and are 5m and 30m long respectively. If the pump is working at 30 rpm, determine the **pressure heads** on the piston **at the beginning, middle and end** of **both suction and delivery** strokes. ( 8 marks )

31) A **single acting** reciprocating pump has a stroke length of 170mm. Suction pipe is 10m long and the **ratio of suction pipe diameter to the piston diameter is  $\frac{3}{4}$** . The water level in the sump is 3.5m below the axis of the pump cylinder and the pipe connecting the sump and pump cylinder is 70mm in diameter. If the crank is running at 60rpm, determine the pressure head on the piston at the beginning, middle and end of the suction stroke. Take friction coefficient  $f = 0.01$ .

32) The length and diameter of a suction pipe of a **single acting** reciprocating pump are 5m and 10cm respectively. The pump has a plunger of diameter 150mm and of stroke length of 300mm. The centre of the pump is 4m above water surface in the pump. The atmospheric pressure head is 10.3 m of water and pump is running at 40 rpm. Determine: i) Pressure head due to acceleration at the beginning of the suction stroke. ii) **Maximum pressure head due to acceleration** iii) Pressure head in the cylinder at the beginning and at the end of the stroke.

33) In a **single acting** reciprocating pump the bore and stroke are 100mm and 150mm respectively. The static head requirements are 4m suction and 18m delivery. If the pressure at the end of delivery is atmospheric calculate the operating speed. The diameter of the delivery pipe is 75mm and the length of the delivery pipe is 24m. Determine the **acceleration head** at  $\theta = 33^\circ$  from the **start of delivery**. ( 6 marks )

34) What is an **air vessel**? Derive an **expression for the percentage work saved by using an air vessel**. ( 8 marks )

35) Determine the percentage of **work saved** in one cycle when an **air vessel is provided** on the delivery side of a **single cylinder single acting reciprocating pump**. ( 8 marks )

36) Calculate the **work saved by fitting an air vessel for a double acting** single cylinder reciprocating pump. ( 8 marks )

37) Calculate the **rate of flow in and out of the air vessel** on the delivery side in a single acting reciprocating pump of 220mm bore and 330mm stroke running at 50 rpm. Also find the **angle of crank rotation** at which there is **no flow into or out of the air vessel**. ( 8 marks )

38) A **double acting** reciprocating pump has an air vessel fitted on the suction pipe. The plunger is 150mm in diameter and 300mm long. The suction pipe is 8m long and 100mm diameter. Determine the **rate of flow into or from the air vessel** at **crank positions of 30°, 90° and 120°** from the inner dead centre. Take speed of the pump as 120rpm.

### **CENTRIFUGAL PUMP**

39) With a neat sketch, explain the construction and working of centrifugal pump.

40) With a neat sketch, explain the types of impellers of centrifugal pump.

41) With a neat sketch explain the types of casing of a centrifugal pump.

42) Sketch and briefly describe the volute and diffusion type pumps. What function is served by the volute chamber in a centrifugal pump? ( 16 marks )

43) Explain the characteristic curves of a centrifugal pump (16 marks )

44) Discuss characteristics of centrifugal pump at constant speed. ( 6 marks )

45) Explain about the performance characteristics of centrifugal pumps. ( 6 marks )

46) Draw the velocity triangle for a centrifugal pump and obtain the expression for work done. ( 8 marks )

47) The impeller of a centrifugal pump has external and internal diameters 500mm and 250mm respectively. Width of outlet 50mm and running at 1200 rpm. It works against a head of 48m. The velocity of flow through the impeller is constant and equal to 3m/s. The vanes are set back at an angle of 40° at outlet. Determine i) inlet vane angle ii) Work done by the impeller on water per second iii) Manometric efficiency.

48) The following data relate to a centrifugal pump: outlet diameter of impeller = 800mm width of impeller vanes at outlet = 100mm Angle of impeller vanes at outlet = 30° The impeller runs at 600rpm and delivers 0.9m<sup>3</sup>/sec under an effective head of 30m. A 450KW motor is used to drive the pump. Determine the manometric, mechanical and overall efficiencies of the pump. Assume water enters the impeller vanes radially at inlet.

49) A centrifugal pump impeller runs at 800 rpm and has outlet vane angle of 60°. The velocity of flow is 2.5m/sec throughout and

diameter of impeller at exit is twice that at inlet. If the manometric head is 20m and the manometric efficiency is 75%. Determine the diameter of the impeller at the exit and the inlet vane angle(8 marks )

50) A centrifugal pump is to discharge  $0.118 \text{ m}^3/\text{sec}$  at a speed of 1450 rpm against a head of 25m. The impeller diameter is 25cm, its width at outlet is 5cm and manometric efficiency is 75% Determine the vane angle at the outer periphery of the impeller and draw its velocity triangles. ( 8 marks )

51) A centrifugal pump running at 1200 rpm has a discharge of  $13 \text{ m}^3/\text{min}$ . The pump has a manometric efficiency of 85% and working against a head of 22m. The impeller has an outlet vane angle of  $40^\circ$ . If the velocity of flow at outlet is  $2.6 \text{ m/s}$ , determine the diameter of the impeller and the width of the impeller at the outlet. ( 16 marks )

52) A centrifugal pump having outer diameter equal to 2 times the inner diameter and running at 1200 rpm works against a total head of 75m. The velocity of flow through the impeller is constant and equal to  $3 \text{ m/s}$ . The vanes are set back at an angle of  $30^\circ$  at outlet. If the outer diameter of the impeller is 600mm and width at outlet is 50mm. Determine i) Vane angle at inlet ii) Work done per second by impeller iii) Manometric efficiency.

53) The impeller of a centrifugal pump has an external diameter of 450mm and internal diameter of 200mm and it runs at 1440rpm. Assuming a constant radial flow through the impeller at  $2.5 \text{ m/sec}$  and that the vanes at exit are set back at an angle of  $25^\circ$ . Determine i) Inlet vane angle ii) The angle, absolute velocity of water at exit makes with the tangent and iii) The work done per N of water.

54) A centrifugal pump delivers water against a net head of 14.5m and a design speed of 1000rpm. The vanes are curved back to an angle of  $30^\circ$  with the periphery. The impeller diameter is 300mm and outlet width 50mm. Determine the discharge of the pump, if manometric efficiency is 95%.

55) A centrifugal pump has 30cm and 60cm diameters at inlet and outlet. The inlet and outlet vane angles are  $30^\circ$  and  $45^\circ$  respectively. Water enters at a velocity of  $2.5 \text{ m/s}$  radially. Find the speed of impeller in rpm and power of the pump, if the flow is  $0.2 \text{ m}^3/\text{s}$ .

56) The internal and external diameters of the impeller of a centrifugal pump are 200mm and 400mm respectively. The pump is running at 1200 rpm, the vane angle of the impeller at inlet and outlet are  $20^\circ$  and  $30^\circ$  respectively. The water enters the impeller radially and the velocity of flow is constant. Determine the work done by the impeller per unit weight of water.(16 marks )

57) A centrifugal pump discharges 2000 Litres of water per second developing a head of 20m when running at 300rpm. The impeller diameter at the outlet and outflow velocity is 1.5m and  $3.0 \text{ m/s}$  respectively. If vanes are set back at an angle of  $30^\circ$  at the outlet, determine i) manometric efficiency ii) Power required by the pump. If

inner diameter is 750mm, find the minimum speed to start the pump.  
( 16 marks )

58) A centrifugal pump has an impeller 500mm in diameter running at 400rpm. The discharge at the inlet is entirely radial. The velocity of the flow at outlet is 1m/sec. The vanes are curved backwards at outlet at  $30^\circ$  to the wheel tangent. If the discharge of the pump is  $0.14\text{m}^3/\text{sec}$ , calculate the impeller power and the torque on the shaft.  
( 16 marks )

59) A centrifugal pump runs at 1000rpm with their vane angles at inlet and outlet as  $20^\circ$  and  $35^\circ$  respectively. The internal and external diameters are 25cm and 50cm respectively. Find the work done per N of water assuming velocity of flow as constant. Water enters radially through the pump.

60) A three stage centrifugal pump has impellers 500mm in diameter and 60mm width at outlet. All the impellers are keyed to the same shaft. The vanes of each impeller are having outlet angle as  $40^\circ$ . The speed of the pump is 400rpm and the total manometric head developed is 25m. If the discharge through the pump is  $0.12\text{m}^3/\text{sec}$ . Find the manometric efficiency.

61) Find the power required to drive a centrifugal pump which delivers  $0.04\text{m}^3/\text{sec}$  of water to a height of 20m through a 15cm diameter pipe and 100m long. The overall efficiency of the pump is 70% and coefficient of friction is 0.15 in the formula  $h_f = 4fV^2/2gd$ .

62) The Head – discharge characteristics of a centrifugal pump is given below: The pump delivers fresh water through a 500m long, 15cm diameter pipeline having friction coefficient of  $f = 0.025$ .

Discharge (lit/sec)	0	10	20	30	40	50
Head (metres)	25.3	25.5	24.5	22.2	18.7	12.0

The static lift is 15m. Neglecting minor losses in the pipe flow, find i) the discharge of the pump under the above conditions ii) driving power of the pump motor. Assume pump efficiency of 72%. ( 16 marks )

63) The following details refer to a centrifugal pump. Outer diameter – 30cm, eye diameter – 15cm Blade angle at inlet  $30^\circ$  Blade angle at outlet  $25^\circ$  speed 1450 rpm. The flow velocity remains constant. The whirl at inlet is zero. Determine the work done per Kg. If the manometric efficiency is 82%, determine the working head. If width at outlet is 2cm. determine the power.  $\eta_o = 76\%$  ( 16 marks )

64) A centrifugal pump with backward curved blades has the following measured performance when tested with water at  $20^\circ\text{C}$ :

Discharge (lit/min)	0	1800	3600	5400	7200	9000	10800
Head (m)	37	35	33	31	28	25	19
Power(KW)	22	27	30	33	35	36	34

Estimate the best efficiency point and the maximum efficiency, also estimate the most efficient flow rate and the resulting head and brake



power if the diameter is doubled and the rotation speed is increased by 50%. ( 10 marks )

65) The impeller of a centrifugal pump is 300mm in diameter and having a width of 50mm at the periphery. It has blades whose tip angles are inclined backwards at  $60^\circ$  from the radius. The pump delivers  $17\text{m}^3/\text{min}$  of water and the impeller rotates at 1000 rpm. assuming that the pump is designed to admit liquid radially. Calculate i) speed and direction of water as it leaves impeller ii) torque exerted by the impeller on water iii) shaft power required iv) lift of the pump. Assume the mechanical efficiency = 95% and the hydraulic efficiency = 75% ( 12 marks )

66) The internal and external diameter of an impeller of a centrifugal pump which is running at 1000rpm are 200mm and 400mm respectively. The discharge through the pump is  $0.04\text{m}^3/\text{sec}$  and velocity of flow is constant and equal to 2.0m/s. The diameters of the suction and delivery pipes are 150mm and 100mm respectively and suction and delivery heads 6m (abs.) and 30m (abs.) of water respectively. If the outlet vane angle is  $45^\circ$  and power required to drive the pump is 16.186KW, determine i) vane angle of the impeller at inlet ii) the overall efficiency of the pump and iii) manometric efficiency of the pump. ( 16 marks )

67) A radial flow impeller has a diameter 25cm and width 7.5cm at exit. It delivers 120 litres of water per second against a head of 24m at 1440rpm. Assuming the vanes block the flow area by 5% and hydraulic efficiency of 0.8, estimate the vane angle at exit. Also calculate the torque exerted on the driving shaft if the mechanical efficiency is 95%.

68) As the total head required to be developed is more than the head developed by each pump. The pumps should be arranged in series. A multistage centrifugal pump has four identical impellers of 40cm in diameter and 2.5cm wide at outlet. The vanes are curved back at the outlet at  $30^\circ$  and reduce the circumferential area by 15% The manometric efficiency is 85% and overall efficiency is 75% Determine the head generated by the pump when running at 1200rpm and discharging  $0.06\text{m}^3/\text{sec}$ . find the shaft power also.

69) A centrifugal pump with an impeller diameter of 0.4m runs at 1450 rpm. The angle at outlet of the backward curved vane is  $25^\circ$  with tangent. The flow velocity remains constant at 3m/s. If the manometric efficiency is 84% determine the fraction of the kinetic energy at outlet recovered as static head. ( 10 marks )

70) The diameter of a centrifugal pump impeller is 300mm and its width is 60mm. The pump delivers 120lit/s with a manometric efficiency 85% The effective outlet vane angle is  $30^\circ$  If the speed of rotation is 1000rpm calculate the specific speed of the pump.

71) Find the number of pumps required and their arrangement to take water from a deep well under a total head of 89m. All the pumps are identical and are running at 800rpm. The specific speed of each

pump is given as 25 while the rated capacity of each pump is  $0.16\text{m}^3/\text{sec}$ .

72) A pump has to supply water which is at  $70^\circ\text{C}$  water at  $90\text{m}^3/\text{min}$  and  $1800\text{rpm}$ . Find the type of pump needed, the power required and the impeller diameter if the required pressure rise for one stage is  $200\text{Kpa}$  and  $1250\text{Kpa}$ . ( 10 marks )

73) Two geometrically similar pumps are running at the same speed of  $750\text{rpm}$ . One pump has an impeller diameter of  $0.25\text{m}$  and lifts the water at the rate of  $30\text{ lit/sec}$  against a head of  $20\text{m}$ . Determine the head and impeller diameter of the other pump to deliver half the discharge.

74) A centrifugal pump running at  $920\text{ rpm}$  and delivering  $0.32\text{m}^3/\text{sec}$  of water against a head of  $28\text{m}$ , the flow velocity being  $3\text{m/s}$ . If the manometric efficiency is  $80\%$  determine the diameter and width of the impeller. The blade angle at outlet is  $25^\circ$ . ( 10 marks)

75) The dimensionless specific speed of a centrifugal pump is  $0.06$ . Static head is  $32\text{m}$ . Flow rate is  $50\text{lit/sec}$ . The suction and delivery pipes are each of diameter  $15\text{cm}$ . The friction factor is  $0.02$ . Total length is  $60\text{m}$  other losses equal 4 times the velocity head in the pipe. The vane are forward curved at  $120^\circ$ . The width is one tenth of the diameter. There is  $7\%$  reduction in flow area due to the blade thickness. The manometric efficiency is  $80\%$ . Determine the impeller diameter if inlet is radial. ( 10 marks )

## **UNIT V - TURBINES**

### **PART A Questions**

1) State the principles on which turbo-machines are based.

**Ans:** Impulse Momentum principle which states that "The impulse force  $F$  acting on a fluid mass ' $m$ ' over a short interval of time  $dt$  is equal to the change in momentum  $d(mv)$  in the direction of force".  
 $F \cdot dt = d(mv)$

2) Write the Euler's equation.

**Ans:** Force exerted by the water  $F = \rho a V_1 (V_{w1} + V_{w2})$

Workdone by the fluid  $= F \times u = \rho a V_1 (V_{w1} + V_{w2}) \times u$

3) What is Hydraulic turbine?

**Ans:** Hydraulic turbines are the machines which convert the hydraulic energy into mechanical energy.

4) What is hydroelectric power?

**Ans:** Hydraulic turbines are the machines which convert the hydraulic energy into mechanical energy. This mechanical energy is converted into electrical energy. So conversion of energy from hydraulic energy to electric energy is called hydroelectric power.

5) Classify the different types of turbine.(or) How are hydraulic turbines classified?

**Ans:** i) According to the action of the water flowing a) Impulse turbine  
b) Reaction turbine

ii) According to the direction of flow of water a) Tangential flow turbine b) Radial flow turbine c) Axial flow turbine d) Mixed flow turbine

iii) According to head and quantity of water a) High head turbine b) medium head turbine c) low head turbine iv) According to the specific speed a) Low specific speed b) medium specific speed c) high specific speed.

6) Classify turbines according to flow.

**Ans:** According to the direction of flow of water a) Tangential flow turbine – Pelton wheel b) Radial flow turbine – Thomson turbine c) Axial flow turbine – Kaplan turbine d) Mixed flow turbine – Francis turbine

7) Classify Turbines according to Head with examples.

**Ans:** According to head and quantity of water a) High head turbine (above 250m) – pelton wheel b) medium head turbine(60m to 250m) – Francis turbine c) low head turbine(less than 60m) – Kaplan turbine

8) What is Impulse turbine? Give an example.

**Ans:** In Impulse turbines, all the energies are converted into kinetic energy. From these, the turbine will develop high kinetic energy power. This turbine is called Impulse turbine. Ex: Pelton Wheel

9) What is Reaction turbine? Give an example.

**Ans:** In a reaction turbine, the runner utilizes both potential and kinetic energies. Here, portion of potential energy is converted into kinetic energy before entering into the turbine. Ex: Francis and Kaplan Turbine

10) Differentiate between Impulse turbine and Reaction turbine.

**Ans:**

S.No.	Impulse Turbine	Reaction Turbine
1.	All the potential energies are converted into kinetic energy by nozzle before entering to turbine runner	Only a portion of potential energy is converted into kinetic energy before entering into the turbine.
2.	Unit is installed above the tailrace	Unit is kept entirely submerged in water below tailrace

- |    |  |   |
|----|--|---|
| 3. | Flow regulation is possible without loss                                 | Flow regulation is possible with loss                 |
| 4. | Water may be allowed to enter a part or whole of the wheel circumference | Water is admitted over the circumference of the wheel |

11) What is tangential flow turbine? Give an example.

**Ans:** The water flows along the tangent to the path of rotation of the runner. Ex: Pelton Wheel

12) What is Radial flow turbine? Give an example.

**Ans:** Water flows along the radial direction and mainly in the plane normal to the axis of rotation as it passes through the runner. It may be either inward radial flow type or outward flow type. Ex: Thomson turbine, Fourneyron turbine.

13) What is Axial flow turbine? Give an example.

**Ans:** In axial flow turbines, water flows parallel to the axis of the turbine shaft. Ex: Kaplan turbine

14) What is mixed flow turbine? Give an example.

**Ans:** In mixed flow turbines, water enters the blades radially and leaves the turbine axially. Ex: Francis turbine

15) Differentiate between inward flow reaction turbine and outward flow reaction turbine.

**Ans:**

S.No.	Inward flow reaction Turbine	Outward flow reaction Turbine
1.	Water enters at the outer periphery, flows inward and towards the centre of the turbine and discharges at the outer periphery.	Water enters at the outer periphery, flows outward and discharges at the outer periphery.
2.	The discharge does not increase	The discharge increases
3.	Easy and effective speed control	Very difficult to speed control
4.	Good for medium and high heads	Good for medium or low heads

16) Differentiate between Francis and Kaplan turbines.

**Ans:**

S.No.	Francis Turbine	Kaplan Turbine
1.	Correct disposition of the guide and moving vanes is obtained at full load only.	Correct disposition of the guide and moving vanes is obtained at any load.
2.	Servomotors are kept outside the turbine shaft.	Servomotors are kept inside the hollow shaft of the turbine runner.
3.	System may have one or two servomotors depending on the size of the unit.	Two servomotors respective of the size of the unit always do governing.
4.	Since only the guide vanes are controlled high efficiency is obtained.	Both guide and runner vanes high efficiency is obtained even at partial loads.

17) What are the main parts of pelton wheel?

**Ans:** i) Outer casing ii) Spear and nozzle iii) Break nozzle iv) penstock v) Runner with buckets vi) Governing mechanism

18) What is the function of spear and nozzle?

**Ans:** The nozzle is used to convert whole hydraulic energy into kinetic energy and delivers high speed jet. To regulate the water flow through nozzle and to obtain a good jet of water, spear or nozzle is arranged.

19) What is meant by hydraulic efficiency of turbine?(or) Define hydraulic efficiency and axial thrust of a roto-dynamic hydraulic machine.

**Ans:** It is defined as the ratio of power developed by the runner to the power supplied by the water jet.  $\eta_h = \text{Power developed by the runner} / \text{power supplied by the water jet}$ .

20) Define hydraulic efficiency and overall efficiency of turbine.

**Ans:** It is defined as the ratio of power developed by the runner to the power supplied by the water jet.  $\eta_h = \text{Power developed by the runner} / \text{power supplied by the water jet}$

Overall Efficiency of the turbine is defined as the ratio of power output of the turbine to the power input to the turbine.  $\eta_o = \text{power output of the turbine} / \text{power input to the turbine}$

21) Draw velocity triangles for pelton wheel turbine.

**Ans:** Refer notes or book

22) What is the function of casing in pelton wheel turbine?

**Ans:** A casing is made of cast iron or fabricated steel plate. It is used to i) prevent splashing of water ii) safeguard against accidents iii) lead the water to tail race.

23) What is break nozzle and mention its function?

**Ans:** If the spear nozzle set is closed, the runner will revolve long time due to inertia. To stop the runner in a short time, a small nozzle is provided which directs a jet of water on the backside of the buckets.

24) What is the function of governing mechanism in pelton wheel turbine?

**Ans:** Governing mechanism is used to regulate the water flow to the turbine at constant level so that the speed of the turbine kept constant. This automatically regulates the quantity of water flowing through the runner in accordance with any variation of load.

25) Define gross head and effective or net head.

**Ans:** Gross head is the difference between the water level at the reservoir and the level at the tailstock. Effective head or Net Head is the head available at the inlet of the turbine.

26) Explain the function of casing in reaction turbines.

**Ans:** The water from the penstock enters the scroll casing which completely surrounds by the runner. The cross sectional area of the scroll casing decreases along the flow direction, area is maximum at inlet and nearly zero at exit, which helps in conversion of hydraulic energy into kinetic energy.

27) What is speed ring or stay ring?

**Ans:** Speed rings consist of an upper and lower ring held together by series of fixed vanes called stay vanes. The number of stay vanes is usually taken as half to direct the water from the scroll casing to the guide vanes and also it resists the load imposed upon it.

28) What is the function of guide vanes or wicket gates in reaction turbines?

**Ans:** The guide vanes direct the water on to the runner at appropriate angles as per design. Also, it is used to regulate the quantity of water supplied to the runner. The guide vanes are airfoil shaped and they may be made of cast steel or stainless steel.

29) What is draft tube? What are the functions of draft tube?

**Ans:** After passing through the runner of a reaction turbine, the water is discharged to the tailrace through a gradually expanding tube called Draft tube. Functions of Draft tube are i) To create a negative suction head at the exit of the runner. ii) to lead the water to the tail race.

30) What are the different types of draft tube?

**Ans:** Different types of draft tube are i) Moody draft tube ii) Straight divergent tube iii) Simple Elbow iv) Elbow with variable cross sections

31) Why does a pelton wheel does not possess any draft tube?

**Ans:** A draft tube is generally not used since the runner operates under approximately atmospheric pressure and the head represented by the elevation of the unit above tail race water cannot be utilized.

32) What are the main components in Kaplan turbine?

**Ans:** i) Scroll Casing ii) Stay ring iii) Guide vanes iv) Draft tube v) Runner

33) What are the significance of unit quantities and specific quantities?

**Ans:** i) To predict the behavior of a turbine working under different conditions ii) Make comparison between the performances of turbine of same type of different sizes. iii) Compare the performance.

34) Define i) unit speed ii) unit discharge iii) unit power.

**Ans:** i) Unit speed is defined as the speed of turbine when working under a unit head. Unit speed  $N_u = N/(H)^{1/2}$

ii) Unit discharge is the theoretical discharge of a turbine when working under unit head. Unit Discharge  $Q_u = Q/(H)^{1/2}$

iii) Unit power is the theoretical power of a turbine when working under unit head. Unit Power  $P_u = P/(H)^{3/2}$

35) Define the specific speed of turbine.

**Ans:** Specific speed is the speed of a geometrically similar turbine which will develop unit power when working under unit head.

36) What is meant by surge tank?

**Ans:** Surge tank is a small reservoir or tank in which the water level rises or falls to reduce the rapid velocity fluctuations in pipe line during start and shut down of a turbine.

37) What are the functions of surge tank?

**Ans:** i) During sudden closure of valve, sudden reduction of rate of flow in the penstock will occur. To avoid this reduction of rate of flow, surge tanks are provided in the upstream of the pipeline. ii) to reduce the rapid velocity fluctuations in pipe line during start and shut down of a turbine.

38) What are the different types of surge tank?

**Ans:** i) simple surge tank ii) Inclined surge tank iii) Differential surge tank

39) A turbine develops 5MW under a head of 20m at 125rpm. What is the specific speed?

**Ans:**  $N_s = N (P)^{1/2} / (H)^{5/4} = 125(5000)^{1/2} / (20)^{5/4} = 208.98$

40) Define flow ratio of reaction radial flow turbine.

**Ans:** Flow ratio is the ratio of the velocity of flow at inlet to the theoretical velocity Speed ratio  $K_f = V_{f1} / (2gH)^{1/2}$

41) Differentiate between turbine and pump.

**Ans:**

S.No.	Turbine	Pumps
1.	Turbine converts hydraulic energy into mechanical energy.	Pump converts mechanical energy into hydraulic energy.
2.	It is energy producing machine.	It is energy absorbing machine.
3.	Flow is decelerated.	Flow is accelerated.
4.	Flow takes place from high pressure side to the low pressure side.	Flow takes place from low pressure side to the high pressure side.

42) The mean velocity of the buckets of the pelton wheel is 10m/s. the jet supplies water at  $0.7\text{m}^3/\text{sec}$  at a head of 30m. The jet is deflected through an angle of  $160^\circ$  by the bucket. Find the hydraulic efficiency. Take  $C_v = 0.98$

**Ans:**  $\eta_h = 2u(v - u) (1 + \cos\phi) / V^2$        $V = C_v (2 \times 9.81 \times 30)^{1/2} = 23.77\text{m/s}$

$\eta_h = 2 \times 10 \times (23.77 - 10) (1 + \cos 20^\circ) / (23.77)^2 = 94.54 \%$

43) A shaft transmits 150KW at 600 rpm. What is the torque in newton - metres?

**Ans:** Torque  $T = 60 \times P / 2\pi N = 60 \times 150 \times 10^3 / (2 \times \pi \times 600) = 2387.324\text{N-m}$

44) What are the different efficiencies of turbine to determine the characteristics of turbine?

**Ans:** i) Hydraulic Efficiency ii) Mechanical Efficiency iii) Overall efficiency



## PART-B Questions

- 1) Obtain an expression for the work done per second by water on the runner of a **pelton wheel** and draw inlet and outlet velocity triangles for a **pelton wheel turbine** and indicate the direction of various velocities. ( 8 marks )
- 2) Show that the overall efficiency of a **hydraulic turbine** is the product of volumetric, hydraulic and mechanical efficiencies. ( 8 marks )
- 3) Derive an expression for **specific speed**. What is the significance of specific speed of turbine?(8 marks )
- 4) Give the **comparison** between **impulse and reaction turbine**. ( 8 marks )
- 5) With the help of neat diagram explain the construction and working of a **pelton wheel turbine**. ( 8 marks )
- 6) Obtain an expression for the work done per second by water on the runner of a **pelton wheel**. Hence derive an expression for maximum efficiency of the pelton wheel giving the relationship between the jet speed and bucket speed. ( 16 marks )
- 7) Derive the **Euler's equation of motion for turbines** and obtain the components of energy transfer with a construction of velocity triangles. ( 16 marks )
- 8) Derive the expression for **efficiency and work done for a pelton wheel** and draw the velocity triangles. ( 16 marks ) (or) Draw inlet and outlet velocity triangles for a pelton turbine and indicate the direction of various velocity components. Also obtain an expression for the work done per second by water on the runner of the Pelton wheel. ( 16 marks )
- 9) A **pelton turbine** is to produce 18MW under a head of 450m running at 480 rpm. If D/d ratio is 10, determine the number of jets required.( 6 marks )
- 10) i) Sketch velocity triangles at inlet and outlet of a **pelton wheel**. ( 6 marks )  
ii) A **pelton wheel** has to be designed for the following data: Power to be developed = 6000KW, Net head available = 300m, speed = 550 rpm, ratio of jet diameter to wheel diameter = 1/10 and overall efficiency = 85%. Find the number of jets, diameter of the jet, diameter of the wheel and the quantity of water required. ( 10 marks )
- 11) A **pelton wheel turbine** having 1.6m bucket diameter develops a power of 3600KW at 400 rpm under a net head of 275m. If the overall efficiency is 88% and the coefficient of velocity is 0.97, find speed ratio, discharge, diameter of the nozzle and specific speed.( 8 marks )
- 12) A **pelton wheel** supplied water from reservoir under a gross head of 112m and the friction losses in the penstock amounts to 20 m of head. The water from penstock is discharged through a single nozzle of diameter of 100mm at the rate of  $0.3\text{m}^3/\text{sec}$ . "Mechanical losses due to friction amounts to 4.3KW of power and shaft power

available is 208KW. Determine velocity of jet, water power at inlet to runner, power loss in nozzles, power lost in runner due to hydraulic resistance. ( 8 marks )

13) A **pelton wheel** has a bucket speed of 10 metres per second with a jet of water flowing at the rate of 700litres/sec under a head of 30metres. The buckets deflect the jet through an angle of  $160^\circ$ . Calculate the power given by water to the runner and the hydraulic efficiency of the turbine. Assume coefficient of velocity as 0.98. ( 8 marks )

14) In a hydroelectric station, water is available at the rate of  $175\text{m}^3/\text{s}$  under a head of 18m. The turbine run at a speed of 150 rpm, with overall efficiency of 82%. Find the **number of turbines** required, if they have the maximum **specific speed** of 460. ( 8 marks )

15) A **pelton wheel** having a mean bucket diameter of 1m and is running at 1000rpm. The net head on the pelton wheel is 700m. If the side clearance angle is  $15^\circ$  and discharge through the nozzle is  $0.1\text{m}^3/\text{s}$ . find i) power available at the nozzle and ii) hydraulic efficiency of the turbine. Take  $C_v = 1$ . ( 8 marks )

16) A **turbine** is to operate under a head of 25m at 200 rpm. The discharge is  $9\text{m}^3/\text{s}$ . If the efficiency is 96%. Determine specific speed of the machine, power generated and type of turbine. ( 8 marks )

17) Determine the hp of the **pelton wheel** with tangential velocity 20m/s, head 50m, discharge  $0.03\text{m}^3/\text{s}$ , side clearance angle is  $15^\circ$ . Take  $C_v$  as 0.975 ( 16 marks )

18) A **pelton wheel** has a mean bucket speed of 10m/s with a jet of water flowing at the rate of  $0.7\text{m}^3/\text{s}$  under a head of 30m. If the buckets deflect the jet through an angle of  $160^\circ$ , calculate the power given by water to the runner and hydraulic efficiency of turbine. Assume the coefficient of velocity of 0.98. ( 16 marks )

19) A **pelton turbine** running at 720rpm uses 300kg of water per second. If the head available is 425m, determine the hydraulic efficiency. The bucket deflects the jet by  $165^\circ$ . Also find the diameter of the runner and jet. Assume  $C_v = 0.97$  and  $\phi = 0.46$ , blade velocity coefficient is 0.9 ( 16 marks )

20) A **single jet pelton wheel** runs at 300rpm, under a head of 510m. The jet dia is 200mm and the deflection inside the bucket is  $165^\circ$ . Assuming that its relative velocity is reduced by 15% due to friction, determine i) water power ii) resultant force on bucket and iii) overall efficiency. ( 12 marks )

21) A **pelton turbine** is required to develop 9000KW when working under a head of 300m the impeller may rotate at 500rpm. Assuming a jet ratio of 10 and an overall efficiency of 85%, calculate i) quantity of water required ii) diameter of the wheel iii) number of jets iv) number and size of the bucket vanes on the runner.

22) The nozzle of a **pelton wheel** gives a jet of 9cm diameter and velocity 75m/s. coefficient of velocity is 0.978. The pitch circle diameter is 1.5m and the deflection angle of the buckets is  $170^\circ$ . The wheel velocity is 0.46 times the jet velocity. Estimate the

speed of the pelton wheel turbine in rpm, theoretical power developed and also the efficiency of the turbine.

23) A dam on a river is being suited for a **hydraulic turbine**. The flow rate is  $1600\text{m}^3/\text{h}$ , the available head is 25m and the turbine speed is to be 460 rpm. Discuss the estimated turbine size and feasibility for a **francis turbine and a pelton wheel**. ( 6 marks )

24) A **turbine** is to operate under a head of 25m at 200rpm, the available discharge is  $9\text{m}^3/\text{s}$  assuming an efficiency of 90%, determine i) specific speed ii) power generated iii) performance under a head of 20m iv) the **type of turbine**.

25) A **pelton wheel** having **semi – circular buckets** functions under a head of 150m and consumes  $0.06\text{m}^3/\text{s}$  of water. If 750mm diameter wheel turns 800rpm, calculate the power available at the nozzle and hydraulic efficiency of the wheel. Assume the coefficient of velocity as unity.

26) The shaft power of a **pelton wheel** the buckets of which are struck by two jets, is 15445Kw. The diameter of each jet is 200mm. If the net head on the turbine is 400m find the overall efficiency of the turbine. Take  $C_v = 1$ .

27) A **double jet pelton wheel** is required to generate 7500Kw When the available head at the base of nozzle is 400m. The jet is deflected through  $165^\circ$  and the relative velocity of the jet is reduced by 15% in passing over the buckets. Determine i) the diameter of each jet ii) total flow iii) force exerted by the jet on buckets in tangential direction. Assume generator efficiency as 95%, overall efficiency as 80%  $C_v$  as 0.97 and  $K_u$  as 0.46.

28) Obtain an expression for power developed in a **reaction turbine**. ( 8 marks )

29) Show that the hydraulic efficiency for a **francis turbine** having velocity of flow through runner as constant is given by relation  $\eta_h = 1 / ( 1 + \{ \frac{1}{2} \tan^2 \alpha / [(1 - \tan \alpha) / \tan \theta] \})$  where  $\alpha$  = guide blade angle  $\theta$  = runner vane angle at inlet The turbine is having radial discharge at outlet. ( 8 marks )

30) The following data are given for a **Francis turbine**: Net Head = 60m, speed  $N = 700\text{rpm}$ , shaft power  $P = 294.3\text{KW}$ , Overall efficiency = 84%, hydraulic efficiency = 93%, flow ratio = 0.2, breadth ratio  $n = 0.1$ , outer diameter of runner = 2 x inner diameter of runner, velocity of flow is constant at inlet and outlet and discharge is radial at outlet. Determine: i) guide blade angle ii) Runner vane angle at inlet and outlet iii) diameter of runner at inlet and outlet iv) width of the wheel at inlet. ( 16 marks )

31) The outer diameter of a **francis turbine** is 1.4m. The flow velocity at inlet is 9.5m/s. The absolute velocity at the exit is 7m/s. The speed of operation is 430rpm. The power developed is 12.25MW, with a flow rate of  $12\text{m}^3/\text{s}$ . Total head is 115m. For shockless entry, determine the angle of the inlet guide vane. Also find the absolute velocity at entrance, the runner blade angle at inlet

and the loss of head in the unit. Assume zero whirl at exit. Also find the specific speed.

( 16 marks )

32) A **Francis turbine** with an overall efficiency of 75% is required to produce 148.25KW power. It is working under a head of 7.62m. The peripheral velocity =  $0.26(2gH)^{1/2}$  and the radial velocity of flow at inlet is  $0.96(2gH)^{1/2}$ . The wheel runs at 150rpm and the hydraulic losses in the turbine are 22% of the available energy. Assuming radial discharge, determine: i) the guide blade angle ii) the wheel vane angle at inlet iii) diameter of the wheel at inlet and iv) width of the wheel at inlet. ( 16 marks )

33) A **reaction turbine** works at 450rpm under a head of 120m. Its diameter at inlet is 120cm and the flow area is  $0.4\text{m}^2$ . The angles made by absolute and relative velocity at inlet are  $20^\circ$  and  $60^\circ$  respectively with the tangential velocity. Determine the volume flow rate, the power developed and the hydraulic efficiency. ( 8 marks )

34) In an **inward radial flow turbine** water enters at an angle of  $22^\circ$  to the wheel tangent to the outer rim and leaves at 4m/s. The flow velocity is constant through the runner. The inner and outer diameters are 300mm and 600mm respectively. The speed of the runner is 300rpm. The discharge through the runner is radial. Find the i) inlet and outlet blade angles ii) taking inlet width as 150mm and neglecting the thickness of the blades, find the power developed by the turbine. ( 16 marks )

35) An **inward flow reaction turbine** having an overall efficiency of 80% is required to deliver 136KW. The head  $H$  is 16m and the peripheral velocity is  $3.3(H)^{1/2}$ . The radial velocity of flow at inlet is  $1.1(H)^{1/2}$ . The runner rotates at 120rpm. The hydraulic losses in the turbine are 15%

the flow available energy. Determine i) diameter of the runner ii) guide vane angle iii) the runner blade angle at inlet and iv) the discharge through the turbine. ( 16 marks )

36) In an **outward flow reaction turbine**, the internal and external diameters are 2m and 2.7m respectively. The turbine speed is 275rpm and the flow rate is  $5.5\text{m}^3/\text{s}$ . The width of the runner is constant at inlet and outlet and equal to 250mm. The head acting on the turbine is 160m. The vanes have negligible thickness and the discharge at the outlet is radial. Determine the vane angles and velocity of flow at inlet and outlet. ( 16 marks )

37) A **vertical reaction turbine** operates under 60m head at 400rpm the area and diameter of the runner at inlet are  $0.7\text{m}^2$  and 1m respectively. The absolute and relative velocities of the fluid entering are  $15^\circ$  and  $60^\circ$  to the tangential direction. Calculate hydraulic efficiency.

38) The inner and outer diameters of an **inward flow reaction turbine** are 50cm and 100cm respectively. The vanes are radial at inlet and discharge is radial. The inlet guide vane angles is  $10^\circ$ ,

assuming the velocity of flow as constant and equal to 3m/s. find the speed of the runner and the vane angle outlet.

39) Design a **Francis turbine** runner with the following data: Net head = 70m speed  $N = 800\text{rpm}$  output power = 400KW hydraulic efficiency = 95% overall efficiency = 85% flow ratio = 0.2, breadth ratio  $n = 0.1$ . Inner diameter is  $1/3$  outlet diameter. Assume 6% circumferential area of the runner to be occupied by the thickness of the vanes. The flow is radial at exit and remains constant throughout.

40) i) A Francis turbine developing 16120KW under a head of 260m runs at 600rpm. The runner outside diameter is 1500mm and the width is 135mm. The flow rate is  $7\text{m}^3/\text{sec}$ . The exit velocity at the draft tube outlet is 16m/sec. Assuming zero whirl velocity at exit and neglecting blade thickness determine the overall and hydraulic efficiency and rotor blade angle at inlet. Also find the guide vane outlet angle. ( 10 marks )

ii) Discuss about draft tube and its types. ( 6 marks )

41) Draw a neat sketch of **Kaplan turbine**, name the parts and briefly explain the working. ( 8 marks ) (or)

Draw a schematic diagram of a **kaplan turbine** and explain briefly its construction and working. Obtain an expression for work done by the runner. ( 8 marks )

42) A **Kaplan turbine** runner is to be designed to develop 7357.5 Kw shaft power. The net available head is 5.5m. Assume that the speed is 2.09 and flow ratio is 0.68 and the overall efficiency is 60%. The diameter of the boss is  $1/3$  of the diameter of the runner. Determine the diameter of the runner, its speed and its specific speed. ( 8 marks )

43) Calculate the diameter and speed of the runner of a **Kaplan turbine** developing 6000kw under an effective head of 5m. Overall efficiency of the turbine is 90%. The diameter of the boss is 0.4 times the external diameter of the runner. The turbine speed ratio is 2.0 and flow ratio is 0.6. ( 10 marks )

44) A **kaplan turbine** working under a head of 20m develops 15MW brake power. The hub diameter and runner diameter of the turbine are 1.5m and 4m respectively. The guide blade angle at the inlet is  $30^\circ$   $\eta_h = 0.9$  and  $\eta_o = 0.8$ . The discharge is radial. Find the runner vane angle and turbine speed. ( 16 marks )

45) i) A **Kaplan turbine** delivers 10MW under a head of 25m. The hub and tip diameters are 1.2m and 3m. Hydraulic and overall efficiencies are 0.9 and 0.85. If both velocity triangles are right angled triangles, determine the speed, guide blade outlet angle and blade outlet angle. (10 marks )

ii) Discuss about construction details of Kaplan turbine with a neat sketch. ( 6 marks )

46) A **kaplan turbine** develops 20000 Kw at a head of 35m and at a rotational speed of 320rpm. The outer diameter of the blades is 2.5m and the hub diameter is 0.85m. If the overall efficiency is 85% and

the hydraulic efficiency is 88%. Calculate the discharge, the inlet flow angle and the blade angle at the inlet.

47) A **Kaplan turbine** working under a head of 20m develops 16000bhp. The outer diameter of the runner is 3.5m and the hub diameter is 1.75m. The guide blade angle at the extreme edge of the runner is  $30^\circ$ . The hydraulic and overall efficiencies of the turbine are 88% and 84% respectively. If the velocity of whirl is zero at outlet, determine: i) Runner vane angles at inlet and outlet at the extreme edge of the runner and ii) speed of the turbine.

48) The hub diameter of a Kaplan turbine working under a head of 12m, is 0.35 times the diameter of the runner. The turbine is running at 100rpm. If the vane angle of the extreme edge of the runner at outlet is  $15^\circ$  and flow ratio is 0.6. find the diameter of the runner, diameter of the boss and the discharge through the runner. The velocity at the whirl at outlet is given as zero. ( 16 marks )

# **ME8351 MANUFACTURING TECHNOLOGY – I**

**OBJECTIVE:**

- To introduce the concepts of basic manufacturing processes and fabrication techniques, such as metal casting, metal joining, metal forming and manufacture of plastic components.

**UNIT I METAL CASTING PROCESSES 9**

Sand Casting : Sand Mould – Type of patterns - Pattern Materials – Pattern allowances –Moulding sand Properties and testing – Cores – Types and applications – Moulding machines– Types and applications; Melting furnaces : Blast and Cupola Furnaces; Principle of special casting processes : Shell - investment – Ceramic mould – Pressure die casting - Centrifugal Casting - CO<sub>2</sub> process – Stir casting; Defects in Sand casting

**UNIT II JOINING PROCESSES 9**

Operating principle, basic equipment, merits and applications of: Fusion welding processes: Gas welding - Types – Flame characteristics; Manual metal arc welding – Gas Tungsten arc welding - Gas metal arc welding – Submerged arc welding – Electro slag welding; Operating principle and applications of: Resistance welding - Plasma arc welding – Thermit welding – Electron beam welding – Friction welding and Friction Stir Welding; Brazing and soldering; Weld defects: types, causes and cure.

**UNIT III METAL FORMING PROCESSES 9**

Hot working and cold working of metals – Forging processes – Open, impression and closed die forging – forging operations. Rolling of metals– Types of Rolling – Flat strip rolling – shape rolling operations – Defects in rolled parts. Principle of rod and wire drawing – Tube drawing – Principles of Extrusion – Types – Hot and Cold extrusion. 40

**UNIT IV SHEET METAL PROCESSES 9**

Sheet metal characteristics – shearing, bending and drawing operations – Stretch forming operations – Formability of sheet metal – Test methods –special forming processes-Working principle and applications – Hydro forming – Rubber pad forming – Metal spinning– Introduction of Explosive forming, magnetic pulse forming, peen forming, Super plastic forming – Micro forming

**UNIT V MANUFACTURE OF PLASTIC COMPONENTS 9**

Types and characteristics of plastics – Moulding of thermoplastics – working principles and typical applications – injection moulding – Plunger and screw machines – Compression moulding, Transfer Moulding – Typical industrial applications – introduction to blow



moulding –Rotational moulding – Film blowing – Extrusion – Thermoforming – Bonding of Thermoplastics.

**TOTAL: 45 PERIODS**

**COURSE OUTCOMES:**

**Upon the completion of this course, the students will be able to**

<b>CO 1</b>	Explain different metal casting processes, associated defects, merits and demerits.
<b>CO 2</b>	Compare different metal joining processes.
<b>CO 3</b>	Summarize various hot working and cold working methods of metals.
<b>CO 4</b>	Explain various sheet metal making processes.
<b>CO 5</b>	Distinguish various methods of manufacturing plastic components.

## Unit I – Metal Casting

### PART A Questions with answer

#### **1 Name the steps involved in making a casting.**

Ans: Steps involved in making a casting are

(1) Pattern making	(2)	Sand mixing and preparation
(3) Core making	(4)	Melting
(5) Pouring	(6)	Finishing
(7) Testing	(8)	Heat treatment
(9) Re-testing		

#### **2 What are the applications of casting?**

Ans: Transportation vehicles (in automobile engine and tractors)

Machine tool structures

Turbine vanes and power generators

Mill housing pump filter and valve

#### **3 Define pattern.**

Ans: A pattern is defined as a model or replica of the object to be cast.

A pattern exactly resembles the casting to be made except for the various allowances.

#### **4 Define mould making.**

Ans: It is a model or form around which sand is packed to give rise to a cavity called as mould cavity, in which molten metal is poured and the casting is produced.

#### **5 Why is a pattern larger than casting ?**

Ans: A pattern is slightly larger than the casting because a pattern carries allowance compensate for metal shrinkage.

#### **6 What do you mean by coreprints in pattern ?**

Ans: To produce seats for the cores in the mould in which cores can be placed, for producing cavity in the casting. Such seats in the mould are called as coreprints.

#### **7 Name the functions of pattern.**

Ans:

Prepare a mould cavity

To produce seats for the cores

To establish the parting line

To minimize casting defects.

### **8 Name the materials for making patterns**

Ans: The common materials of which the patterns are made are as follows:

- (1) Wood                      (2) Metal                      (3) Plastic
- (4) Plaster                  (5) Wax

### **9 List the various alloys and metal used in pattern.**

Ans: The various metals and alloys employed for making patterns are :

- (a) Aluminium and its alloys                      (b) Steel
- (c) Brass                      (d) Cast iron
- (e) White metal

### **10 Explain wax moulding.**

Ans: After being moulded, the wax pattern is not taken out; rather the mould is inverted and heated and the molten wax comes out or gets evaporated, hence there is no chance of the mould cavity getting damaged while removing the pattern.

### **11 List the allowances of pattern.**

Ans: The following allowances are provided on the pattern :

- Shrinkage or contraction allowance
- Machining allowance
- Draft or taper allowance
- Distortion allowance
- (e) Rapping or shake allowance

### **12 List the three forms of contraction.**

Ans: Contraction takes place in three forms

- Liquid contraction
- Solidifying contraction
- Solid contraction

### **13 Shrinkage of metal depends on what factors?**

Ans: The shrinkage of metal depends on the following factors :

- The metal to be cast
- Pouring temperature of the molten metal
- Dimensions of the casting
- Method of moulding

### **14 What do you mean by finish allowance?**

Ans: Machining allowance or finish allowance is the amount of dimension on a casting which is made oversized to provide stock for machining.

### **15 What are the factors on which amount of machining depends?**

Ans: Factors affecting machining are  
Metal of casting  
Machining method used  
Casting method used  
Shape and size of the casting  
Amount of finish required on the machined portion

**16 Why is a taper allowance used?**

Ans: Draft allowance or taper allowance is given to all vertical faces of a pattern for their easy removal from sand without damaging the mould.

**17 When does warpage occur ?**

Ans: Warpage occurs when it is in irregular shape.  
It is of U or V-shape  
The arms having unequal thickness.  
One portion of the casting cools at a faster rate than the other.

**18 How do you eliminate warpage ?**

Ans: To eliminate this defect, an opposite distortion is provided on the pattern, so that the effect is balanced and correct shape of the casting is produced

**19 Enlist the factors affecting selection of types of pattern.**

Ans : The type of pattern to be used for a particular casting will depend on following factors  
Quantity of casting to be produced  
Size and shape of the casting  
Type of moulding method  
Design of casting

**20 Name any four types of pattern.**

Ans: The various types of patterns which are commonly used are as follows :

- |                                   |                                |
|-----------------------------------|--------------------------------|
| (1) Single piece or solid pattern | (2) Two piece or split pattern |
| (3) Loose piece pattern           | (4) Cope and drag pattern      |
| (5) Gated pattern                 |                                |

**21 Write the significance of loose moulding.**

Ans: Some patterns embedded in the moulding sand cannot be withdrawn, hence such patterns are made with one or more loose pieces for their easy removal from the moulding box.

**22 Name and give use of the pattern in which number of casting are made at a time**

Ans: Gated pattern by using gated patterns number of casting can be made at a time, hence they are used in mass production system.

**23 Piston rings are made by .....pattern**

Ans : Match plate pattern

These patterns are made in two pieces i.e. one piece mounted on one side and the other on the other side of the plate, called as match plate.

**24 What is the difference between sweep and segmental pattern?**

Ans: The main difference between them is that, a sweep is given a continuous revolving motion to generate the required shape, whereas a segmental pattern is a portion of the solid pattern itself and the mould is prepared in parts by it.

**25 Why are patterns coloured ?**

Ans: Patterns are provided with certain colours and shade for following reasons:

To identify quickly the main pattern body and different pattern parts.

To indicate the type of the metal to be cast.

To identify loose pieces, core prints, etc.

To visualise machined surfaces, etc.

**26 Selection of mould materials depends on.....**

Ans: selection depends on following factors

cost of the material

Quality of casting required

Number of casting required

Shape and size of the casting

Material to be cast, etc.

**27 What are the types of moulding sand?**

Ans: All types of sands used in the foundry can be grouped as:

1. Natural sand    2. Synthetic sand    3. Special sands

**28 Why is synthetic sand better than natural sand?**

Ans:

It requires less proportion of binder.

Higher refractoriness and permeability.

Properties can be easily controlled.

Refractory grain size is more uniform.

**29 Name the different types of special sand.**

Ans: Types of special sand are

- |                  |                 |                  |
|------------------|-----------------|------------------|
| (1) Green sand   | (2) Loam sand   | (3) Core sand    |
| (4) Parting sand | (5) Facing sand | (6) Backing sand |

**30 Define black sand**

Ans : It is the sand which backs up the facing sand and does not come in direct contact with the pattern. This sand has black colour and hence, sometimes called as black sand.

**31 Define green strength.**

Ans : A mould which has adequate green strength will retain its shape and does not distort or collapse, even after the pattern has been removed from the moulding box.

**32 Define permeability.**

Ans : The sand must be porous to allow the gases and steam generated within the moulds to be removed freely. This property of sand is known as permeability or porosity.

**33 Name the constituents of moulding sand.**

Ans : The main constituents of moulding sand are :

(1)	Sand	(2)	Binder
(3)	Additives	(4)	Water

**34 Classify binders and name the types in it**

Ans :

Organic binders

(a)	Linseed oil	(c)	Dextrin
(b)	Molasses	(d)	Pitch

Inorganic binders

(a)	Clay,	(b)	Sodium silicate
-----	-------	-----	-----------------

Portland cement

**35 Name the types of clay binders**

Ans: Clay binder which is most widely used have following types:

(a)	Bentonite	(b)	Fire clay	(c)	Limonite
(d)	Ball clay	(e)	Kaolonite		

**36 Additives are used so as to.....**

Ans: 1) To enhance the existing properties.

To develop certain other properties like resistance to sand expansion defects, etc.

**37 What do you mean by coal dust?**

Ans: It reacts chemically with the oxygen present in the sand pores and thus, produces a reducing atmosphere at the mould metal interface and prevents oxidation of the metal.

**38 Functions of sand preparation are.....**

Ans:

To develop optimum properties in the moulding sand.

To obtain even distribution of sand grains throughout the bond.  
To add suitable amount of water to activate clay binder.  
To deliver sand at the suitable temperature.

**39 Define Muller.**

Ans: It is a mechanical mixer used for mixing sand ingredients in dry state.

**40 Name various methods of sand testing.**

Ans:

- |     |                       |     |                     |
|-----|-----------------------|-----|---------------------|
| (1) | Moisture content test | (2) | Clay content test   |
| (3) | Permeability test     | (4) | Grain fineness test |

**41 Name the factors affecting permeability test**

Ans: permeability depends on the following factors:

Grain shape and size  
Grain distribution  
Binder and its contents  
Water amount in the moulding sand  
Degree of ramming

**42 Enlist the functions of core.**

Ans: Core provides a means of forming the main internal cavity for hollow casting.  
Core provides external undercut feature.  
Cores can be inserted to obtain deep recesses in the casting.  
Cores can be used to increase the strength of the mould.

**43 Define Core.**

Ans: Core is a sand shape or form which makes the contour of a casting for which no provision has been made in the pattern for moulding.

**44 Difference between core sand and mould sand.**

Ans: The main difference is that core sand has very low clay content and larger grain size.

**45 Core sand mixture consists of...**

Ans: Core sand mixture consists of sand, 1% core oil, 1% cereal and 2.5 to 6% of water.

**46 Name the core sand ingredients.**

Ans: Ingredients are

- |     |                       |     |              |
|-----|-----------------------|-----|--------------|
| (1) | Granular refractories | (2) | Core binders |
| (3) | Water                 | (4) | Additives    |

**47 What does core making consists of?**

Ans: Core making basically consists of following steps:

- |                           |                            |
|---------------------------|----------------------------|
| (1) Core sand preparation | (2) Core making            |
| (3) Core baking           | Core finishing or dressing |
| Setting the cores         | (4)                        |

**48 Define core driers.**

Ans: The special shapes, which support the green sand cores having curved surfaces, are known as core driers.

**49 List various types of core.**

Ans: Their main types are as follows

- |                     |                   |     |
|---------------------|-------------------|-----|
| (1) Horizontal core | (2) Vertical core | (3) |
| (4) Balanced core   | (5) Ram up core   | (6) |
| Drop core           |                   |     |

**50 What is core box?**

Ans: Core box is a pattern for making cores. They are employed for ramming cores in them. Core boxes provide the required shape to the core sand.

**51 Name the types of core boxes.**

Ans:

- |                                |                          |
|--------------------------------|--------------------------|
| (1) Half core box              | (2) Dump core box        |
| (3) Split core box             | (4) Strickle core box    |
| (5) Gang core box              | (6) Loose piece core box |
| Left and right hand core boxes |                          |

**52 Why do we use a core prints ?**

Ans: Core prints are basically extra projections provided on the pattern. They form core seats in the mould when pattern is embedded in the sand for mould making. Core seats are provided to support all the types of cores.

**53 Name the types of core prints.**

Ans: Core prints are of the following types:

- |                           |                          |
|---------------------------|--------------------------|
| (i) Horizontal core print | (ii) Vertical core print |
| (iii) Balanced core print | (iv) Cover core print    |

**54 Define mould.**



Ans: When the pattern is removed, a cavity corresponding to the shape of the pattern remains in the sand which is known as mould or mould cavity.

### **55 What is loam moulding?**

Ans: In this, a rough structure of component is made by hand using bricks and loam sand.

The sand used is known as loam sand or loam mortar.

### **56 Explain in short shell moulding.**

Shell moulding is suitable for thin walled articles.

It consists of making a mould that has two or more thin shell like parts consisting of thermosetting resin bonded sand.

### **57 Name any six hand mould tools**

Ans: A number of hand tools are

- |                         |                 |             |
|-------------------------|-----------------|-------------|
| (1) Shovel              | (2) Hand riddle | (3) Rammers |
| (4) Lifters or cleaners | (5) Draw spike  | (6) Bellow  |

### **58 Functions of moulding machine.**

Ans: The main functions of moulding machines are:

Ramming of moulding sand.

Rolling over or inverting the mould through 180°

Rapping of pattern.

Removing the pattern from the mould.

### **59 Name the types of moulding machine.**

Ans: Following are the types of moulding machines:

- |                               |     |                        |
|-------------------------------|-----|------------------------|
| (a) Squeeze moulding machines | (b) | Jolt moulding machines |
| (c) Jolt-squeezing machines   | (d) | Sand slinger           |

### **60 Difference between permanent mould casting and sand casting.**

Ans: The main difference between permanent mould casting and sand casting is that, in this the mould is permanent which is neither destroyed nor remade after each cast.

### **61 Name the type of die casting machine.**

Ans: The main types of die-casting machines are:

Hot chamber die-casting

Cold chamber die-casting

**62 Classify centrifugal casting.**

Ans: Centrifugal casting processes can be classified as:

- (a) True centrifugal casting (b) Semi-centrifugal casting  
Centrifuging

**63 What do you mean by shaking out operation?**

Ans: After solidification of casting, the mould are broken to obtain the final casting. This operation is known as shake out operation, which may be performed manually or mechanically.

**64 Operations performed after shaking are.....**

Ans: The various operations which are performed after shake out are as follows:

Removal of dry sand cores.

Removal of gates, risers, runners, etc.

Removal of unwanted metal projections, fins, etc.

Removal of adhering sand and oxide, scale from the casting surface.

**65 Define snagging.**

Ans: The operation of removal of unwanted metal projections and fins is called as snagging.

**66 Name defects occurring in casting.**

Ans:

- |                       |     |                         |     |           |
|-----------------------|-----|-------------------------|-----|-----------|
| (1) Blow holes        | (2) | Porosity                | (3) | Shrinkage |
| (4) Inclusions        | (5) | Hot tears or hot cracks |     |           |
| Misrun and cold shuts |     |                         |     |           |

**67 Name the inspection methods of casting .**

Ans:

- |                           |     |                         |
|---------------------------|-----|-------------------------|
| (1) Pressure test         | (2) | Magnetic particle test  |
| (3) Dye penetrant test    | (4) | Radiographic inspection |
| (5) Ultrasonic inspection | (6) | Visual inspection       |

**PART - B Questions**

- 1 (i) Describe the preparation of sand moulding process  
(ii) Explain the various types of pattern used in Mould Making.

- 2 (i) Classify the materials used for pattern making and write about them.  
(ii) What are the allowances given while making Pattern? Explain

- 3 (i) Classify the different types of moulding sand and explain.

(ii) Explain the method of moulding sand testing

4 (i) Describe the various properties required for the moulding sand.

(ii) Explain types of cores and its application

5 (i) Identify and Explain the various steps involved in sand core manufacturing.

6 (i) Explain squeeze Jolting machine with neat sketch.

(ii) Explain sand Slinger machine with neat sketch.

7 (i) Explain the Jolting Machine with neat sketch.

(ii) Explain construction and operation of Blast furnace with necessary sketch

8 (i) Describe the constructional feature of cupola furnace.

(ii) Describe the operation of Cupola furnace with necessary sketch.

9 (i) Enumerate the steps in sequence for producing Shell Moulding.

(ii) Explain lost wax - Investment casting processes with neat sketch

10 (i) Explain ceramic moulding with a sketch.

(ii) With the help of neat Sketch, describe in detail, the process of producing components by pressure die casting.

11 (i) Describe with a neat sketch of cold chamber die casting machine.

(ii) Describe the procedure of making castings by the true centrifugal casting and write its advantages and disadvantages.

12 (i) Briefly describe hot chamber die casting process.

(ii) Describe any one type of Centrifugal casting with neat diagram

13 (i) Explain how the pipes and cylinder liners are made by centrifugal casting process

14 (i) Explain stir casting method with a sketch.

(ii) Name any five casting defects and Explain the remedies for those defects.

## **Unit II – Metal Joining**

### **PART A Questions with answer**

#### **1 Define weldability.**

Ans: Weldability is defined as the capacity of a material to be welded under fabrication conditions imposed in a specific and suitably designed structure and to perform satisfactorily in the intended service.

#### **2 State requirement of a good weldability.**

Ans: A metallic material with adequate weldability should fulfil the following requirements:

Have full strength and toughness after welding.

Contribute to good weld quality even with high dilution.

Have unchanged corrosion resistance after welding.

Should not embrittle after stress relieving.

#### **3 How is welding classified?**

Ans: Welding is classified as

Gas welding

Arc welding

Resistance welding

Solid state welding

Thermo-chemical welding processes

Radiant energy welding processes

#### **4 Name the applications of welding.**

Ans: Applications of welding are

Aircraft                      construction

Automobile                Pressure vessels and tanks

Buildings                    equipment

Rail road                    Pippings and pipelines

#### **5 Write in short about gas welding.**

Ans: Gas welding is a fusion-welding or non-pressure welding method. It joins the metals, by using combustion heat of oxygen/air and fuel gas (acetylene, hydrogen, propane or butane) mixture.

#### **6        Name the types of gas welding.**

Ans: Following are the types of gas welding.

(a)    Oxy-acetylene welding                      (b)            Air-acetylene welding

(c)    Oxy-hydrogen welding                      (d)            Pressure gas welding

**7 Explain the principle of oxy-acetylene welding.**

Ans: When acetylene, in correct proportion, is mixed with oxygen in a welding torch and ignited, then the flame resulting at the tip of the torch is sufficiently hot to melt and join the parent metals.

**8 Name the types of flames.**

Ans: The generated flames are classified into following three types

Neutral flame (Acetylene and oxygen in equal proportion)

Oxidising flame (Excess of oxygen)

Reducing flame or carburising flame (Excess of acetylene)

**9 Explain neutral flame.**

Ans: The flame has a nicely defined inner cone which is light blue in colour and surrounded by an outer flame envelope.

**10 What are the metals welded using neutral flame?**

Ans: A neutral flame is mostly used for the welding of:

Mild steel	Cast iron
Aluminium	Stainless steel
Copper	

**11 How do we obtain oxidizing flame using neutral flame?**

Ans: If, after the neutral flame has been established, the oxygen supply is further increased then oxidising flame will be developed.

**12 How does the flame of an oxidizing flame look?**

Ans: It is recognised by the small white cone which is shorter, much bluer in colour and more pointed than neutral flame.

**13 Where is oxidizing flame used?**

Ans: An oxidising flame is used for :

Copper-base metals

Zinc-base metals

Ferrous metals such as manganese steel, cast iron, etc.

**14 Define carburizing flame.**

Ans: If the amount of oxygen supplied to the neutral flame is reduced, then the generated flame will be a carburising flame or reducing flame i.e more content of acetylene.

**15 Name the metals welded by carburising flame.**

Ans: This flame is generally used for:

Welding of low alloy steel rods  
Non-ferrous metals  
High carbon steel

**16 Write down the methods of welding.**

Ans: There are three typical methods that may be used which are as follows:

Leftward or fore-hand welding method  
Rightward or back-hand welding method  
Vertical welding method

**17 What do you mean by filler metal?**

Ans: Filler metal is the material which is added to the weld pool to assist in filling the gap.

**18 Explain the function of flux in welding.**

Ans: While welding, if the metal is heated in air then the oxygen from air combines with the metal to form oxides. This results in poor quality, low weld strength hence, to avoid this difficulty a **flux** is employed during welding. It prevents the oxidation of molten metal.

**19 What are the disadvantages of flux.**

Ans: Fluxes used in welding produces fumes that are irritating to the eyes, nose, throat and lungs.

**20 Give the applications of gas welding.**

Ans: Gas welding is most widely used for the following purposes:  
Joining thin materials.  
Joining most ferrous and non-ferrous metals.  
In automobile and aircraft industries.

**21 What is arc welding?**

Ans: Electric arc welding is a fusion welding process in which welding heat is obtained from an electric arc between an electrode and the workpiece.

**22 Define arc length and arc crater.**

Ans: The distance between the centre of arc of the electrode tip and the bottom of arc crater is called as **arc length**. A small depression is formed in the base of the metal which is called as **arc crater**.

**23 Name the equipments of gas welding**

Ans: The most commonly used equipments for arc welding are as follows:

- |     |                    |     |            |     |                      |
|-----|--------------------|-----|------------|-----|----------------------|
| (a) | A.C or D.C machine | (b) | Wire brush | (c) | Cables and connector |
|     |                    |     | Chipping   |     | Wire                 |
| (d) | Earthing clamps    | (e) | hammer     | (f) | brush                |

(g) Helmet  
Hand gloves, apron, etc.

(h) Safety goggles (i) Cable lug

#### **24 What are the functions of a coating on electrode?**

Ans: The coating improves penetration and surface finish.

Suitable coating will improve metal deposition rates.

It limits spatter, produces a quite arc and easily removes slag.

Core wire melts faster than the covering, thus forming a sleeve of the coating which constricts and produces an arc with high concentrated heat.

Coating saves the welder from the radiations.

#### **25 Name the types of arc welding.**

Ans: The main types of arc welding are as follows:

- |                           |                                |
|---------------------------|--------------------------------|
| (a) Carbon arc welding    | (b) Shielded metal arc welding |
| (c) Submerged arc welding | (d) Gas tungsten arc welding   |
| (e) Gas metal arc welding | (f) Electro slag welding       |
| (g) Plasma arc welding    | (h) Flux cored arc welding     |

#### **26 Define SMAW.**

Ans: It is an arc welding process where coalescence is produced by heating the workpiece with an electric arc set up between the flux coated electrode and the workpiece.

#### **27 What is submerged arc welding ?**

Ans: It is an arc welding process where coalescence is produced by heating, with an electric arc set up between bare metal electrode and workpiece.

#### **28 Explain in short plasma arc welding.**

Ans: It is an arc welding process where coalescence is produced by the heat obtained from a constricted arc set up between a tungsten electrode and the water cooled nozzle or the workpiece. The process employs two inert gases i.e. one forms the plasma arc and the second shields the plasma arc.

Filler rod may or may not be added and pressure is not required for welding.

#### **29 Write about special feature of flux cored welding.**

Ans: The electrode is **flux cored** i.e. flux is contained within the hollow electrode. The flux cored electrode is coiled and supplied to the arc as a continuous wire. The flux inside the wire provides the necessary shielding of the weld pool.

### **30 Give the applications of flux cored welding**

Ans: Applications of flux core welding are

- |     |                                    |     |                               |
|-----|------------------------------------|-----|-------------------------------|
| (1) | Bulldozer blades, main frames      | (2) | Rotating frames for cranes    |
| (3) | Tractor frames, punch press frames | (4) | Bridge girders, furnace tubes |
| (5) | Diesel engine chassis, etc.        |     |                               |

### **31 Explain resistance welding and its filler metal.**

Ans: Resistance welding is a process where coalescence is produced by the heat obtained from resistance offered by the workpiece to the flow of electric current in a circuit of which the workpiece is a part and by the application of pressure. Filler metal (rod) is not required during the process.

### **32 What are the factors affecting resistance welding?**

Ans: Four factors are involved in operation of resistance welding:

Amount of current passing through the workpiece.

The pressure that electrodes transfer to the workpiece.

Time during which current flows.

Area of electrode tip in contact with the workpiece.

### **33 Write the applications of resistance welding**

Ans: This process is used for:

Joining of sheets, bars, rods and tubes.

Welding of aircraft and automobile parts.

Making of cutting tools, fuel tanks of cars, tractors, etc.

### **34 Name the types of resistance welding**

Ans: Resistance welding process includes following methods:

- |                                       |                    |     |                         |
|---------------------------------------|--------------------|-----|-------------------------|
| (a)                                   | Spot welding       | (b) | Seam welding            |
| (c)                                   | Projection welding | (d) | Percussion welding      |
| (e)                                   | Flash butt welding | (f) | Resistance butt welding |
| (g) High frequency resistance welding |                    |     |                         |

### **35 What is adhesive bonding?**

Ans: Adhesive bonding is the process of joining materials by using adhesives. The term adhesive includes substances such as glues, cements and other bonding agents.

### **36 Write the main steps of adhesive bonding**

Ans: Main steps in adhesive bonding are

- |     |                       |     |                                       |
|-----|-----------------------|-----|---------------------------------------|
| (1) | Surface Preparation   | (2) | Applying the primer                   |
| (3) | Applying the adhesive | (4) | Assembling adhesive coated components |
| (5) | Curing the assembly   | (6) | Testing of the joints                 |



**37 Give various mediums of applying adhesives.**

Ans: Medium of applying the adhesive on the surfaces to be joined are as follows:  
Liquid, Tape, Film, Solution, Powder, Paste

**38 Name types of adhesives.**

Ans: The most commonly used adhesives are as follows:  
Thermoplastic adhesives  
Thermosetting adhesives

**39 Explain thermoplastic adhesives.**

Ans: Thermoplastic type adhesives soften at high temperature. They are easy to use and are employed as, air drying dispersions, emulsions or solutions that achieve their strength through the evaporation of the solvent.

**40 Explain thermosetting adhesives.**

Ans: Thermosetting adhesives, once hardened cannot be remelted and a broken joint cannot be rebounded by heating also. These types of adhesives cure or harden by chemical reactions like polymerisation, condensation, vulcanisation or oxidation caused by the addition of a catalyst; heat, pressure, radiations, etc.

**41 Name any four synthetic adhesives and their applications.**

Ans:

Phenolic	Structural bonding, plywood
Acrylic	Bonding of plastics, glass
Epoxy	Structural bonding, concrete repair, construction industries
Olefin polymers	Laminating, packaging, book-binding
Polyurethane	Bonding of flexible to non-flexible substrate
Urea	Plywood, furniture

**42 Give the applications of adhesive bonding.**

Ans: Adhesive bonding are used in following industries:

(a) Automotive	(b) Aircraft	Packaging
(d) Furniture	(e) Ship-building	Book-binding
(g) Shoe and apparel	(h) Medical and dental	Electrical

- (j) Railroad (k) Tape, etc.

**43 Define soldering and classify it.**

Ans: It is defined as a group of joining processes where coalescence is produced by heating to a suitable temperature and by using a filler metal having a liquidus not exceeding  $427^{\circ}\text{C}$  and below the solids of base metals. Soldering is classified as Soft solder, Hard solder.

**44 Define soft and hard soldering.**

Ans: **Soft soldering** is used in sheet metal work for joining parts that are not exposed to the high temperature action and not subjected to excessive loads and forces.

**Hard soldering** used solders which melt at higher temperatures and are stronger than those used in soft soldering.

**45 What is brazing?**

Ans: It is defined as a group of joining processes where coalescence is produced by heating to a suitable temperature and by using a filler metal having a liquidus above  $470^{\circ}\text{C}$  and below the solids of the base metal.

**46 Name the methods of brazing.**

Ans: There are various brazing methods such as:

- |                     |                      |
|---------------------|----------------------|
| o Torch brazing     | o Resistance brazing |
| o Immersion brazing | o Furnace brazing    |

**47 What do you mean by bronze welding?**

Ans: Bronze welding does not mean the welding of bronze, but it is a welding using bronze filler rod.

**48 Name different defects in weld.**

Ans: Some common weld defects are listed below:

- |                               |                            |     |
|-------------------------------|----------------------------|-----|
| (a) Cracks                    | (b) Distortion             | (c) |
| (d) Porosity and blow holes   | (e) Undercutting           | (f) |
| (g) Spatter                   | (h) Poor fusion            |     |
| (i) Poor weld bead appearance | (j) Incomplete penetration |     |

**PART - B Questions**

1 (i) Describe various types of welding joints with neat sketch and list out the types of edge preparation before Welding Process.

2 (i) Distinguish between Gas Welding and Arc Welding.

(ii) Distinguish between MIG and TIG Welding

- 3 (i) List out the types of arc welding process and list out the arc welding Equipments and selection factors for power sources  
(ii) Describe with neat sketch the various components of Oxy-Acetylene gas welding equipment
- 4 (i) Explain the various types of oxy-acetylene flames with sketches.  
(ii) Explain the Manual Metal Arc Welding Process with neat sketch.
- 5 Explain about the equipment and operation of GTAW process.  
(ii) Explain about the Advantages and Disadvantages of GTAW
- 6 (i) Explain Gas metal Arc Welding Process with Neat diagram  
(ii) Explain the Advantages, Disadvantages and Application of Gas Metal Arc Welding Process
- 7 (i) Describe the submerged arc welding process with neat diagram  
(ii) State its advantages and application of submerged arc welding process.
- 8 (i) Describe the process of Electro Slag Welding and mention their major application.  
(ii) Explain the Resistance spot welding Process with a neat sketch
- 9 (i) Explain with neat sketch the principle of resistance welding.  
(ii) Differentiate between upset welding and flash welding.
- 10 (i) Explain the Advantages, Disadvantages and limitation of Resistance Welding Process  
(ii) Explain in detail the Plasma Arc Welding process and write its applications and demerits
- 11 (i) Explain Thermit welding Process with neat sketch.  
(ii) Briefly explain the principle of operation advantages and limitations of Electron beam Welding.
- 12 (i) Explain the principle and application of Friction Welding Process.  
(ii) Explain the principle of Friction Stir Welding.
- 13 (i) Explain the advantages of Friction Stir Welding.  
(ii) Compare and Contrast Brazing and Soldering Process.
- 14 (i) Classify and enumerate the various welding defects with causes of occurrences and describe a method of detecting cracks on a weld surface

## **UNIT III Metal Forming Processes**

### **PART A Questions with answer**

**1. What is mechanical working?**

Mechanical working of a metal is a simply plastic deformation performed to change the dimensions, properties and surface conditions with the help of mechanical pressure.

**2. Define cold and hot working.**

Mechanical working of metals above the recrystallization temperature, but below the melting or burning point is known as hot working whereas; below the recrystallization temperature is known as cold working.

**3. What are the four major drawbacks of hot working?**

- As hot working is carried out at high temperatures, a rapid oxidation or scale formation of metal.
- Due to the loss of carbon from the surface of the steel piece being worked, the surface layer loses its strength.
- Close tolerance cannot be obtained.
- Hot working involves excessive expenditure on account of high tooling cost.

**4. What is the purpose of rolling?**

The main purpose of rolling is to convert larger sections such as ingots into smaller sections, which can be used directly in as rolled state or stock for working through other process.

**5. What are the types of rolling mills?**

According to the number and arrangement of the rolls, rolling mills are classified as follows:

- |                           |                            |
|---------------------------|----------------------------|
| 1. Two-high rolling mill  | 2. Three-high rolling mill |
| 3. Four-high rolling mill | 4. Tandem rolling mill     |
| 5. Cluster rolling mill   | 6. Planetary rolling       |

**6. What is the difference between a bloom and a billet?**

A bloom has a square cross section with minimum size of 150x150 mm and a billet is smaller than bloom and it may have any square section from 38 mm upto the size of a bloom.

**7. Explain cluster rolling mill.**

It is a special type of fourhigh rolling mill. In this, each of the two working rolls is backed up by two or more of the larger back up rolls.

**8. What is the main function of planetary rolling mill?**

The main feature of this mill is that, it reduces a hot slab to a coiled strip in a single pass.

**9. Define extrusion.**

The extrusion process consists of compressing a metal inside a chamber to force it out through a small opening which is called as die.

**10. What is impact extrusion?**

The raw material is in slug form which have been turned from a bar or punched from a strip. By using punch and dies, the operation is performed. The slug is placed in the die and struck from top by the punch operating at high pressure and speed.

**11. Name the method of extrusion.**

The different methods of extrusion are hot extrusion and cold extrusion.

**12. How is hot extrusion sub-divided?**

Hot extrusion process is subdivided as follows:

- (a) Direct or forward extrusion      (b) Indirect or backward extrusion  
(c) Tube extrusion

**13. Which extrusion requires less force and define it.**

As compared to direct extrusion, less total force is required in indirect extrusion.

In this type, the ram or plunger used is hollow and as it presses the billet against the back wall of the closed chamber, the metal is extruded back into the plunger.

**14. What is forging?**

Forging is the process of shaping heated metal by the application of sudden blows (hammer forging) or steady pressure (press forging) and makes use of the characteristic of plasticity of the material.

**15. How is forging classified?**

According to the equipment's utilised for forging, they are classified as follows:

1. Smith die (Open die) forging;
  - (a) Hand forging
  - (b) Power forging
2. Impression die (Closed die) forging:

- (a) Drop forging                      (b) Press forging  
(c) Machining or upset forging      (d) Roll forging

**16. Differentiate between hydraulic and mechanical press.**

Sr. No.	Press forging	Hammer/Drop forging
1.	Press forging is slow as compared to hammer forging, but the reduction in the size of heavy parts is comparatively rapid.	Hammer forging is fast process, but a large number of blows are applied in rapid succession for reduction in the size of heavy parts.
2.	In press forging there is no restriction of the component.	In hammer forging there is a restriction of the component size.

**17. Differentiate between press and drop forging.**

S.No.	Hot rolling	Cold rolling
1.	Metal is fed into the rolls after being heated above recrystallisation temperature.	Metal is fed into the rolls when its temperature is below recrystallisation temperature.
2.	Hot rolled metal does not show work hardening effect.	Cold rolled metal shows work hardening effect.
3.	Coefficient of friction between the rolls and stock is higher.	Coefficient of friction between rolls and stock is relatively lower.

**18. Name the typical forging operations.**

A typical smith forging operations are as follows:

1. Upsetting      2. Drawing out or drawing down      3. Cutting
4. Bending
5. Punching and Drifting      6. Setting down      7. Fullering      8. Welding

**19. What is upsetting?**

It is a process through which the cross-section of a metal piece is increased with a corresponding increase in its length.

**20. Name the opposite process to upsetting process.**

Drawing out is exactly a reverse process to that of upsetting. It is employed when a reduction in thickness, width of a bar is desired with a corresponding increase in its length.

**21. Explain fullering.**

Fullering is also called as spreading. Fullering the metal along the length of the work piece is done but working separate sections. In this, the axis of the work piece is positioned perpendicular to the width of the flat die.

**22. How are seamless tubes produced?**

Seamless tubing is a popular and economical raw stock for machining because it saves drilling and boring of parts. The piercing machine consists of two tapered rolls, called as piercing rolls.

**23. What is Sejournet process?**

That extrusion process which is based both on the use of a lubricant in a viscous condition at extrusion temperature and on a separation between the lubrication of the chamber wall and die is called Sejournet process.

**24. Name the process of seamless tubes and Why is it used?**

Roll piercing is a method of producing seamless tubing is a popular and economical raw stock for machining because it saves drilling and boring of parts.

**25. Define surface defects.**

Surface defects include defects like scale, rust, cracks, scratches, gouges, etc. It occurs due to the impurities and inclusions in the original cast material and different conditions related to material preparation and rolling operation.

**26. Name the defects in internal structural defects.**

These types of defects include following defects:

- |                  |                  |            |
|------------------|------------------|------------|
|                  | i) Wavy edges    | ii) Zipper |
| cracks           |                  |            |
| iii) Edge cracks | iv) Alligatoring |            |
|                  | v) Folds         | vi)        |

Laminations

**27. Define swaging.**

Rotary swaging is a process of reducing the cross-sectional shape of bars, rods, tubes or wires by a large number of impacting blows with one or more pairs of opposed dies.

**28. Compare hot and cold working.**

Sr. No.	Hot working	Cold working
1.	Hot working is carried out above the recrystallisation temperature but below the melting point, hence deformation of metal and recovery takes place simultaneously.	Cold working is carried out below the recrystallisation temperature and as such there is not appreciable recovery of metal.
2.	During the process, residual stresses are not developed in the metal.	During the process, residual stresses are developed in the metal.
3.	Because of higher deformation temperature used, the stress required for deformation is less.	The stress required to cause deformation is much higher.

S.No.	Hot rolling	Cold rolling
1.	Metal is fed into the rolls after being heated above recrystallisation temperature.	Metal is fed into the rolls when its temperature is below recrystallisation temperature.
2.	Hot rolled metal does not show work hardening effect.	Cold rolled metal shows work hardening effect.
3.	Coefficient of friction between the rolls and stock is higher.	Coefficient of friction between rolls and stock is relatively lower.

### **PART - B Questions**

- (i) Explain hot working and cold working processes.
  - (ii) Explain various forging operation
- (i) Explain the steps involved in drop forging with neat sketches. With suitable sketches describe open die forging.
  - (ii) Formulate the advantages and limitations of closed die forging.
  - (iii) Explain the Precision forging Process with neat sketch and also compare with Closed Die Forging process
- (i) Explain flashless forging operation.
  - (ii) Explain about Impression die forging
- (i) Explain in detail about the defects occurred in forging operations.
  - (ii) Draw a simple sketch showing rolling process and make a short note on deformation of grains in rolling
- Describe the ring rolling and thread rolling process



7. Classify and write notes on various rolling stand arrangement in detail.

8.

9. (i) Discuss the types of Rolling mills.

(ii) Discuss the types defects in rolled parts.

10. A 300 mm wide strip 25 mm thick is fed through a rolling mill with two powered rolls each of radius = 250 mm. the work thickness is to be reduced to 22 mm in one pass at a roll speed of 50 rev/min. the work material has a flow curve defined by  $K = 275 \text{ MPa}$  and  $n = 0.15$  and the coefficient of friction between the rolls and the work is assumed to be 0.12. Determine if the friction is sufficient to permit the rolling operation to be accomplished. if so, calculate the roll force, torque and horsepower.

11. Explain in detail about wire drawing

12. Explain with neat sketches the process of tube drawing of metals.

13. (i) Explain with a neat sketch the process of Rod Drawing.

(ii) Explain about Hot and Cold Extrusion

14. Explain the forward and backward extrusion process

15. Analyze and Sketch variation in pressure during the Extrusion process by direct and indirect methods.

16. (i) Compare direct and indirect Extrusion process

(ii) Write short notes on impact extrusion and hydro static extrusion. With neat diagram explain the process of forward extrusion. Explain also how hollow sections can be produced in this process

#### **UNIT IV Sheet Metal Processes**

##### **PART A Questions with answer**

**1. Name the different sheet metal.**

- |                      |                    |
|----------------------|--------------------|
| 1. Black iron        | 2. Galvanised iron |
| 3. Aluminium Sheets: | 4. Copper Sheets:  |
| 5. Stainless steel   | 6. Tin platyes     |

**2. Name the different hand tools used in sheets metals.**

- |                           |                       |           |
|---------------------------|-----------------------|-----------|
| 1. Hammers                | 2. Mallet             | 3. Swages |
| 4. Tongs                  | 5. Punches and shears | 6. Stakes |
| 7. Tri square and scribes | 8. Wing compass       |           |

**3. Name the operations of sheet metal working**

- Shearing
- Bending
- Drawing
- Forming

**4. Explain Shearing**

It is process of cutting a straight line across a strip, sheet or bar shearing process has three important stages;

- 1) Plastic deformation
- 2) Fracture ( Crack propagation)
- 3) Shear

Shearing is performed either by using hand or by using machines also.

**5. Explain Bending**

The bending operation involves stretching of metal on the outer surface and compressing it on inner surface along a neutral axis which unchanged. Sheet metal can be bent by hammering on a base by hand or by bending machines.

**6. Explain drawing**

Drawing operation is used to produce thin walled hollow shapes in sheet metal. It is carried out by using a die and punch on a press machine. If the drawn length is more than the width then the operation is called as deep drawing.

**7. Explain forming**

For safety purpose, the edges of the sheet metal products are formed or folded. Also formed edges provide stiffness to the components so that they will not retain their shapes during handling.

**8. What is press working?**

Press working is a chip less manufacturing process by which various components are produced from sheet metal.

**9. Why are press machines preferred?**

Press machines are preferred for mass production of similar components, because for each component separate tool is required and the cost of every press tool is very high as compared to the cost of other cutting tools.

**10. What is the difference between manually and power operated press?**

The main difference between manually operated press and power press is that, the former moves by means of a screw and the latter by means of a crankshaft.

**11. What is clutch and flywheel?**

Clutch and flywheel: Flywheel is used to store the energy, which is required to maintain the constant speed of the ram whereas, clutch is used to engage or disengage the drive shaft with the flywheel

**12. Define trimming.**

It is used for cutting unwanted excess material from the periphery of a previously formed workpiece.

**13. Define shaving**

It is almost similar to trimming, but only small amount of material is removed during the operation as compared to trimming

**14. Define lancing**

In this operation, there is a cutting of the sheet metal through a small length and bending this small cut portion downwards

**15. Explain embossing.**

With the help of this operation, specific shapes or figures are produced on the sheet metal. It is used for decorative purposes of giving details like names, trade marks, specifications, etc, On the steel metal.

**16. What is clearance?**

During metal cutting, the shape of the punch is similar to die opening except that, it is smaller on each side. This difference in dimensions between die and punch (making members of a die set) is known as clearance.

**17. How is clearance applied in blanking operation?**

In blanking operation, where blank is the desired part, the die opening size is same as blank size and the punch size is obtained by subtracting the clearance from the die opening size.

**18. Why is angular clearance provided?**

Angular clearance is provided to enable the blank to clear the die easily and fall freely out of the die block. If the angular clearance is not provided, the punched blank would remain stuck in the die block.

**19. What is die space and press adjustment?**

Die space: The available surface for mounting the die and punch components in the press.

Press adjustment: The distance through which the ram can be lowered below its shut height position.

**20. What do you mean by shallow and deep drawing?**

If depth cup is upto half its diameter then the process is called as shallow drawing and if the depth of the drawn cup exceeds the diameter, it is called as deep drawing.

**21. Define drawing force:**

the force required to draw a shell is given by,

$$F = \pi d t \sigma_y (D/d - C), N$$

Where ,d= Finished shell diameter,

t = Blank thickness,

$\sigma_y$ =yield Strength in tension,

D = Blank diameter,

C = Constant varies between 0.6to 0.7.

**22. What is stretch forming?**

Stretch forming is used for forming smoothly contoured parts or those having double curvatures on the same curved surface out of large and thin sheets of metal.

**23. Name the methods of stretch forming.**

Stretch forming can be done by using two methods.

- 1) Form block method
- 2) Mating die method

**24. Give the advantages of stretch following.**

- There is no direct bending of the sheet; hence chances of cracks are reduced.
- Components can be stretched in single operation only.
- Plastic deformation is because of pure tension only.
- It is suitable for mass production.
- Cost of tooling is low.

**25. Define formability.**

Formability represents the response and suitability of the material for forming processes.

**26. Name the ways on which hydro forming can be carried out?**

Hydro forming is a drawing process which can be carried out in two ways:

- 1) Hydro - mechanical forming
- 2) Electro - hydraulic forming

**27. Name the tests to judge the suitability of metal.**

To judge the suitability of a metal and to find the temperature range for forging, various special tests have been introduced which are as follows:

- 1) Bend test
- 2) Stretch flanging test
- 3) Shrink flanging test

**28. What is explosive forming and how is it classified?**

Explosive forming makes use of the pressure wave generated by an explosion in a fluid, for applying the pressure against the wall of the die. The explosives are used in the form of rod, sheet, granules, stick, liquid, etc. According to the placement of the explosive (charge) the operations are divided in two categories:

- 1) Stand off operation
- 2) Contact operation.

**29. Why is explosive forming process used?**

Explosive forming process is used for the following operations:

- Blanking
- Embossing
- Coining
- Drawing
- sizing
- Expanding
- Cutting, etc.

**30. Name the explosives and how are they divided?**

Explosives can be solid (TNT- trinitro toluene), liquid (Nitroglycerine), or gaseous

(oxygen and acetylene mixtures).

Explosives are divided into two classes; Low Explosives in which the ammunition burns rapidly rather than exploding, hence pressure build up is not large, and High Explosives which have a high rate of reaction with a large pressure build up. Low explosives are generally used as propellants in guns and in rockets for the propelling of missiles.

**31. Name the certain die materials for explosive forming.**

Kirksite and plastic faced dies are employed for light forming operations; tool steels, cast steels, and ductile iron for medium requirements.

**32. What is rubber pad forming?**

Rubber pad forming process is also known as Marform process. It is metal working process where sheet metal is pressed between a die and rubber block.

**33. Give the applications of rubber pad forming.**

Applications of rubber pad forming:

- This process is used for producing flanged cylindrical and rectangular cups, spherical domes, shells with parallel or tapered walls.
- Also used for producing variety of unsymmetrical shapes.

**34. What does a magnetic pulse forming consists of?**

A basic magnetic pulse forming circuit consists of:

- Energy storage capacitor
- Switch
- Power supply
- Coil

**35. Explain shot peening.**

Shot peening process consists of throwing a blast of metal shots on to the surface of a component. The blast may be thrown either by using air pressure or by a wheel rotating at high speed. This high velocity metal blast shot provides a sort of compression over the surface of a component. This increases the strength and hardness of the surface and also its fatigue resistance.

**36. Where is metal spinning used?**

Usually, spinning process is used for making cup shaped articles which are symmetrically such as pressure vessels, refinery equipment's, tanks, etc.

**37. Define roll forming**

Roll forming process consists of feeding a continuous metal strip through a series of rolls whereby it is gradually formed into required shapes.

**38. What are the operations performed on press brake?**

Press brake can perform operations like bending, forming, blanking, piercing, notching, embossing, wiring, etc. by using simple dies.

**39. What is super plastic forming operation?**

Superplastic forming is a metalworking process for forming sheet metal. It works upon the theory of superplasticity, which means that a material can elongate beyond 100% of its original size.

**40. Give the difference between punching and blanking.**

**Blanking:** It is the cutting operation of a flat metal sheet. The article punched out is known as blank. Blank is the required product of the operation and the metal left behind is considered as a waste.

**Punching:** It is similar to blanking; only the main difference is that, the hole is the desired product and the material punched out to form a hole is considered as a waste.

**41. How is hydroforming is similar to rubber forming?**

In both the sheet metal working processes sheet metal is pressed between a die and rubber block. Under pressure, the rubber and sheet metal are driven into the die and conform to its shape by forming the part.

**42. What do you mean by minimum bend radius?**

It is the radius of curvature on inside surface of the bend. If the bend radius is too small, then cracking of a material on the outer tensile surface takes place. To prevent any damage to punch and die, the bend radius should not be less than 0.8mm.

**43. Define limiting drawing ratio.**

It is the ratio of finished shell diameter (d) to the radius of bottom corner.

**PART - B Questions**

1. (i) Summarizes the sheet metal characteristics
2. (i) Write a short notes on sheet bending and perforating operation  
(ii) Explain the important factors of Bending operations.
3. Write short notes on the following Shearing, Blanking, clearance in shearing and Spring back in bending
4. Explain the different types of bending process
5. (i) Explain various sheet metal drawing operations with sketches

(ii) Describe with a neat sketch any two type of stretch forming operations

6. (i) Explain the Formability of sheet metals and formability test methods

(ii) Compare Conventional forming with high strain rate forming technique.

7. (i) Explain with a neat sketch hydro forming

(ii) Describe Rubber Pad Forming with suitable sketch.

8. (i) Explain Metal spinning operation with a diagram.

(ii) Summarise the advantages and application of metal spinning

9. (i) Formulate the process variables in Explosive Forming and explain explosive forming with sketch.

(ii) Describe Magnetic Pulse Forming with a neat sketch.

10. (i) Explain peen forming with sketch.

(ii) Describe super plastic forming and Explain with neat sketch.

11. (i) Explain Micro forming.

(ii) Describe the die cutting and slitting operations.

12. (i) Describe the nibbling and notching operations

(ii) Explain in detail the Coining and Embossing Process.

13. (i) Point out the advantages and limitation of compound dies over progressive dies

(ii) Analyse the reasons to provide proper clearance between the punch

14. (i) Differentiate single die and multiple operation die with neat sketch

(ii) Discuss the advantages and limitations of single and multiple die operation

## **UNIT V Manufacture of Plastic Components**

### **PART A Questions with answer**

**1. Name the characteristic of polymer.**

The important characteristics of polymers are

1) Light weight

2) High Corrosion resistance.

3) Low density.

4) Low thermal and electrical properties.



5) Low mechanical properties (can be improved by fibre reinforcement of plastics)

**2. On what basis are polymers classified and how are they classified?**

According to mechanical response at high temperatures, polymers are classified into two major categories:

- 1) Thermoplastic polymers (Soften when heated and harden when cooled)
- 2) Thermosetting polymers (Soften when heated and permanently hardened when cooled).

**3. Give the mechanism of thermosetting polymers.**

These plastics are formed by condensation polymerisation. During initial heating, covalent cross-links are formed which anchor the chains together and resist the vibrational and rotational chain motions at high temperature. If heated to excessively high temperature, there occurs severance of these crosslink bonds leading to polymer degradation.

**4. Differentiate thermosetting and thermoplastic polymers**

Sr. No.	Thermoplastics	Thermosetting
1.	They are formed by addition polymerisation	They are formed by condensation.
2.	They are linear polymers composed of chain molecules.	They are composed of three dimensional network of cross-linked molecules.

**5. Where are the thermosetting polymers used?**

These polymers are used in

- (i) Vulcanised rubbers
- (ii) Epoxides
- (iii) Phenolic
- (iv) Polyester resins
- (v) Urea formaldehyde, etc.

**6. Define Degree of polymerization**

It is the number of repetitive units present in one molecule of a polymer.

$$\text{Degree of polymerisation} = \frac{\text{Molecular weight of a polymer}}{\text{Molecular weight of a single monomer}}$$

**7. Define Isomerism.**

It is a phenomenon where different atomic configurations are responsible for the formation of same configuration.

**8. Define Oligo-polymers.**

Oligo polymers or oligomers are polymers that have very short chains with molecular weight in order of 100g/mol. They are mainly liquids or gases.

**9. Define High polymers.**

Polymers which have a very high molecular weight ranging between 10,000 and 1,000,000 g/mol. are known as High-polymers. They are mainly solids.

**10. Give the three methods of mechanism of polymerisation:**

There are three general methods or mechanisms of polymerisation:

(1) Addition Polymerisation, (2) Copolymerisation, (3) Condensation polymerisation

**11. Define addition polymerization.**

The Polymer is produced by adding a second monomer to the first, a third monomer to this dimer and so on till the long polymer chain is terminated. This process is called as addition polymerisation.

**12. Define copolymerisation and give its example.**

It is the addition polymerisation of two or more different monomer forming copolymers. Example: Styrene and butadiene combine to give a copolymer of butadiene - styrene, a rubber used in tyres.

**13. Define condensation polymerisation and give its other name.**

Condensation polymerisation is also known as step-growth polymerisation.

It is the formation of polymers by step wise intermolecular chemical reactions that normally involve at least two different monomers.

**14. Why are additives used and enlist its advantages?**

Additives used to improve the properties and performance of polymers.

Advantages of additives when added to the polymers are:

- (i) Improve mechanical properties.
- (ii) Reduce the cost.
- (iii) Improve the thermal processing such as moldability.
- (iv) Improve the appearance and aesthetic properties.

**15. Enlist some polymer additives.**

The following mentioned are the various polymer additives used in practice :

- (1) Filler material (2) Plasticizers (3) Stabilizers
- (4) Colorants (5) Flame retardants (6) Reinforcements (7) Lubricants.

**16. Why are fillers used?**

- (1) It improves the compressive and tensile strengths of the polymer.
- (2) Reduces the cost of the final product.
- (3) Improvement in the thermal and dimensional properties of the polymers.

**17. Why are plasticizers used?**

- (1) They improve the ductility, flexibility and toughness of the polymer.
- (2) Hardness and stiffness are reduced.
- (3) During moulding, plasticizers control the flow of the polymer.

**18. Why are stabilizers used?**

- (1) They prevent deterioration of polymer due to environmental effects.
- (2) Also prevent deterioration due to ultraviolet radiation.
- (3) Help to extend the life of the finished product.

**19. Name the methods of processing thermoplastics.**

Thermoplastics can be processed to their final size and shape with the help of following processes:

- (1) Injection moulding (plunger and screw type) (2) Rotational moulding
- (3) Blow moulding (4) Film blowing
- (5) Sheet forming process.

**20. What are the applications of injection moulding?**

Typical parts produced by this process are cups, chairs, toys, containers, knobs, automobile parts (car dash-board, car handles, etc), air conditioner parts, plumbing fittings, electrical fittings, etc. This process is used for making components which consists of complex threads. Production of intricate shapes and thin walled parts like radiator fan can be done by this process.

**21. Give the types of injection moulding.**

- (1) Ram or Plunger type Injection Moulding
- (2) Screw type Injection Moulding

**22. Enlist the types of blow moulding.**

There are various types of blow moulding process which are as follows:

- (1) Injection blow moulding
- (2) Extrusion blow moulding

**23. Where is blow moulding used ?**

1. Blow moulding process is mainly used for making cosmetic packaging, food and water bottles, pipes, floats, toys, doll bodies and many other articles.
2. It is also used for making hollow containers, automobile fuel tanks, boat fenders, heater ducts and hollow industrial parts like drum.

**24. Explain film blowing**

In this process, a thin walled tube is extruded vertically as shown in fig. 5. 6 and expanded into a balloon like shape by blowing air through the centre of extrusion die until the desired film thickness is obtained

**25. What is the difference between rolling and calendering?**

The main difference between rolling and calendering is that, in calendering there is appreciable thickening after the material has reached minimum thickness at the roll gap and the pre-calendered material is not in the sheet form.

**26. Give the application of calendering?**

- (1) Vinyl, polyethylene, cellulose acetate films, shower curtain, tapes, trays, ATM cards, lamination, and transparent films used for packaging.

**27. Define extrusion moulding .**

Extrusion process is a continuous process in which the hot plasticized material forced through the die opening of required shape.

**28. Name the three sections of screw.**

The screw has three different sections which are as follows

- (1) Feed section      (2) Trition or melting section,      (3) Pumping section

**29. Write the applications of extrusion moulding.**

- (1) The extrusion moulding process is used for producing solid rods, pipes or tubes of U, J, Y or other sections.
- (2) Also used for extrusion of candy canes, chewing gums, drinking straws, plumbing pipes, door insulation seals, optical fibers, plastic coated wires, window frames, sheets, strips for electrical applications, etc.

**30. Explain thermoforming.**

It is a series of processes for forming thermoplastic sheet or film over a mould with the application of heat and pressure.

**31. List the advantages of thermoforming.**

- Initial set-up cost is low.
- Time required for set-up is low.
- Production cost is low.
- During the process less thermal stresses are produced.

- Intricate shapes are easily formed.

**32. Give the application of compression moulding.**

(1) Compression moulding is used for making flatwares, gear, buttons, buckles, knobs, handles, dishes, container taps and fittings  
(2) Also used for moulding of electrical and electronic components, washing machine agitators and housings.

**33. What is gate moulding?**

This is the process of forming articles in a closed mould, where the fluid plastic material is conveyed into the mould cavity under pressure from outside of the mould.

**34. Name the processing methods of plastics?**

(1) Plug and ring forming      (2) pressure forming  
(3) Draw forming                  (4) reaction injection moulding  
(RIM)  
(5) Drape forming

**35. Define pressure forming?**

In this method, the heated plastics sheet is formed into the required shape between a pair of male and female dies. In this process vacuum is not used.

**36. What is draw forming?**

This process is similar to deep drawing process for metal. A heated blank of plastics sheet is plated over a die and held firmly by holding plates. A punch is pressed down into the die cavity to the material into the die and around its own body.

**37. Explain the drape forming?**

It is the simplest of all methods of forming. It consists of draping the heated plastics sheet over the contours of a male form, followed by pressure and cooling.

**38. Define reaction injection moulding?**

RIM is the different forms the conventional injection moulding process as the molten polymer is not injected into a mould but a mixture of two or more monomers (reactants) are forced into a mould cavity. The chemical reaction takes places between the mixture and the heat is generated. This generated heat is used to form a plastics polymer that solidifies and produces thermoset components.

**PART - B Questions**

- (i) Explain the types of Plastics
- (ii) State the purpose of the following plastics 1. plasticizers 2. Filler 3. Stabilizers

2. (i) Discuss about a few Commercial Plastics
3. (i) List out and write the various processes of joining plastics  
(ii) Summarize the various differences between thermoplastics and thermosetting plastics .
4. (i) Explain the injection blow moulding process.  
(ii) Enumerate injection moulding of plastic products
5. (i) Describe the process Equipment for Injection moulding  
(ii) Explain the working principle of plunger and screw type injection machines.
6. (i) Explain Positive, semi positive and flash type Compression  
(ii) State the typical industrial applications of Transfer moulding.
7. (i) Explain transfer moulding. Discuss its advantages and limitations.  
(ii) Explain the process of compression moulding with neat diagram
8. (i) Compare blow moulding and rotational moulding  
(ii) Explain the Process Rotational moulding
9. (i) Explain the Extrusion blow moulding process  
(ii) Describe the Blown-film Extrusion process
10. (i) Explain the calendaring process.  
(ii) Enumerate and write about various methods of bonding thermoplastics
11. (i) Discuss in detail the various thermosetting and thermoplastic compound and their application  
(ii) Describe any two types of thermoforming process.
12. Explain various types of thermoforming method shaping thermoplastics
13. Explain the structure of thermo plastic and thermosetting plastics.
14. Explain the various methods of Bonding of Thermoplastic.

# **EE8353 ELECTRICAL DRIVES AND CONTROLS**

## **EE8353 ELECTRICAL DRIVES AND CONTROLS L T P C 3 0 0 3**

### **OBJECTIVES:**

- To understand the basic concepts of different types of electrical machines and their performance.
- To study the different methods of starting D.C motors and induction motors.
- To study the conventional and solid-state drives

### **UNIT I INTRODUCTION 8**

Basic Elements – Types of Electric Drives – factors influencing the choice of electrical drives – heating and cooling curves – Loading conditions and classes of duty – Selection of power rating for drive motors with regard to thermal overloading and Load variation factors

### **UNIT II DRIVE MOTOR CHARACTERISTICS 9**

Mechanical characteristics – Speed-Torque characteristics of various types of load and drive motors – Braking of Electrical motors – DC motors: Shunt, series and compound - single phase and three phase induction motors.

### **UNIT III STARTING METHODS 8**

Types of D.C Motor starters – Typical control circuits for shunt and series motors – Three phase squirrel cage and slip ring induction motors.

### **UNIT IV CONVENTIONAL AND SOLID STATE SPEED CONTROL OF D.C. DRIVES 10**

Speed control of DC series and shunt motors – Armature and field control, Ward-Leonard control system - Using controlled rectifiers and DC choppers –applications.

### **UNIT V CONVENTIONAL AND SOLID STATE SPEED CONTROL OF A.C. DRIVES 10**

Speed control of three phase induction motor – Voltage control, voltage / frequency control, slip power recovery scheme – Using inverters and AC voltage regulators – applications.

**TOTAL: 45 PERIODS**



**COURSE OUTCOMES:**

**On Successful completion of this course, student should be able to**

CO1	Acquiring knowledge on the basics of Electrical drives.
CO2	Explain the performance characteristics and different types of DC and AC Drives.
CO3	Illustrate the concept of starting methods of DC and AC Drives.
CO4	Explain the concept of speed control of DC Drives.
CO5	Explain the concept of speed control of AC Drives.
CO6	Explain the concept of speed control of AC Drives with power electronics application.

**TEXT BOOKS:**

1. Nagrath .I.J. & Kothari .D.P, "Electrical Machines", Tata McGraw-Hill, 2006
2. Vedam Subrahmaniam, "Electric Drives (Concepts and Applications)", Tata McGraw-Hill, 2010

**REFERENCES:**

1. Partab. H., "Art and Science and Utilisation of Electrical Energy", Dhanpat Rai and Sons, 2017
2. Pillai.S.K "A First Course on Electric Drives", Wiley Eastern Limited, 2012
3. Singh. M.D., K.B.Khanchandani, "Power Electronics", Tata McGraw-Hill, 2006.

## **UNIT 1 – INTRODUCTION**

### **PART A**

#### **1. What is meant by electrical drives?(Dec 2016)**

Systems employed for motion control are called "Drives" and many employ any of the prime movers such as, diesel or petrol engines, gas or steam turbines, hydraulic motors and electric motors for supplying mechanical energy for motion control. Drives employing electrical motors are known as "Electrical drives".

#### **2. What are the different types of drives? (Dec 2014)**

1. Group drive
2. Individual drive
3. Multi motor drive

#### **3. What are the different types of electrical drives? (June 2013)**

1. DC drives
2. AC drives

#### **4. What are the advantages of electric drives?**

i. They have flexible control characteristics. The steady state and dynamic characteristics of electrical drives can be shaped to satisfy load requirements.

ii. Drives can be provided with automatic fault detection systems. Programmable logic controllers and computers can be employed to automatically control the drive operations in a desired sequence.

iii. They are available in wide range of torque, speed and power.

iv. It can operate in all the four quadrants of speed-torque plane. Electric braking gives smooth deceleration and increases life of the equipment compound to other forms of braking

v. Control gear enquired for speed control, starting and braking is usually simple and easy to operate

#### **5. Mention the different factors for the selection of electric drives.(May 2013) (May 2014) (April 2017)**

1. Steady state operation requirements
2. Transient operation requirements
3. Requirements related to the source cost, maintenance needs, life
4. Capital and running
5. Environment and location
6. Reliability

#### **6. What are the parts of electrical drives?(May 2014)**

1. Electrical motors and load
2. Power modulator
3. Sources
4. Control unit
5. Sensing unit

## **7. What are the applications of electrical drives?**

1. Paper mills
2. Electric traction
3. Cement mills
4. Steel mills

## **8. What is an individual drive?( May 2015)**

If a single motor is used to drive a single machine and all the connected mechanisms belonging to the same machine then the system is called individual drive system

Eg: Lathe machine .

## **9. What are the disadvantages of an individual drive?**

Initial cost is high.

## **10. What are the advantages of group drive?**

- a) Initial cost is less
- b) Less space is required in group drive
- c) Maintenance cost is less
- d) Group drive system is useful because all operation are stopped simultaneously

## **11. Define continuous duty.**

Continuous duty is defined as the load that may be carried by the machine for an indefinite time

without the temperature rise of any part exceeding the maximum permissible value.

## **12. What are the different types of classes of duty?**

- |   |  |                           |
|---|--|---------------------------|
| ✓ | Continuous duty                                    | Main classes of<br>duties |
| ✓ | Short time duty operation of motor                 |                           |
| ✓ | Intermittent periodic duty                         |                           |
| ✓ | Intermittent periodic duty with starting           |                           |
| ✓ | Intermittent periodic duty with starting & braking |                           |
| ✓ | Continuous duty with intermittent periodic loading |                           |
| ✓ | Continuous duty with starting & braking            |                           |
| ✓ | Continuous duty with periodic load changes         |                           |

## **13. Define equivalent current method.**

The motor selected should have a current rating more than or equal to the equal current. It is also necessary to check the overload capacity of the motor. This method of determining the power rating of motor is known as the equivalent current method.

## **14. What is meant by cooling time constant?(April 2017)**

It is defined as the ratio between C and A. Cooling time constant is denoted as  $\tau = C/A$

Where  $C$  = amount of heat required to rise the temperature of the motor body by 1 degree Celsius in  $^{\circ}\text{C}$ ,  $A$  = amount of heat dissipated by the motor per unit time per degree Celsius in  $\text{J/S}/^{\circ}\text{C}$ .

(or)

Cooling time constant is defined as the time required to cool the machine down to 0.368 times the initial temperature rise above the ambient temperature.

**15. What are the assumptions made while performing heating & cooling calculation of an electric motor?**

The machine is considered to be a homogeneous body having a uniform temperature gradient. All the points at which heat generated have the same temperature. All the points at which heat is dissipated are also at same temperature.

Heat dissipation taking place is proportional to the difference of temperature of the body and surrounding medium. No heat is radiated.

The rate of dissipation of heat is constant at all temperatures.

**16. What are the factors that influence the choice of electrical drives? (Dec 2011) (NOV 2015)**

1. Shaft power & speed 2. Power range 3. Speed range 4. Starting torque 5. Efficiency 6. Starting torque 7. Influence on the supply network 8. Maintenance 9. Special competence 10. Total purchase cost 11. Cost of energy losses 12. Influence on power supply 13. Environment 14. Availability 15. Accessibility 16. Nature of electric supply 17. Nature of load 18. Types of drive 19. Electrical Characteristics 20. Service cost 21. Service capacity & rating

**17. Why the loss at starting is not a factor of consideration in a continuous duty motor?**

While selecting a motor for this type of duty it is not necessary to give importance to the heating

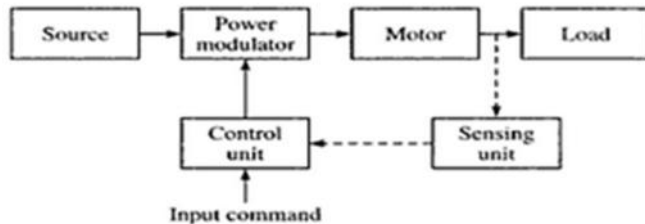
Caused by losses at starting even though they are more than the losses at rated load. This is because the motor does not require frequent starting it is started only once in its duty cycle and the losses during starting do not have much influence on heating.

**18. What is meant by “short time rating of motor”? (Dec 2016) (April 2017)**

Any electric motor that is rated for a power rating  $P$  for continuous operation can be loaded for a

short time duty ( $P_{sh}$ ) that is much higher than  $P$ , if the temperature rise is the consideration.

**19. Draw the basic block diagram of Electric drive.( Dec 2013)(Nov 2015)**



**20. What is meant by “load equalization”?**

In the method of “load Equalization” intentionally the motor inertia is increased by adding a

flywheel on the motor shaft, if the motor is not to be reversed. For effectiveness of the flywheel, the motor should have a prominent drooping characteristic so that on load there is a considerable speed drop.

**21. What is short time duty?**

In short time duty the period of operation is so short that the temperature rise of the motor does not reach its final steady value and the period of rest is so long that the motor returns to cold conditions.

**22. Write the expression for thermal overload factor. (Dec 2014)**

$$\frac{Q_k}{Q} = \frac{1}{1 - e^{-\frac{t_k}{T_H}}} = K_T$$

$K_T$ - Thermal Overload Factor

**23. What is heating time constant?**

Heating time constant is defined as the time taken by the machine to attain 0.632 of its final steady temperature rise. The heating time constant of the machine is the index of the time taken by the machine to attain its final steady temperature rise.

## **24. How heating occurs in motor drives? (May 2015)**

An electric motor has various power losses, mainly copper losses in the winding and core losses due to the hysteresis losses and eddy current losses, in the core. These losses appear in the form of heat. The mechanical losses due to the friction and windage also contribute to such heat development

### **PART-B**

1. Explain the factors governing the selection of motors.(Dec 2014)(May 2015) (Dec 2016)
2. Discuss in detail the determination of power rating of motors.(May 2014)(Nov 2015)
3. Explain the different types of loading of drives.
4. Describe the simplifications based on which the heating and cooling calculations of an electric motor are made. Derive the heating and cooling curves of an electrical machine.  
(Dec 2011) (Nov 2013)(Dec 2014) (May 2015)
5. Write a brief note on classes of duty for an electric motor and also compare the D.C and A.C drives.(Dec 2013) (May 2014) (Dec 2014)(Nov 2015) .(Dec 2016) (April 2017)
6. Draw the typical temperature rise-time curve and derive the equation for temperature rise in an electric drive.
7. The enclosure of 20 KW motor is equivalent to a cylinder of 70 cm diameter and 100 cm length. The motor weighs 500kg. Assuming the specific heat is  $700^{\circ}\text{J/kg}^{\circ}\text{C}$  and that the peripheral surface of the enclosure of the motor alone is capable of heat dissipation of  $12.5\text{W/sq.m}^{\circ}\text{C}$ , calculate the heating time constant of the motor and its final temperature rise. Efficiency of motor is 80%.
8. Explain in detail about the various types of electric drives.(May 2013)(Nov 2015)(May 2015)
9. Select the power rating of a motor of 750 rpm, which has the following load pattern.  
A constant speed drive operating at a speed of 500 rpm has a cyclic loading as given below,  
  
a) 200Nm for 10 minutes      b) 300 Nm for 20 minutes  
  
c) 150 Nm for 20 minutes      d) No Load for 10 minutes
10. Estimate the power rating of the motor.(Dec 2013)(Nov 2015)
11. Draw the pattern of temperature rise characteristic under steady state for (i)Short time duty (ii) intermittent duty and explain the equivalent current method of estimating motor rating. (Dec2016)
12. A motor has a thermal time constant of 45 minutes. When the motor runs continuously on full load, its final temperature rise is

80°C. (i) What is the temperature rise after 1 hour if the motor runs continuously on full load? If the temperature on one hour rating is 80°C, find the maximum steady state temperature at this rating. (Dec 2016)

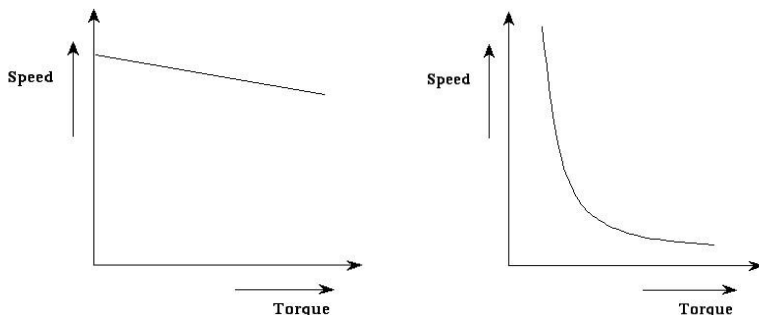
## **UNIT 2 - DRIVE MOTOR CHARACTERISTICS**

### **PART A**

**1. What is meant by mechanical characteristics?**

A curve is drawn between speed-torque. This characteristic is called mechanical characteristics.

**2. Draw the speed-torque characteristics of dc shunt motor and series motor. (May2013) (May 2014) (April 2017)**



**3. A series motor should never be started without some mechanical load why?**

When the load current  $I_a$  falls to a small value, speed becomes dangerously high. Hence a series

motor should never be started without some mechanical load.

**4. What is rheostatic braking?**

In rheostatic braking the armature is disconnected from the supply and is connected as a variable resistance. The braking is controlled by varying the series resistance.

**5. What are the different types of dc motor?**

1. DC series motor, 2. Shunt motor, 3. Compound motor,
4. Separately excited de motor

## **6. What is meant by electrical characteristics?**

A curve is drawn torque and armature current. It is known as electrical characteristics.

## **7. What is the relation between speed and flux of a dc motor?**

The speed of a dc motor is inversely proportional to field flux.

## **8. What is the application of dc motor?**

DC shunt motor:- 1. For driving constant speed operations, 2. Machine tools, 3. Lathes, 4. Blowers and fans, 5. Centrifugal pumps 6. Reciprocating pumps

DC series motor:- 1. Electric locomotives 2. Rapid transit systems, 3. Trolley cars, 4. Cranes and hoists, 5. Conveyors

DC compound motor:- 1. Elevators. 2. Air compressors 3. Rolling mills 4. Heavy planers

## **9. A dc shunt motor is called as constant speed motor-why? (May 2015, 2017)**

The drop in speed from no-load full-load is small; hence the dc shunt motor is also called as constant speed motor.

## **10. What is mean by braking?**

Whenever an electric drive is disconnected from the supply, the speed of the driving motor gradually decreases and becomes zero. Braking is a generic term used to describe a set of operating conditions for electric drive systems. It includes rapid stopping of the electric motor holding the motor shaft to a specific position, maintaining the speed to a desired value of preventing the motor from over speeding.

### **12. What are the two types of braking?**

1. Mechanical braking
2. Electrical braking

### **12. What is meant by mechanical braking?**

In mechanical braking, the frictional force between the rotating parts and brake drums provide the required brake.

### **13. What is meant by electric braking? (DEC 2016)**

In electric braking, the motor is made to work as generator. So it produces a negative slip and negative torque (braking torque). This is



achieved by suitably changing the electrical connections of the motor.

**14. What are the different types of electric braking?(Nov2013)(May 2013) ( May 2015)**

1. Regenerative braking
2. Dynamic braking
3. Plugging

**15. What are the advantages of electric braking?**

1. High efficient method
2. Low maintenance
3. Braking is very smooth

**16. What is meant by regenerative braking? (Dec 2011)**

In the regenerative braking operation, the motor operates as a generator, while it is still connected to the supply. Here, the motor speed is greater than the synchronous speed. Mechanical energy is converted into electrical energy, part of which is returned to the supply and rest of the energy is lost as heat in the winding and bearings.

**17. What is meant by dynamic braking?**

When an electric motor rotates, a kinetic energy of the motor is converted into electric energy. This energy is dissipated in resistive elements.

**18. What is meant by plugging? (Dec 2014)**

The plugging operation can be obtained by changing the polarity of the motor. For a machine, the phase sequence of the starter windings and dc machines the polarities of the field or armature terminals.

**19. What are the disadvantages of dc machine?**

1. Higher cost
2. Higher rotor inertia
3. EMI problems
4. Maintains problems with commutator

and brushes. 5. Do not permit a machine to operate in dirty and explosive environments.

**20. What are the advantages of squirrel cage induction motor?**

1. Rugged
2. Cheaper
3. Lighter
4. More efficient
5. Less maintenance
6. Can operate in explosive and dirty environment.

**21. What are the two types of rotors in three phase induction motors? (April 2017)**

1. Squirrel cage rotor
2. Slip ring rotor

**22. What is the necessity of braking?(Dec 2014)**

The quickness and accuracy of braking techniques determine the productivity and quality of the manufactured goods. Control the motor for our optimum requirement.

**23. What are different methods of Braking of DC series motor? (Nov 2015)**

1. Regenerative braking
2. Dynamic braking
3. Plugging

**24. A 220V DC shunt motor having the armature current of 10A, runs at 1500 rpm. Find the armature current if the source voltage drops to 150V. Assume the load torque is constant.(Nov 2015)**

Given,  $V_1=220V$ ,  $I_{a1}=10A$ ,  $V_2=150V$ ,  $I_{a2}=?$ .

For Shunt motor,  $I_{a1}\Phi_1 = I_{a2}\Phi_2$  (since torque is constant)

$$\Phi_2 = (150/220) \Phi_1 \text{ -----} \rightarrow \Phi_2 = 0.68 \Phi_1.$$

Therefore,

$$I_{a2} = (10 \Phi_1 / 0.68 \Phi_1) = 14.67A.$$

**25. What are the various components of load torque?(Dec 2016)**

- Components: Friction, Windage, Torque required to do useful work
- Friction at zero speed is called stiction or static friction
- Coulumb ( $T_c$ )+Friction at stand still ( $T_s$ )+Viscous Friction ( $B\omega_m$ )
- Windage torque is proportional to square of speed  $T_w = C(\omega m^2)$
- $T_L = T_L + B\omega_m + T_c + C(\omega m^2)$

**PART – B**

1. Explain about the speed-torque characteristics of a DC Compound Motor with suitable graph and equations.(Dec 2013)

2. Draw and explain the speed torque characteristics of dc series motor and three phase induction motor. **(Dec 2011)**

3. Explain about the speed-torque characteristics of a DC Shunt Motor with suitable graph and

equations. **(Dec 2014)**

4. Explain about the quadrant diagram of speed-torque characteristics for a motor driving hoist load. **(Nov 2015) (May 2015)(Dec 2016)**

5. Explain how an induction motor is brought to stop by (i) Plugging and (ii) dynamic braking.

**(Dec 2014)**

6. Explain the various methods of braking of induction motors. **(June 2014)**

7. Describe the speed Torque characteristics of DC Dynamic braking of three phase induction motor. **(May 2014)**

8. Explain speed- torque characteristics of different types of load with graph. **(May 2013)**

9. A 220 V dc series motor runs at 1200 rpm (clockwise) and takes an armature current of 80 A when driving a load with constant torque. Armature resistance is  $0.05\Omega$  and field resistance is  $0.05\Omega$ . Find the magnitude and direction of motor speed and armature current if the motor terminal voltage is reversed and number of turns in field winding is reduced to 80%. Assume linear magnetic circuit. **(Dec 2013)**

10. Explain various methods of braking of DC Series Motors with neat diagrams. **(May 2013)**

11. Discuss the dynamic braking of DC shunt motor. **(Nov 2015) (April 2017)**

12. List the advantages and disadvantages of Electrical braking over mechanical braking. Discuss any one method of electrical braking of DC machines. **(May 2015) (Dec 2016)**

13. For drives, classify the types of load torques available and sketch few speed torque curves of typical loads. **(Dec 2016)**

## UNIT 3 - STARTING METHODS

### PART A

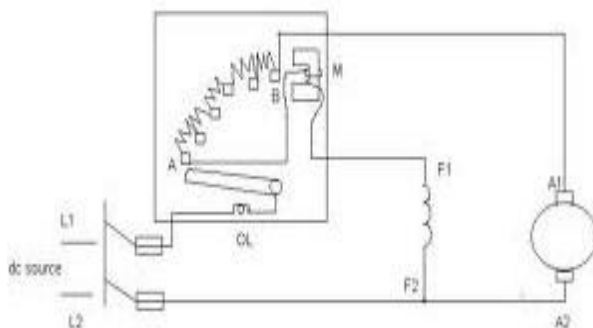
**1. What are the different types of DC motor starters?(May 2014)  
(Dec 2014)**

Three point starter, four point starter for shunt & compound motors and two point starter for series motor.

**2. What is the basic principle of primary resistance starter used in 3 phase induction motor?(May 2014)**

The purpose of the resistance is to reduce the supply voltage. This reduced voltage given to motor terminals, this reduced voltage limits the starting current.

**3. Draw the basic starter arrangement for shut motor starting.  
(Dec 2013)**



**4. What are the starters used for Squirrel cage induction motor?**

Direct on Line starter, Auto transformer starter, Star - Delta starter, Stator resistance starter for Squirrel cage induction motor.

**5. Which type of starter is used for slip ring induction motor?**

Rotor resistance starter for slip ring induction motor.

**6. What are the applications of DC motor?**

Electric traction, Machine tool, Steel mills, Textile mills.

## **7. What are the advantages of DC drives?**

Lower cost, Reliability, Simple control.

## **8. Give some advantages and disadvantages of D.O.L starter.**

Advantages: Highest starting torque, Low cost, Greatest simplicity

Disadvantages: The inrush current of large motors may cause excessive voltage drop in the weak power system. The torque may be limited to protect certain types of loads.

## **9. What is the objective of rotor resistance starter in Three phase induction motor? (June 2013) (May 2015) (April 2017)**

To include resistance in the rotor circuit there by reducing the induced rotor current at starting. This can be implemented only on a slip ring induction motor.

## **10. Give the prime purpose of a starter for motors. (May 2015, 2017)**

When motor is switched on to the supply, it takes about 5 to 8 times full load current at starting. This starting current may be of such a magnitude as to cause objectionable voltage drop in the lines. So Starters are necessary.

## **1. Why motor take heavy current at starting?**

When 3 phase supply is given to the stator of an induction motor, magnetic field rotating in space at synchronous speed is produced. This magnetic field is cut by the rotor conductors, which are short circuited. This gives to induced current in them. Since rotor of an induction motor behaves as a short circuited secondary of a transformer whose primary is stator winding, heavy rotor current will require corresponding heavy stator balancing currents. Thus motor draws heavy current at starting.

## **12. Mention the Starters used to start an three phase Induction motor. (Dec 2011)&(Dec 2014)**

\*D.O.L Starter (Direct Online Starter)

\* Star-Delta Starter

\*Auto Transformer Starter

\*Rotor Resistance starter (slip ring IM)

\*Stator Rotor Starter (Rotor Resistance Starter)

### 13. What are the protective devices in a DC/AC motor Starter.

\*Over load Release (O.L.R) or No volt coil    \*Hold on Coil    \*Thermal Relays    \*Fuses(Starting

/Running)    \* Over load relay

### 14. What are the advantages of star delta starter?

a) Cheaper than the auto-transformer starter    b) Simple arrangement  
c) Commonly employed for

both small and medium size motors.

### 15. What is the function of Over Load Release present in the starter?

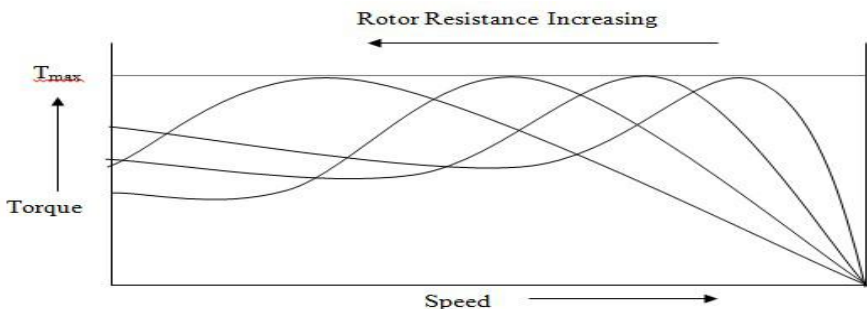
If there is any overload conditions, then the motor will draw large current. This large current will flow through the over load release coil. Due to this, the electromagnet gets energized and pulls the

iron piece upward which short circuits the coils of the hold on electromagnet. The hold on electromagnet gets de-energized and therefore the starter arm returns to the off position, thus protecting the motor against overload.

### 16. What is the function of No-Load release present in the starter?

If there is no load or low load, the speed of DC series motor will be dangerously high. During this condition, no load release makes the control arm to return to OFF position and prevent the motor from over speeding.

### 17. Draw the Speed-Torque characteristics of an Induction motor with various values of Rotor Resistance.



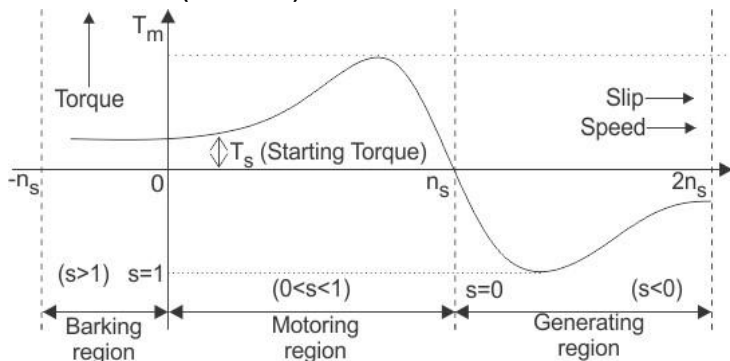
**18. What is the necessity of starters of electric motors? (Dec 2011)(Nov 2015) (April 2017)**

The main necessity of starters is to reduce the high starting current while starting the motor. In case of dc motor,  $I_a = (V - E_b) / R_a$ , at the time of starting  $E_b = 0$ . Therefore when the motor is directly connected to the supply lines a heavy current will flow through the armature conductors will create serious problems. Hence the starter is necessary to protect the motor.

**19. State the basic principle in DOL for 3- phase induction motor. (May 2013)**

The simplest form of motor starter for the induction motor is the Direct On Line starter. The Direct On Line Motor starter (DOL ) consists a Circuit Breaker, Contactor and an overload relay for protection. For this type of starter input directly given to the motor through protection arrangement.

**20. Draw the torque slip characteristics of an Induction motor (or) Draw the mechanical characteristics of three phase induction motor (Dec 2013)**



**21. What are the problems of heavy inrush current at the time of starting?**

Problems of heavy inrush current at the starting time,

- Heavy sparking at the commutator and even flashovers.
- Damage to the armature windings.
- Damage to rotating parts of the motor.
- Large dip in supply voltage.

**22. Mention the advantages of four point starter over three point starter. (Nov 2015)**

- Three point starter cannot be used for variable speed motor drive. This is because if too much resistance is cut in by the field rheostat, the field current is reduced very much. So that it is unable to create enough electromagnetic pull to overcome the spring tension. In four point starter the HOLD ON coil has been taken out of the shunt field circuit and has been connected directly across the line through a line resistance.
- It is used for compound motor starting applications.

**23. Why DC motors should not be started without starters? (Dec 2016)**

At the time of starting of motor it is at rest and no back e.m.f. is generated. On application of full voltage, armature winding draws a heavy current due to small armature resistance. This high armature current may damage the armature windings, commutator and brushes. To prevent high armature current during the starting of motors, variable resistance is connected in series with the armature winding. The starting resistance is reduced as the motor speeds up. The resistance is cut off fully when the motor attains full speed. This arrangement is known as starter. Hence starter is required for DC motor.

**24. What type of protection is provided in the starters used for three phase Induction motor? (Dec 2016) (April 2017)**

To protect the Induction motor from the high-starting current, there are different starting methods available like reduced voltage, rotor resistance, DOL, [star-delta starter](#), auto transformer, soft starter, etc. And, for protecting the motor from Phase-to-phase and phase-to-ground faults, single phasing, over and under voltage, voltage and current unbalance, under frequency, various protection equipments like relays, circuit breakers, contractors and various drives are implemented. These are some of the protection systems for three-phase induction motors against starting inrush currents, overheating and single phasing faults with the use of microcontroller for low-level applications.

**PART – B**

1. What is the need for starters? Draw a neat schematic diagram of a three point starter and explain its working. (May/June 2013) (Nov 2015) (May 2015) (Dec 2016) (April 2017)

2. Draw a neat schematic diagram of a four point starter and explain its working. (AU Nov/Dec 2011) (May 2014) (May 2015)



3. Explain with neat circuit diagram, the star-delta starter method of starting squirrel cage induction motor. **(May 2015)(Dec2016)**

4. Explain different methods of starting of DC Motors.

5. Explain with neat diagram the starting of three phase slip ring induction motor. **(AU Nov/Dec 2011) (April 2017)**

6. Draw and explain the push-button operated direct-on line starter for three phase induction motor.

7. Draw and explain the auto-transformer starter for three phase induction motor. **(May/June 2013)**

8. Explain the different starting methods for three phase squirrel cage induction motor **(Nov/Dec` 2013)(Nov 2015)**

9. A Starter is required for a 220 V shunt motor. The maximum and minimum range of current values are 50 A and 30 A respectively. Find the number of sections of starter resistance required and resistance of each section. The armature resistance of the motor is  $0.5\Omega$  **(Nov/Dec 2013)**.

10. Explain with Diagram, Construction, and Working of Rotor resistance starter? **(June 2014) (Dec 2014)**

#### **UNIT 4**

#### **CONVENTIONAL AND SOLID STATE SPEED CONTROL OF D.C. DRIVES**

#### **PART A**

**1. Write down the speed equation of a dc motor.**

$$N = (V - I_a R_a) / K_b \phi$$

Where, V = applied voltage,  $I_a$  = armature current,  $R_a$  = armature resistance,  $\phi$  = flux,

$K_b$  = constant

**2. What are the methods of speed control of dc motors? (Dec 2014)**

Armature resistance control, Flux control, Voltage control.

**3. What is meant by armature resistance control?(or) Why is armature voltage control used below rated speed? (Dec 2013) (May 2014)**

A controller resistance is connected in series with armature. By varying the controller resistance  $R$ , the potential drop across the armature is varied. Therefore, the motor speed also varied. This method of speed control only applicable for speed less than no load speed.

#### **4. What are the advantages and disadvantages of armature resistance control of dc shunt**

**motor? (Nov 2015)**

Advantages: Simple method of speed control.

Disadvantages: This method is highly inefficient, because more power is wasted in controller resistance. Change in speed with Change in load becomes large.

#### **5. What is meant by flux control method?**

The speed of the dc motor can be controlled by varying the field flux. This can be increasing the speed of the motor above its rated speed, because the speed is inversely proportional to the field flux.

#### **6. What are the methods of speed control of dc series motor?**

Variable resistance in series with motor, Flux control method- Field diverter, Armature diverter,

Tapped field control, Paralleling field coils and Series parallel control.

#### **7. What is meant Ward Leonard system?**

The speed of the dc shunt motor can be controlled by above and below rated speed using the system. It consists of motor generator set. The armature voltage control can be achieved by varying the field of the dc generator. The flux control can be achieved by varying the field of the dc motor.

#### **8. What are the advantages and disadvantages of Ward Leonard system?**

**Advantages:-**

1. Full forward and reverse speed can be achieved. 2. A wide range of speed control is possible.

3. Power is automatically regenerated to the ac line to the mg set when speed is reduced. 4. Short

time overload capacity is large. 5. The armature current is smooth.

**Disadvantages:**

1. High initial cost. 2. Overall efficiency low (less than 80%, because of the additional MG set).

3. Costly foundation and large amount of space is required. 4. This produces noise. 5. It requires

frequent maintenance

**9. What are the methods of speed control of induction motors?**

Stator voltage control, frequency control, Pole changing method, Cascaded control, slip power control.

**10. What are the advantages of solid state drives over conventional drives? (Dec 2011)**

Wide variation of speed control is possible, less maintenance, simple, reliable, high efficiency,

low initial cost and faster response.

**11. What is meant by electric-drive system?**

An electric drive together with its working machines constitutes an electric drive system.

**13. What are the different types of drives?**

Single phase dc drives, three phase dc drives, Chopper drives.

**14. What are the two basic methods of speed control of dc motors?**

The dc motor speed controlled by (1) armature voltage control (below base speed) (2) Flux control (above base speed).

**15. What are the different types of single phase dc drives?**

Single phase half controlled dc drives. Single phase full controlled dc drives.

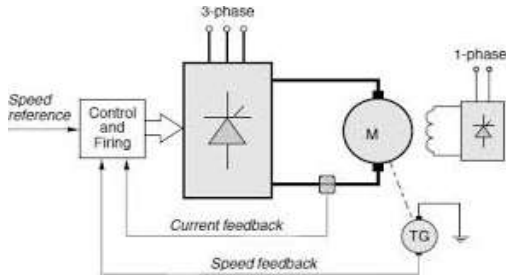
**16. What are the advantages of dc chopper drives? (Dec 2014)**

Dc chopper drives has the advantages of high efficiency, flexibility in control, light weight, small size, quick response and regeneration down to very low speed.

**17. What are the different methods of speed control of induction motors?**

1. Stator voltage control
2. Voltage/frequency control (v/f)
3. Rotor resistance control
4. Slip energy recovery control.

**18. Draw the block diagram of phase controlled rectifier fed DC drives. (May 2013)**



**19. What is stator voltage control?**

Three phase induction motor speed can be controlled by varying the stator voltage. This stator

voltage can be varied by using ac voltage controllers. This method of speed control of induction motor is called as stator voltage control. It is also called soft start.

**20. What is frequency control?**

$$N = 120f/P$$

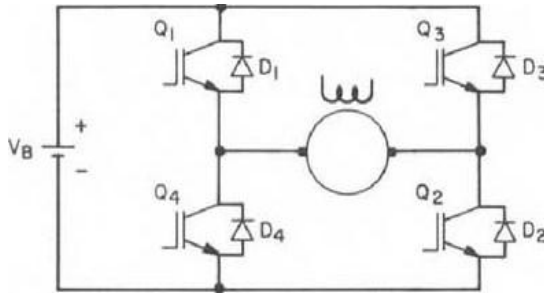
where  $N$  = speed of the motor,  $f$  = supply frequency,  $p$  = number of poles

Speed of the induction motor can be controlled by varying supply frequency as speed is directly proportional to supply frequency.

**21. What is meant by duty cycle? (Dec 2011) (May 2014)**

Duty cycle is defined as the ratio between on time of chopper and total time of chopper  $\alpha = T_{on}/T$

**22. Draw the basic circuit for chopper controlled separately excited dc motor driv (Nov 2013)**



**23. List the Application of Chopper fed DC drives. (Nov 2015) (April 2017)**

Cranes, traction applications, Battery operated vehicles, Subway cars.

**24. Give the limitation of field control? (May2015) (April 2017)**

1. The Speed control below normal rated speed is not possible as flux can be increased only upto its rated value.
2. As flux reduces, speed increases. But high speed affects the commutation making motor operation unstable. So there is limit to the maximum speed above normal possible by this method.

**25. What are the main applications of Ward Leonard system? (May2015)**

- Full forward and reverse speed can be achieved.
- A wide range of speed control is possible.
- Power is automatically regenerated to the ac line to the mg set when speed is reduced.
- Short time overload capacity is large.
- The armature current is smooth.

**26. List the various methods of conventional and solid state speed control of DC motors. (Dec 2016)**

Conventional methods:

Armature resistance control, Flux control, Voltage control, Ward-Leonard Speed control.

Solid State Methods:

Using Controlled and uncontrolled rectifiers, using Choppers.

**27. Compare armature control and field control. (Dec 2016)**

Parameters	Armature control	Field control
Speed control is achieved by	Varying armature voltage	Varying field current
Air gap flux	Remains constant	Does not remain constant
Torque	Remains constant	Changes
Range of Speed	0 to rated speed	Above the rated speed
Efficiency	Low	High
Power loss in external Rheostat	High	Less

**PART – B**

1. Explain with neat sketch the chopper control method of speed control of DC Motors. **(April 2017)**

2. Explain with neat sketches about the DC Shunt Motor speed control by using single phase fully controlled bridge converter.

3. Discuss the Ward-Leonard speed control system with a neat circuit diagram. Also mention its advantages and disadvantages. **(Dec 2011) (May 2013) (May 2014) (Dec 2014)(Nov 2015)(May 2015)(Dec2016) (April 2017)**

4. Explain with neat sketch the operation of a chopper fed DC Series motor drive. Also derive the expression for average motor current. **(Dec 2014)(Dec2016)**

5. Explain first quadrant chopper control of separately excited motor for continuous conduction. **(May 2013)**

6. Explain in detail the single phase semi-converter speed control for DC drive for separately excited motor.

7. A 500V series motor having armature resistance and field resistance of  $0.2 \Omega$  and  $0.3 \Omega$  respectively runs at 500 rpm when taking 70A. Assuming unsaturated field, find out its speed when field diverter of  $0.684 \Omega$  is used constant load torque.

8. A 220V DC Shunt Motor takes 5A on no load and runs at 750 rpm. The resistances of armature and field winding are 0.2 ohms and 110 ohms respectively. Calculate the speed when the motor is loaded

and taking a current of 50A. Assume the armature reaction weakens the field by 3%.**(Dec 2014)**

9. With neat circuit diagram, explain chopper fed four quadrant dc drive.**(Dec 2013)(Nov 2015)**

10. Explain the operation of single phase full converter fed separately excited dc motor drive**(Nov2013)(May 2014)**

11. Explain with neat sketch the chopper control method of speed control of DC Motors.

12. A 220V DC shunt motor having a field flux of 0.8 wb, runs at a speed of 900rpm. Find the speed of the motor, if the field flux reduced to 0.6wb by field resistance control method. **(Nov 2015).**

13. A 220V, 1200rpm,  $1\Phi$  full converter fed separately excited DC motor having a armature resistance and current of  $0.25\Omega$  and 40 A respectively. For the delay angle of  $30^\circ$ , find the speed of the motor. Consider motor constant  $K_a\Phi=0.18$  N/rpm. **(Nov 2015)**

14. Explain the single phase half wave converter drive speed control for DC drive with waveforms.**(May 2015)**

15. A 220V, 70A dc Series motor has combined resistance of armature and field resistance of 0.12 ohm. Running on no load with field winding connected to a separate source it gave the following magnetization characteristics at 600rpm:

$I_f(A)$	10	20	30	40	50	60	70	80
$V_t(v)$	64	118	150	170	184	194	202	210

Motor is controlled by chopper with a source voltage = 220V. Calculate (i) Motor speed for a duty ratio of 0.6 and motor current of 60A. (ii) Torque for a speed of 400 rpm and duty ratio of 0.65.**(May 2015)**

16. A 220V dc shunt motor takes 22 A at rated voltage and runs at 1000 rpm. Its field resistance is 100 ohms and armature circuit resistance is 0.1 ohms. Compute the value of additional resistance required in the armature circuit to reduce the speed to 800 rpm when the load torque is proportional to speed. **(Dec 2016).**

## UNIT 5

### CONVENTIONAL AND SOLID STATE SPEED CONTROL OF A.C. DRIVES

#### PART A

**1. What are the different types of slip power control system? (April 2017)**

Kramer system, Scherbius system.

**2. What is meant by slip power?**

The portion of air gap power, which is not converted into mechanical power, is called slip power. Slip power is nothing but multiplication of slip ( $s$ ) and air gap power ( $P_{ag}$ )

Slip power =  $s (P_{ag})$

**3. What are the advantages of slip power recovery system?**

The slip power can be recovered and fed back to the supply. The overall efficiency also improved.

**4. What are the different types of slip power recovery system?(Nov 2015) (April 2017)**

These are classified two types.

Kramer system

\*Conventional Kramer system

\*Static Kramer system

Scherbius system

\*Conventional Scherbius system

\*Static Scherbius system

\* DC link static Scherbius system

\*Cycloconverter      0Scherbius system

**5. What is meant by Kramer system?**

The Kramer system is only applicable for sub-synchronous speed operation because the slip power is fed back to the supply.



## **6. What are advantages of conventional Kramer method?**

✓ The main advantage of this method is that any speed, within the working range, can be obtained instead of only two or three, as with other methods of speed control.

✓ If the rotor converter is over excited, it will take a leading current which compensates for the lagging current drawn by SRIM & hence improves the power factor of the system.

## **7. What is the function of static Kramer system?**

The slip power is converted into dc by diode bridge rectifier and the DC voltage is converted into AC by line commutated inverter and fed back to supply. As the slip power can flow only in one direction, static Kramer drive offers speed control below synchronous speed only.

## **8. Define slip power control. What is meant by slip power recovery system?**

In slip ring induction motor the rotor power (slip power) can be recovered and fed back to supply or can be used to supply an additional motor which is mechanically coupled to the main motor. This type of drive is known as slip power recovery system and improves overall efficiency of the system.

## **9. What is the function of conventional Kramer System?**

In conventional Kramer system, the slip power is converted into dc by rotary converter. The dc voltage is fed to dc motor. The dc motor is coupled with slip ring induction motor. The speed of the SRIM can be controlled by varying the field regulator of the dc motor.

## **10. Where static Kramer drive is used?**

In large power pump and fan type drives, where speed control within narrow range and below synchronous speed.

## **11. What are the advantages of static Kramer system?**

i. The drive system is very efficient and the converter power rating is low, because it has to handle only the slip power.

ii. The drive system has dc machine-like characteristics and the control is very simple.

## **12. What are applications of static Scherbius drive system?**

Multi-MW, variable speed pumps/generators. 2. Flywheel energy storage system.

### **13. What are the advantages and disadvantages of static Scherbius drive?**

#### **Advantages:**

- a) In this method, the problem of commutation near synchronous speed disappears.
- b) The cyclo-converter can easily operate as a phase-controlled rectifier, supplying dc current in the rotor and permitting true synchronous machine operation.
- c) The near-sinusoidal current waves in the rotor, which reduce harmonic loss, and a machine over excitation capacity that permits leading power factor operation on the stator side. So the line's power factor is unity.
- d) The cyclo-converter is to be controlled so that its output frequency tracks precisely with the slip frequency.

#### **Disadvantages:**

- a) The cyclo-converter cost increases, b) The control of the Scherbius drive is somewhat complex.

### **14. Compare conventional method of Kramer and Scherbius system(May 2015)**

Kramer Method	Scherbius Method
This system consists of SRIM, Rotary converter and dc motor and Induction generator	This system consists of SRIM, Rotary converter and dc motor
Here, the return power is Mechanical	Here, the return power is mechanical
Less cost.	More cost

### **15. What are advantages of stator voltage control?**

- 1. The control is very simple
- 2. More compact and less weight
- 3. Its response time is quick
- 4. This is an economical method

### **16. What are the variable frequency AC drive applications?(May 2013)**

VFDs are used in applications ranging from small appliances to the largest of mine mill drives and compressors.

### **17. What is slip?**

The difference between the synchronous speed  $N_s$  and the actual speed  $N$  of the rotor is known as slip.

### **18. What are the various methods available for speed control of three phase induction motor?**

**(Dec 2011) (Nov 2015)**

Induction motors are of two types - Squirrel-cage motor and Wound-rotor motor. There are various types of speed control methods of induction motor. These are –

(i) Pole Changing, (ii) Stator Voltage Control, (iii) Supply Frequency Control, (iv) Rotor Resistance Control, (v) Slip Power Recovery.

### **19. What are applications of three phase AC voltage controllers?(Dec 2011) (Dec 2014)**

- Lighting / Illumination control in ac power circuits.
- Induction heating.
- Industrial heating & Domestic heating.
- Transformer tap changing (on load transformer tap changing).
- Speed control of induction motors (single phase and poly phase ac induction motor control).
- AC magnet controls.

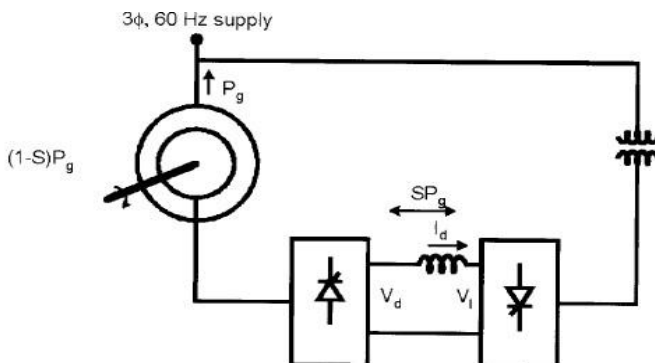
### **20. How can the direction of rotation of three phase induction motor be reversed. (Dec 2011)**

The direction of rotation of three phase induction motor can be reversed by interchanging the input phase sequence from RYB to RBY.

### **21. State the applications where stator voltage control is employed for three phase induction motor. (Dec 2013)**

Fans, Centrifugal pumps, Compressor and etc.

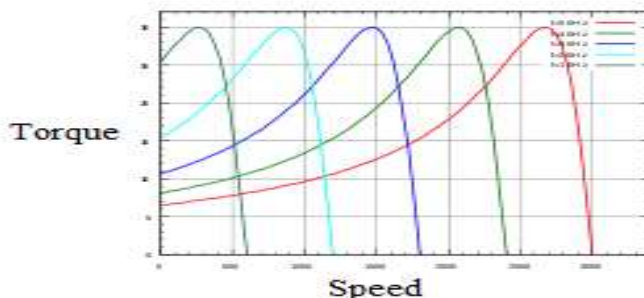
**22. Draw the block diagram of conventional scherbius system. (May 2013)**



**23. What is meant by v/f control? (May 2014)(Dec 2016) (April 2017)**

If the ratio of voltage to frequency is kept constant; the flux also remains constant. At low frequency, the air gap flux is reduced due to the drop in the stator impedance and the voltage has to be increased to maintain the torque level. This type of control is usually known as volts/hertz (v/f) control. The voltage at variable frequency can be obtained from three-phase inverter or cyclo-converter.

**25. Draw a neat sketch of Speed- Torque Characteristics of Induction Motor with v/f control.(Dec 2014)**



**25. What are the conventional methods of control of three phase induction motor from the stator side? (May 2014)**

Stator voltage control, primary resistance starter

**26. What is the function of an inverter? (May 2015)**

The inverters are basically d.c to a.c converters. The inverters are used to convert a d.c input voltage to a symmetrical a.c output voltage of desired magnitude and frequency.

**27. Mention the advantages of squirrel cage induction motor over a DC motor. (Dec 2016)**

Low Cost, Wide Speed Variation, High power factor, Reliable operation.

**PART-B**

1. Draw the power circuit arrangement of three phase variable Frequency inverter for the speed control of three phase induction motor and explain its working.
2. Explain the V/f control method of AC drive with neat sketches.
3. Discuss the speed control of AC motors by using three phase AC Voltage regulators.
4. Explain in detail about Slip power recovery scheme. **(Dec 2011) (Dec 2014) (Nov 2015) (May 2015) (Dec 2016) (April 2017)**
5. Explain the different methods of speed control used in three phase induction motors.
  - a. Control from stator side
  - b. Control from the rotor side **(May 2014)**
6. Explain the working of following methods with neat circuit diagram.
  - i) Kramer system, ii) Scherbius system **(Dec 2013) (May 2013) (May 2014)**
7. (i) Explain the operation of Pole changing method of speed control. **(May 2015)**  
(ii) Explain the pole amplitude modulation method.
8. Explain in detail about the various methods of solid state speed Control techniques by using inverters. **(Dec 2014)**
9. Explain the solid state stator voltage control technique for the Speed control of three phase induction motor. **(April 2017)**
10. Explain the constant torque mode and constant power mode of operation of voltage source inverter fed induction motor drive with necessary diagrams. **(Dec 2013)**
11. Explain the rotor resistance control employed in 3 $\Phi$  induction motor. **(Nov 2015)**

12. Describe the variable voltage variable frequency method of speed control of 3 phase induction motors for full range of speed. **(Dec 2016)**