



KCG
COLLEGE OF TECHNOLOGY

QUESTION BANK

SIXTH SEMESTER

B.E-Mechanical Engineering)

(Anna University Regulation 2013)

DEPARTMENT OF MECHANICAL ENGINEERING

KCG COLLEGE OF TECHNOLOGY

CHENNAI – 600097

VISION OF THE COLLEGE

KCG College of Technology aspires to become a globally recognized centre of excellence for science, technology & engineering education, committed to quality teaching, learning, and research while ensuring for every student a unique educational experience which will promote leadership, job creation, social commitment and service to nation building

MISSION OF THE COLLEGE

- Disseminate knowledge in a rigorous and intellectually stimulating environment
- Facilitate socially responsive research, innovation and entrepreneurship
- Foster holistic development and professional competency
- Nurture the virtue of service and an ethical value system in the young minds

VISION OF THE DEPARTMENT

The Department of Civil Engineering envisions becoming a global centre of excellence founded on effective teaching and innovative research, producing competent engineering graduates to serve the nation

MISSION OF THE DEPARTMENT

- To effectively teach engineering fundamentals and modern technology.
- To identify, analyze and find sustainable solutions to infrastructural challenges facing the nation.
- To contribute to the holistic development of the students.

PROGRAM OUTCOMES (POs)

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Engineering graduates will be able to:

- 1. Engineering Knowledge:** Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization to the solution of complex engineering problems.
- 2. Problem analysis:** Identify, formulate, review research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences.
- 3. Design/development of solutions:** Design solution for complex engineering problems and design systems components or process that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal, and environmental considerations.
- 4. Conduct investigations of complex problems:** Use research- based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.
- 5. Modern tool usage:** Create, select, and apply appropriate techniques, resources, and modern engineering and IT tools including prediction and modeling to complex engineering activities with an understanding of the limitations.
- 6. The engineer and society:** Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal and cultural issues and the consequent responsibilities relevant to the professional engineering practice.
- 7. Environmental and sustainability:** Understand the impact of the professional engineering solutions in societal and environmental contexts and demonstrate the knowledge of, and need for sustainable development.
- 8. Ethics:** Apply ethical principles and commit to professional ethics and responsibilities and norms of the engineering practice.
- 9. Individual and team work:** Function effectively as an individual, and as a member or leader in diverse teams, and in multidisciplinary settings.
- 10. Communication:** Communicate effectively on complex engineering activities with the engineering community and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions.
- 11. Project management and finance:** Demonstrate knowledge and understanding of the engineering and management principles and apply these to one's own work, as a member and leader in a team, to manage projects and in multidisciplinary environments.
- 12. Life-Long learning:** Recognize the need for, and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change.

PROGRAM EDUCATIONAL OBJECTIVES (PEOs)

PEO1: Graduates of the programme, will continuously update their domain knowledge for continuous professional development with focus on research & development and industry interaction.

PEO2: Graduates of the programme will create innovations in providing solution for sustainable built environment.

PEO3: Graduates will be familiar with modern engineering software tools and equipment to analyze complex civil engineering problems.

PEO4: Graduates of the programme will involve in the research world to meet the practical challenges.

PEO5: Graduates of the programme will be professional civil engineers with ethical and societal responsibility.

PROGRAMME SPECIFIC OUTCOMES(PSOS)

PSO 1 : Model, analyze, design and realize physical systems, components or process by applying principles of three core streams of Mechanical Engineering, i.e.Design, Manufacturing, Thermal and Fluid Engineering.

PSO 2 : Apply the knowledge of Auto CAD, SolidWorks, ANSYS,CNC, Simulation softwares, MATLAB, Machine tool practices, Material & Machine testing, Fluid & Thermal machinery to solve real time Mechanical Engineering problems

PSO 3 : Engage in lifelong learning and follow professional ethics, codes and standards of Professional practices.

ME 6603 FINITE ELEMENT ANALYSIS

Syllabus of the course

UNIT 1 : INTRODUCTION

Historical Background – Mathematical Modeling of field problems in Engineering – Governing Equations – Discrete and continuous models – Boundary, Initial and Eigen Value problems– Weighted Residual Methods – Variational Formulation of Boundary Value Problems – Ritz Technique – Basic concepts of the Finite Element Method.

Unit 2 : ONE-DIMENSIONAL PROBLEMS

One Dimensional Second Order Equations – Discretization – Element types- Linear and Higher order Elements – Derivation of Shape functions and Stiffness matrices and force vectors- Assembly of Matrices - Solution of problems from solid mechanics and heat transfer. Longitudinal vibration frequencies and mode shapes. Fourth Order Beam Equation –Transverse deflections and Natural frequencies of beams.

Unit 3 : TWO DIMENSIONAL SCALAR VARIABLE PROBLEMS

Second Order 2D Equations involving Scalar Variable Functions – Variational formulation –Finite Element formulation – Triangular elements – Shape functions and element matrices and vectors. Application to Field Problems - Thermal problems – Torsion of Non circular shafts –Quadrilateral elements – Higher Order Elements.

Unit 4 : TWO DIMENSIONAL VECTOR VARIABLE PROBLEMS

Equations of elasticity – Plane stress, plane strain and axisymmetric problems – Body forces and temperature effects – Stress calculations - Plate and shell elements.

Unit 5 : ISOPARAMETRIC FORMULATION

Natural co-ordinate systems – Isoparametric elements – Shape functions for isoparametric elements – One and two dimensions – Serendipity elements – Numerical integration and application to plane stress problems - Matrix solution techniques – Solutions Techniques to Dynamic problems --- Introduction to Analysis Software.

E. Content Beyond Syllabus

1. 1D,2D and 3D analysis using Software
2. 1D and 2D analysis of a component using FEA

Text Book(s)

T 1: Reddy. J.N., “An Introduction to the Finite Element Method”, 3rd Edition, Tata McGraw-Hill, 2005.

T 2: Seshu, P, “Text Book of Finite Element Analysis”, Prentice-Hall of India Pvt. Ltd., New Delhi, 2007.

R 1: Rao, S.S., “The Finite Element Method in Engineering”, 3rd Edition, Butterworth Heinemann, 2004

R 2: Logan, D.L., “A first course in Finite Element Method”, Thomson Asia Pvt. Ltd., 2002.

R 3: Robert D. Cook, David S. Malkus, Michael E. Plesha, Robert J. Witt, “Concepts and Applications of Finite Element Analysis”, 4th Edition, Wiley Student Edition, 2002.

R 4: Chandrupatla & Belagundu, “Introduction to Finite Elements in Engineering”, 3rd Edition, Prentice Hall College Div, 1990.

R 5: Bhatti Asghar M, "Fundamental Finite Element Analysis and Applications", John Wiley & Sons, 2005 (Indian Reprint 2013).

UNIT :1 INTRODUCTION

Planned Hour	Description of Portion to be Covered	Relevant CO Nos	Highest Cognitive level**	Delivery method	Reference Materials
1	Historical Background	1	K2	Lecture with Discussion	T1,R2,OL1
2	Mathematical Modeling of field problems in Engineering	1	K3	Lecture with Discussion	T1,R2,OL1
3	Governing Equations	1	K3	Lecture with Discussion	T1,R2,OL1
4	Discrete and continuous models	1	K3	Lecture with Discussion	T1,R2,OL1
5	Boundary, Initial and Eigen Value problems	1	K3	Lecture with Discussion	T1,R2,OL1
6	Weighted Residual	1	K2	Lecture with Discussion	T1,R2,OL1

	Methods				
7	Variational Formulation of Boundary Value Problems–	1	K3	Lecture with Discussion	T1,R2,OL1
8	Ritz Technique	1	K2	Lecture with Discussion	T1,R2,OL1
9	Basic concepts of the Finite Element Method.	1	K2	Lecture with Discussion	T1,R2,OL1

UNIT 2: ONE DIMENSIONAL PROBLEMS

Planned Hour	Description of Portion to be Covered	Relevant CO Nos	Highest Cognitive level**	Delivery method	Reference Materials
9	One Dimensional Second Order Equations –	2	K2	Lecture with Discussion	T1,R2,OL1
10	Discretization – Element types- Linear and Higher order Elements	2	K2	Lecture with Discussion	T1,R2,OL1
11	Derivation of Shape functions	2	K3	Lecture with Discussion	T1,R2,OL1
12	Derivation of Stiffness matrices and force vectors-.	2	K3	Lecture with Discussion	T1,R2,OL1
13	Assembly of Matrices Solution of problems from solid Mechanics	2	K3	Lecture with Discussion	T1,R2,OL1
14	Assembly of Matrices Solution of problems from heat	2	K3	Lecture with Discussion	T1,R2,OL1

	transfer				
15	Longitudinal vibration frequencies and mode shapes.	2	K2	Lecture with Discussion	T1,R2,OL1
16	Fourth Order Beam Equation	2	K2	Lecture with Discussion	T1,R2,OL1
17	Transverse deflections	2	K3	Lecture with Discussion	T1,R2,OL1
18	Natural frequencies of beams.	2	K3	Lecture with Discussion	T1,R2,OL1

ME 6603 FINITE ELEMENT ANALYSIS

Unit –I Part-A (Two Marks)

1.Distinguish between classical methods and FEM

Classical method	FEM
1.Exact solutions are formed and exact solutions are obtained	Exact equations are formed and approximate solutions are obtained
2. Applicable for few standard cases	Applicable for all cases
3.problems with material and geometric non linearities can not be handled	There is no difficulty in FEM

2. Distinguish between FDM and FEM

FDM	FEM
1. Pointwise approximation	Piecewise approximation
2. it will not give values other than nodal points	It will give values at any point
3. FDM needs more nodes to get accurate results	Less nodes are required

3. What is meant by node or joint? (May '14)

Node is a selected finite point at which basic unknown is to be determined in the finite element analysis

4. What are the two types of nodes (May 2012)

External nodes and internal nodes.

External nodes are those which occur on the edges/ surface of the elements and they may be common to two or more elements. nodes, 1 and 2 in one dimensional element, nodes 1 to 9 in 10 noded triangular element and nodes 1 to 8 in 9 noded lagrangian element are external nodes.

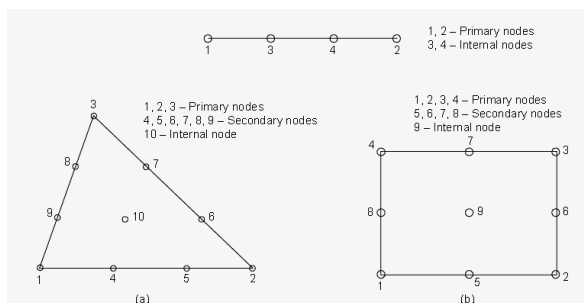
These nodes may be further classified as (i) Primary nodes and (ii) Secondary nodes.

Primary nodes occur at the ends of one dimensional elements or at the corners in the two or three dimensional elements.

Secondary nodes occur along the side of an element but not at corners

Internal nodes are the one which occur inside an element. They are specific to the element selected i.e .there will not be any other element connecting to this node.

Such nodes are selected to satisfy the requirement of geometric isotropy while choosing interpolation functions.



5. What is meant by finite element?

A small units having definite shape of geometry and nodes is called finite element.

6. What is the basis of finite element method?

Discretization is the basis of finite element method. The art of subdividing a structure in to convenient number of smaller components is known as discretization.

7. What are the types of boundary conditions?

Essential boundary condition or Primary boundary conditions- Field variables at nodal points Ex. $X=0$ $y=0$ (in beam problem)

and Natural Boundary condition or Secondary boundary conditions- derivative of the field variables at nodal points Ex. $x=0$ $dy/dx=0$ or $d^2y/dx^2=0$

8. State the methods of engineering analysis?

Experimental methods, Analytical methods, Numerical methods or approximate methods

9. What are the types of element?

1D element, 2D element, 3D element

10. State the three phases of finite element method.

Preprocessing, Analysis, Post Processing

11. What are structural and non-structural problems?

In structural problems, displacement at each nodal point is obtained. By using these displacement solutions, stress and strain in each element can be calculated.

In non-structural problems, temperatures or fluid pressure at each nodal point is obtained. By using these values, properties such as heat flow, fluid flow, etc., for each element can be calculated

12. What is meant by post processing?

Analysis and evaluation of the solution result is referred to as post processing. Postprocessor computer program help the user to interpret the result by displaying them in graphical form.

13. Name the variation methods.

Ritz method. Ray-Leigh Ritz method.

14. What is meant by degrees of freedom?

Number of independent variables required to define the motion of a body.

15. What is meant by discretization and assemblage?

The art of subdividing a structure in to convenient number of smaller components is known as discretization. These smaller components are then put together. The process of uniting the various elements together is called assemblage.

16. What is Rayleigh-Ritz method?

It is integral approach method which is useful for solving complex structural problem, encountered in finite element analysis. This method is possible only if a suitable function is available.

17. Name the weighted residual method

Point collocation method, Sub domain collocation method, Least square method and Galerkin's method.

20. What is Rayleigh-Ritz method? (May '14)

Rayleigh-Ritz method is an integral approach method which is useful for solving complex structural problems encountered in finite element analysis. This method is possible only if a suitable functional is available.

21. What are the steps involved in Weighted residual method?

- 1) Assume Trial function
- 2) Apply boundary conditions and find residue
- 3) Determine the unknown parameters in the assumed trial function in such a way that the residuals are minimum.

22. State the principle of minimum potential or Principle of total stationary potential

For conservative systems, of all the kinematically admissible displacement fields, those corresponding to equilibrium extremize the total potential energy. If the extremum condition is a minimum, the equilibrium state is stable

23. What are the steps involved in Rayleigh Ritz Method

The Rayleigh–Ritz (R–R) method consists of three basic steps:

Step 1: *Assume a displacement field.* Let the displacement field be given by $\{\phi(x) + \sum c_i N_i\}$, $i = 1, 2, \dots, n$, where N_i are the shape functions and c_i are the as yet undetermined coefficients. It is observed that the assumed displacement field should satisfy both the essential boundary conditions of the problem and internal compatibility, i.e. there should be no kinks, voids, etc. within the structure.

Step 2: *Evaluation of the total potential.* For the system under consideration, evaluate the total potential Π_p consistent with the assumed displacement field in Step 1 above.

Step 3: *Set up and solve the system of equations.* By virtue of the PSTP, the total potential will be stationary with respect to small variations in the displacement field. The variations in the displacement field in our case are attained by small variations in the coefficients c_i . Thus we have

$$\frac{\partial \Pi_p}{\partial c_i} = 0, \quad i = 1, 2, \dots, n \quad (3.41)$$

24. Name any four FEA softwares.

ANSYS, NASTRAN, COSMOS and CATIA

25. List the two advantages of post processing.

Required result can be obtained in graphical form. Contour diagrams can be used to understand the solution easily and quickly.

26. Differentiate boundary value problem and initial value problem

A differential equation is said to describe a boundary value problem if the dependent variable and its derivatives are required to take specified values on the boundary.

An initial value problem is one in which the dependent variable and possibly its derivatives are specified initially.

PART B

1. The following differential equation is available for a physical phenomenon (OR)

$$\frac{d^2 y}{dx^2} + 50 = 0 ; 0 \leq x \leq 10$$

The Trial function is $y = a_1 x(10 - x)$

The boundary conditions are : $y(0) = 0$
 $y(10) = 0.$

Find the value of the parameter a_1 by the following

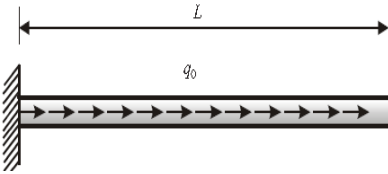
- (i) Least square method**
- (ii) Galerkin's method.**

2. The differential equation of a physical phenomenon is given by $d^2 y/dx^2 + y = 4x$, $0 \leq x \leq 1$. The boundary conditions are $y(0) = 0, y(1) = 1$. Obtain one term approximate solution by using Galerkin's method of weighted residuals.

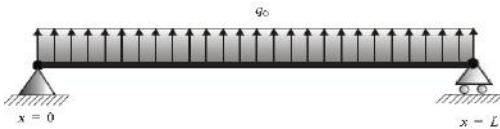
3. Find the displacement field using Rayleigh Ritz

$$AE \frac{d^2 u}{dx^2} + q_0 = 0$$

with the boundary conditions $u(0) = 0, \left. \frac{du}{dx} \right|_{x=L} = 0.$



4. Find the deflection field using Rayleigh Ritz method

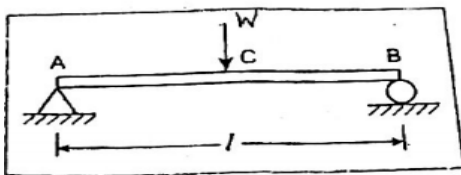


5. A physical phenomenon is governed by the differential equation .
 $(d^2w/dx^2) - 10x^2 = 5$ for $0 \leq x \leq 1$. The boundary conditions are given by . Assuming a trial function

$w(x) = a_0 + a_1x + a_2x^2 + a_3x^3$ $w(0) = w(1) = 0$. Determine using Galerkin method the variation of 'w' with respect to x.

6. Derive the variational formulation of axial load problem

7. Determine the deflection at midspan by using Rayleigh Ritz method and compare with exact solution



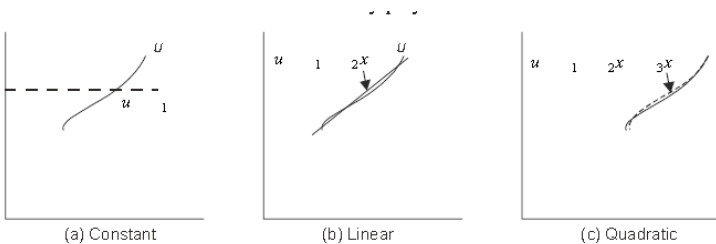
8. The differential equation of a physical phenomenon is given by $\frac{d^2y}{dx^2} + y = 4x$, $0 \leq x \leq 1$. The boundary conditions are $y(0)=0, y(1)=1$. Obtain one term approximate solution by using Galerkin's method of weighted residuals.

Unit - II ONE DIMENSIONAL PROBLEMS

1. Why are polynomial type of interpolation functions preferred over trigonometric functions?

Polynomials are commonly used as shape functions. There are two reasons for using them

1. They are easy to handle mathematically i.e. Differentiation and integration of polynomial is easy
- 2) Using polynomial any function can be approximated reasonably well. If a function is highly non linear, higher order polynomials are used.



2. Distinguish between 1D bar element and 1D beam element.

One dimensional bar element has axial deformation (u) and the element stiffness matrix is 2×2 . One dimensional beam element has transverse deformation and rotation (y, θ) and the element stiffness matrix is 4×4 .

3. What is truss element?

The truss elements are the part of a truss structure linked together by point joint which transmits only axial force to the element.

4. What is the difference between static and dynamic analysis?

Static analysis: The solution of the problem does not vary with time is known as static analysis. Example: Stress analysis on a beam.

Dynamic analysis: The solution of the problem varies with time is known as dynamic analysis. Example: Vibration analysis problems.

5. What are the h and p versions of finite element method?

It is used to improve the accuracy of the finite element method. In h version, the order of polynomial approximation for all elements is kept constant and the numbers of elements are increased. In p version, the numbers of elements are maintained constant and the order of polynomial approximation of element is increased.

6. What are the methods are generally associated with the finite element analysis?

Force method, Displacement or stiffness method.

7. Explain stiffness method.

Displacement or stiffness method, displacement of the nodes is considered as the unknown of the problem. Among them two approaches, displacement method is desirable.

8. What is Aspect ratio?

1. It is defined as the ratio of the largest dimension of the element to the smallest dimension. In many cases, as the aspect ratio increases the in accuracy of the solution increases. The conclusion of many researches is that the aspect ratio
2. should be close to unity as possible.

9. Differentiate between global and local axes.

Local axes are established in an element. Since it is in the element level, they change with the change in orientation of the element. The direction differs from element to element.

Global axes are defined for the entire system. They are same in direction for all the elements even though the elements are differently oriented.

10. Distinguish between potential energy function and potential energy functional

If a system has finite number of degree of freedom (q_1, q_2 , and q_3), then the potential energy expressed as,

$$\pi = f(q_1, q_2, \text{and } q_3)$$

11. What are the types of loading acting on the structure?

Body force (f) Traction force (T) Point load (P)

12. Define the body force

A body force is distributed force acting on every elemental volume of the body Unit: Force per unit volume.

Example: Self weight due to gravity

13. Define traction force

Traction force is defined as distributed force acting on the surface of the body. Unit: Force per unit area.

Example: Frictional resistance, viscous drag, surface shear

14. What is point load?

Point load is force acting at a particular point which causes displacement.

15. Write down the general finite element equation.

$$F = Ku$$

Where F- force vector and K is stiffness matrix and u is displacement vector

16. What are the classifications of coordinates?

Global coordinates, Local coordinates and Natural coordinates

17. What is Global coordinate system?

The points in the entire structure are defined using coordinates system is known as global coordinate system.

18. What is natural coordinates?

A natural coordinate system is used to define any point inside the element by a set of dimensionless number whose magnitude never exceeds unity. This system is very useful in assembling of stiffness matrices.

19. What are the characteristic of shape function?

- It has unit value at one nodal point and zero value at other nodal points.
- The sum of shape function is equal to one.

20. How do you calculate the size of the global stiffness matrix?

Global stiffness matrix size = Number of nodes X Degrees of freedom per node

21. If a displacement field in x direction is given by $u: 2x^2 + 4y^2 + 6xy$. Determine the Strain in x direction.

$$u: 2x^2 + 4y^2 + 6xy$$

$$\text{Strain, } \epsilon = \partial u / \partial x = 4x + 6y$$

22. Give examples for essential (forced or geometric) and non-essential boundary conditions. (AU DEC 2010)

The geometric boundary conditions are displacement, slope, etc. the natural boundary conditions are bending moment, shear force, etc.

23. Write down the expression of shape function N and displacement u for one dimensional bar element.

$$U = N_1 u_1 + N_2 u_2$$

$$N_1 = 1 - X/l$$

$$N_2 = X/l$$

24. Write down the expression of stiffness matrix for one dimensional bar element.

$$[K] = \frac{AE}{l} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

25. Write down the expression for longitudinal vibration of bar element

$$AE \frac{d^2 U}{dx^2} + \rho A \omega^2 U = 0$$

$$[K]\{U\} = \omega^2 [M]\{U\}$$

$$[k]^e = \frac{AE}{\ell} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$[m]^e = \frac{\rho A \ell}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

26. What is meant by dynamic analysis?

Analyzing the structure subjected to excitation force varying with time is known as dynamic analysis.

27. why polynomial radiation type heat transfer is not considered for thermal analysis?

The radiation heat flux is proportional to the fourth power of absolute temperature, which causes the problem to be a non linear. So this mode of heat transfer is not considered.

28. Write consistent and lumped mass matrix of a two noded bar element
Lumped mass matrix

$$[m^e]_{\text{lumped}} = \begin{bmatrix} \frac{\rho A \ell}{2} & 0 \\ 0 & \frac{\rho A \ell}{2} \end{bmatrix} \quad \text{for a bar element}$$

and consistent mass matrix

$$= \frac{\rho A \ell}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

29. Write lumped and consistent mass matrix for beam elements

$$[m]^e = \frac{\rho A \ell}{420} \begin{bmatrix} 156 & & & & \text{Symmetric} \\ 22\ell & 4\ell^2 & & & \\ 54 & 13\ell & 156 & & \\ -13\ell & -3\ell^2 & -22\ell & 4\ell^2 & \end{bmatrix}$$

$$[m^e]_{\text{lumped}} = \begin{bmatrix} \frac{\rho A \ell}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\rho A \ell}{2} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

30. Write the governing equation of heat transfer problem of composite wall

$$\frac{d}{dx} \left(k \frac{dT}{dx} \right) + Q = 0$$

$$T|_{x=0} = T_0 \quad q|_{x=L} = h(T_L - T_\infty)$$

31. Write the governing equation of pin fin problem

$$\frac{k_e}{\ell_c} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + \frac{h\ell_c}{3t} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} = \frac{hT_\infty \ell_c}{t} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

32. What are the two types of dynamic analysis?

Wave Propagation – the excitation is an impact or blast force and analysis of stress wave propagation in the structure . Ex. Crash analysis of car ,Impact analysis of missile

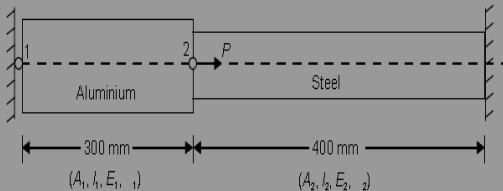
Structural dynamics – the entire structure is subjected to excitation and the response is studied over a period of time. Off shore structures subjected to wave loading, Automotive crankshaft vibration

PART B

1. Determine the nodal displacement at node 2, stresses in each material and support reactions in the bar due to applied force $P=400 \times 10^3$ and temperature rise of 30°C . Given

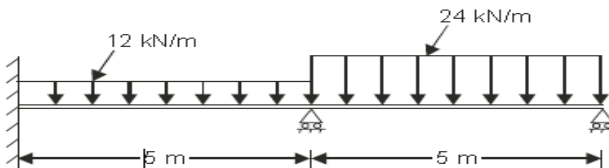
$$\begin{aligned} A_1 &= 2400 \text{ mm}^2 & A_2 &= 1200 \text{ mm}^2 \\ I_1 &= 300 \text{ mm} & I_2 &= 400 \text{ mm} \\ E_1 &= 0.7 \times 10^5 \text{ N/mm}^2 & E_2 &= 2 \times 10^5 \text{ N/mm}^2 \end{aligned}$$

and $\alpha_1 = 22 \times 10^{-6}/^\circ\text{C}$ $\alpha_2 = 12 \times 10^{-6}/^\circ\text{C}$

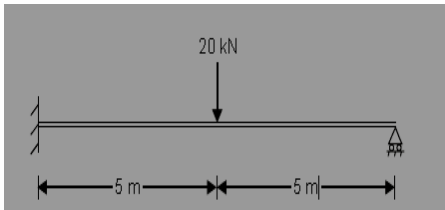


2. Analyse the beam by finite element method and determine the end reactions. Also determine the deflections at mid spans given.

$$E = 2 \times 10^5 \text{ N/mm}^2 \text{ and } I = 5 \times 10^6 \text{ mm}^4$$

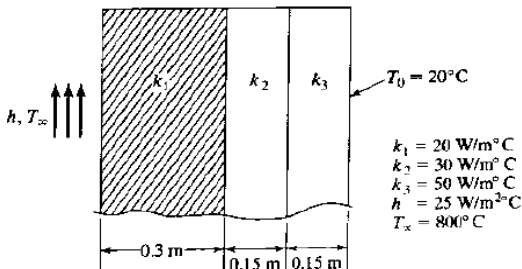


3. A beam of length 10m fixed at one end supported by a roller at the other end carries a 20 kN concentrated load at the center of the span. $E = 20\text{GPa}$ and moment of inertia as $24 \times 10^{-6} \text{ m}^4$ determine: 1. Deflection under load 2. Shear force and bending moment at mid span and 3. Reaction at supports

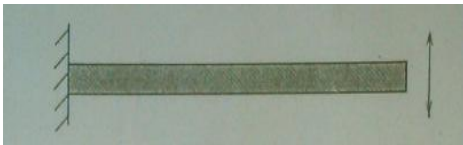


4. A physical phenomenon is governed by the differential equation . The boundary conditions are given by $w(0)=0=w(1)$ by taking a two term trial solution as $w(x)=C_1f_1(x)+ C_2f_2(x)$ with $f_1(x)=x(x-1)$ and $f_2(x)=x^2(x-1)$.Find the solution of the problem using the Galerkin's method

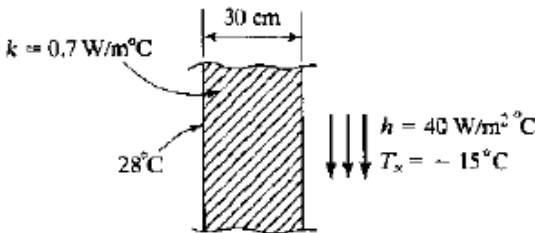
5. A composite wall consists of three materials. outer wall temperature is 20deg C. Convection heat transfer takes place at inner wall. $T_a=800\text{deg.C}$. $h=25\text{W/m}^2\text{C}$. Determine temperature distribution



6. Determine the natural frequency of the transverse vibration of the cantilever beam



7. A brick wall (Fig. P10.1) of thickness $L = 30$ cm, $k = 0.7$ W/m $^{\circ}$ C. The inner surface is at 28° C and the outer surface is exposed to cold air at -15° C. The heat-transfer coefficient associated with the outside surface is $h = 40$ W/m 2 . $^{\circ}$ C. Determine the steady state temperature distribution within the wall and also the heat flux through the wall. Use a two-element model.



8. Determine the natural frequency of a simply supported beam

UNIT 3 TWO DIMENSIONAL SCALAR VARIABLE PROBLEMS

1. How do you define two dimensional elements?

Two dimensional elements are defined by three or more nodes in a two dimensional plane. The basic element useful for two dimensional analysis is the triangular element.

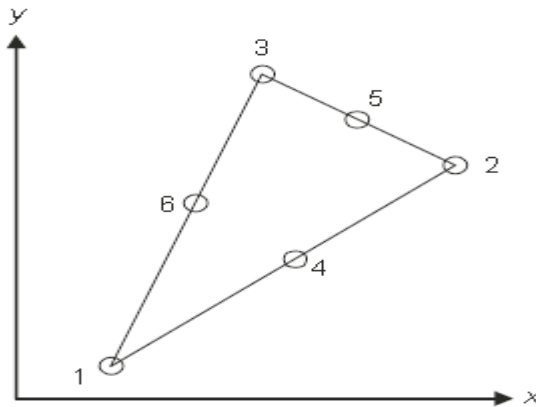
2. What is CST element?

Three noded triangular elements are known as CST. It has six unknown displacement degrees of freedom ($u_1, v_1, u_2, v_2, u_3, v_3$).

v3). The element is called CST because it has a constant strain throughout it.

3. What is LST element?

Six noded triangular elements are known as LST. It has twelve unknown displacement degrees of freedom. The displacement function for the elements are quadratic instead of linear as in the CST.



(a) Typical LST element

$$u = a_0 + a_1x + a_2y + a_3x^2 + a_4xy + a_5y^2$$

$$\frac{du}{dx} = a_1 + 2a_3x + a_4y$$

Strain is linear

4. What is QST element?

Ten noded triangular elements are known as Quadratic strain triangle. It is also called as cubic displacement triangle.

5. What is meant by plane stress analysis?

A thin planar body subjected to in-plane loading on its edge surface is said to be in plane stress. Plane stress is defined to be a state of stress in which the normal stress and shear stress directed perpendicular to the plane are assumed to be zero. A ring press fitted on a shaft is an example.

6. Define plane strain analysis.

Plane strain is defined to be a state of strain normal to the xy plane and the shear strains are assumed to be zero.

7. Write the strain displacement matrix for CST Element

$$\therefore [B] = \frac{1}{2A} \begin{bmatrix} b_1 & b_2 & b_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_1 & c_2 & c_3 \\ c_1 & c_2 & c_3 & b_1 & b_2 & b_3 \end{bmatrix}$$

where

$$\begin{array}{ll} b_1 = y_2 - y_3 & c_1 = x_3 - x_2 \\ b_2 = y_3 - y_1 & c_2 = x_1 - x_3 \\ b_3 = y_1 - y_2 & c_3 = x_2 - x_1 \end{array}$$

8. Give examples for plane stress, plane strain and axisymmetric problems.

Plane stress – Thin disc subjected to in-plane forces. Ex. Ring press fitted on a shaft

Plane strain. Strain in z direction is negligible – Dam wall

Axisymmetric – Boiler subjected to internal steam pressure

9. Differentiate between LST and CST

CST	LST
Strain is constant	Strain is linear
Ex.Three node triangular element	Ex.Six noded triangular element
Stiffness matrix is 6x6	Stiffness matrix is 12x12

10. Write the constitutive law matrix for plane stress, plane strain and axisymmetric problems

The stress – strain relationship matrix(B) is called constitutive law matrix

Plane stress condition

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{1 - \nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1 - \nu}{2} \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

Plane strain condition

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{(1 + \nu)(1 - 2\nu)} \begin{bmatrix} 1 - \nu & \nu & 0 \\ \nu & 1 - \nu & 0 \\ 0 & 0 & \frac{1}{2} - \nu \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

Axi-symmetric condition

$$\begin{Bmatrix} \sigma_r \\ \sigma_z \\ \sigma_\theta \\ \tau_{rz} \end{Bmatrix} = \frac{E}{(1+\mu)(1-2\mu)} \begin{pmatrix} 1-\mu & \mu & \mu & 0 \\ & 1-\mu & \mu & 0 \\ & & 1-\mu & 0 \\ & & & \frac{1-2\mu}{2} \end{pmatrix} \begin{Bmatrix} \varepsilon_r \\ \varepsilon_z \\ \varepsilon_\theta \\ \gamma_{rz} \end{Bmatrix}$$

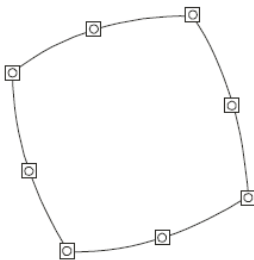
11. Write the shape functions of 3 noded CST element

$$N_1 = 1/2A \{ (X_2Y_3 - X_3Y_2) + (Y_2 - Y_3)X + (X_3 - X_2)Y \}$$

$$N_2 = 1/2A \{ (X_3Y_1 - X_1Y_3) + (Y_3 - Y_1)X + (X_1 - X_3)Y \}$$

$$N_3 = 1/2A \{ (X_1Y_2 - X_2Y_1) + (Y_1 - Y_2)X + (X_2 - X_1)Y \}$$

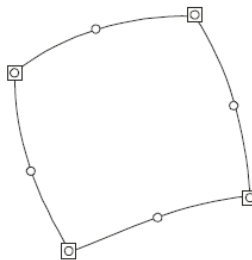
12. Distinguish between isoparametric, subparametric and superparametric elements



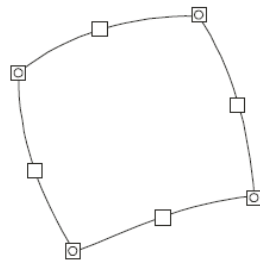
○ Nodes used for defining geometry

□ Nodes used for defining displacement

(a) Isoparametric



(b) Superparametric



(c) Subparametric

13. What is the purpose of natural coordinate system?

- A Coordinate system which permits the specification of a point within the element by a set of dimensionless number whose magnitude never exceeds unity.

-

14. What are the ways a three dimensional problems can be reduced to a 2D element problems

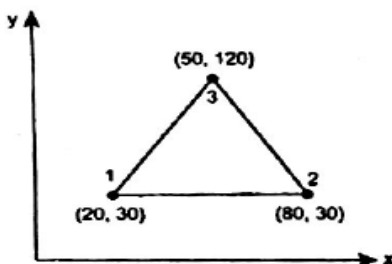
By selecting axis symmetric element, 3D problems can be reduced as 2D problems

15. Write down the stiffness matrix equation for two dimensional CST elements. Stiffness matrix

$$K = tAB^TDB$$

B^T -Strain displacement, D -Stress strain matrix A Area of the triangle, t thickness of the triangle

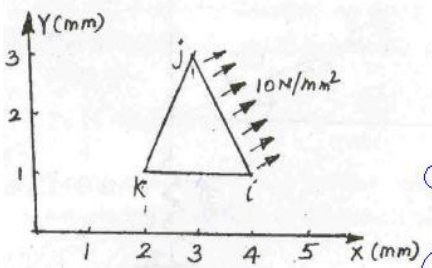
1.Determine the stiffness matrix for a CST element.The coordinates are given in mm.Assume plane strain condition.E=200GPa. $\nu=0.25$. $t=10\text{mm}$



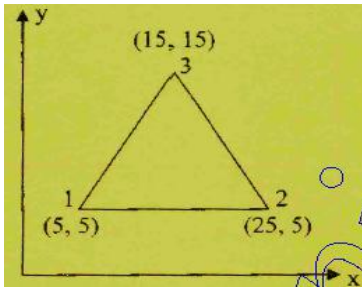
2. The x,y coordinates of nodes i,j and k of a triangular element are given by (0,0), (3,0) and (1.5,4) respectively. Evaluate the shape functions N_1, N_2 and N_3 at an interior point $P(2,2.5)$ mm of the element. Evaluate the strain displacement relation matrix for the above element and explain how stiffness matrix is obtained assuming scalar variable problem.

3. Derive the shape function of a CST Element

4. The triangular element shown in fig. is subjected to a constant pressure of 10 N/mm^2 along the edge ij. Assume $E = 200 \text{ GPa}$. Poisson ratio $\mu = 0.3$. and thickness of the element $= 2 \text{ mm}$. The coefficient of thermal expansion of the material is $\alpha = 2 \times 10^{-6} / ^\circ\text{C}$ and $\Delta t = 50^\circ\text{C}$. Determine the constitutive matrix and nodal force vector

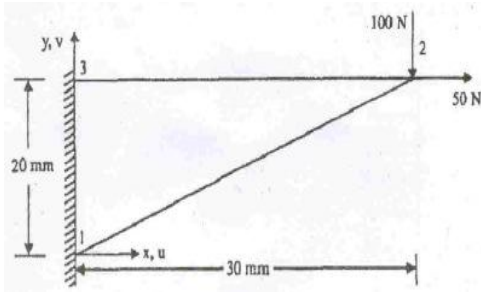


5. For the plane strain element the nodal displacements are $U_1 = 0.005 \text{ mm}$, $U_2 = 0.002 \text{ mm}$, $U_3 = 0 \text{ mm}$, $U_4 = 0.004 \text{ mm}$, $U_5 = 0.004 \text{ mm}$, $U_6 = 0 \text{ mm}$. Determine element stresses. Take $E = 200 \text{ GPa}$ and $\mu = 0.3$. Use unit thickness for plane strain condition.



6. The (x, y) coordinates of nodes i, j and k of a triangular element are given by $(0, 0)$, $(3, 0)$ and $(1.5, 4)$ mm respectively. Evaluate the shape functions N_1, N_2 and N_3 at an interior point $P(2, 2.5)$ mm for the element. For the same element obtain strain displacement matrix

7. Calculate the displacements and stress in the triangular plate fixed along one edge and subjected to concentrated load at its free end. Take $E = 70 \text{ GPa}$. Thickness of plate $= 10 \text{ mm}$. Poisson's ratio $= 0.3$



8. Derive the shape functions of quadratic quadrilateral element using natural coordinate systems with mid node.

UNIT 4 TWO DIMENSIONAL VECTOR VARIABLE PROBLEMS

1. What is the purpose of Isoparametric element?

It is difficult to represent the curved boundaries by straight edges finite elements. A large number of finite elements may be used to obtain reasonable resemblance between original body and the assemblage.

2. What is axisymmetric element?

Many three dimensional problem in engineering exhibit symmetry about an axis of rotation such type of problem are solved by special two dimensional element called the axisymmetric element

3. What are the conditions for a problem to be axisymmetric?

The problem domain must be symmetric about the axis of revolution All boundary condition must be symmetric about the axis of revolution All loading condition must be symmetric about the axis of revolution

4. Give the stiffness matrix equation for an axisymmetric triangular element. Stiffness matrix

$$N_1 = N_2 = N_3 = \frac{1}{3}$$

$$\bar{r} = \frac{r_1 + r_2 + r_3}{3}$$

$$K = 2\pi r A B^T D B$$

5. Write down the stress strain relationship matrix for an

axisymmetric triangular element

$$\mathbf{B} = \begin{bmatrix} \frac{z_{23}}{\det \mathbf{J}} & \frac{z_{31}}{\det \mathbf{J}} & \frac{z_{12}}{\det \mathbf{J}} & 0 & 0 & 0 \\ \frac{N_1}{r} & \frac{N_2}{r} & \frac{N_3}{r} & 0 & 0 & 0 \\ \frac{r_{32}}{\det \mathbf{J}} & \frac{r_{13}}{\det \mathbf{J}} & \frac{r_{21}}{\det \mathbf{J}} & \frac{z_{23}}{\det \mathbf{J}} & \frac{z_{31}}{\det \mathbf{J}} & \frac{z_{12}}{\det \mathbf{J}} \\ 0 & 0 & 0 & \frac{r_{32}}{\det \mathbf{J}} & \frac{r_{13}}{\det \mathbf{J}} & \frac{r_{21}}{\det \mathbf{J}} \end{bmatrix}$$

6. What are the conditions for a problem to be axisymmetric?

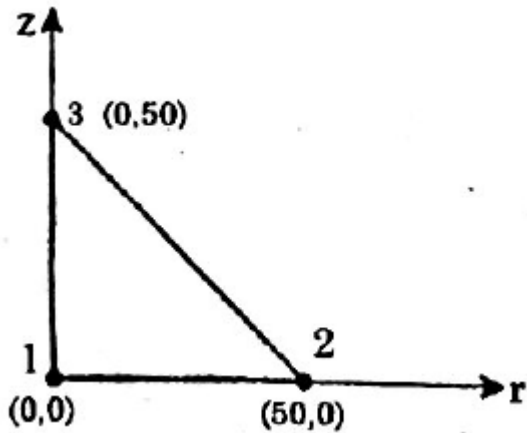
The problem domain must be symmetric about the axis of revolution All boundary condition must be symmetric about the axis of revolution All loading condition must be symmetric about the axis of revolution

7. What is simple natural coordinate?

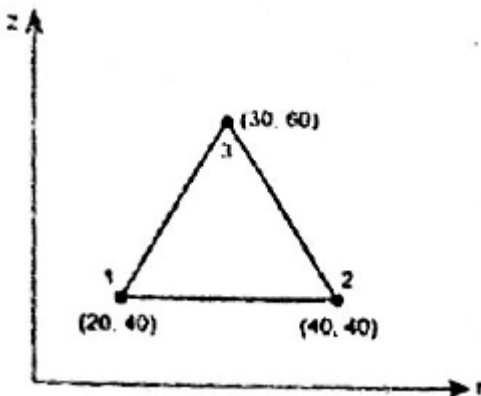
A simple natural coordinate is one whose value never exceeds one.(between -1 and 1.)

PART B

1.Derive the stiffness matrix for the axisymmetric element. Take $E=2.1 \times 10^5 \text{ N/mm}^2$. $\nu=0.25$.The coordinates are in mm



2. The nodal coordinates for an axisymmetric triangular element are given in Fig. Evaluate the strain displacement matrix



UNIT 5 ISOPARAMETRIC FORMULATION

1. Write down stiffness matrix equation for 4 noded isoparametric quadrilateral elements.

$$\mathbf{u} = \mathbf{Nq}$$

$$\boldsymbol{\epsilon} = \mathbf{Bq}$$

$$\mathbf{k}^e = t_e \int_{-1}^1 \int_{-1}^1 \mathbf{B}^T \mathbf{D} \mathbf{B} \det \mathbf{J} d\xi d\eta$$

where \mathbf{k}^e is evaluated using Gaussian quadrature.

2. What is the purpose of Isoparametric element?

It is difficult to represent the curved boundaries by straight edges finite elements. A large number of finite elements may be used to obtain reasonable resemblance between original body and the assemblage.

3. Write down the shape functions for 4 noded rectangular elements using natural coordinate system.

$$N_1 = \frac{1}{4}(1 - \xi)(1 - \eta)$$

$$N_2 = \frac{1}{4}(1 + \xi)(1 - \eta)$$

$$N_3 = \frac{1}{4}(1 + \xi)(1 + \eta)$$

$$N_4 = \frac{1}{4}(1 - \xi)(1 + \eta)$$

5. what is Jacobian ?

Jacobian is short form of determinant of Jacobian matrix which is a transformation matrix from natural coordinate system to cartesian coordinate system.

6. State the advantages of natural coordinate system.

The two dimensional element may be a rectangle or quadrilateral or may have curved shape. But when represented in the natural coordinate system they all will be represented by a straight square sides only. This enables to formulate the shape functions quite easily, which can be used to represent the geometry as well.

7. Write one point and two point formula

One point formula

$$\int_{-1}^1 f(\xi) d\xi \approx w_1 f(\xi_1)$$

Two point formula

$$\int_{-1}^1 f(\xi) d\xi \approx w_1 f(\xi_1) + w_2 f(\xi_2)$$

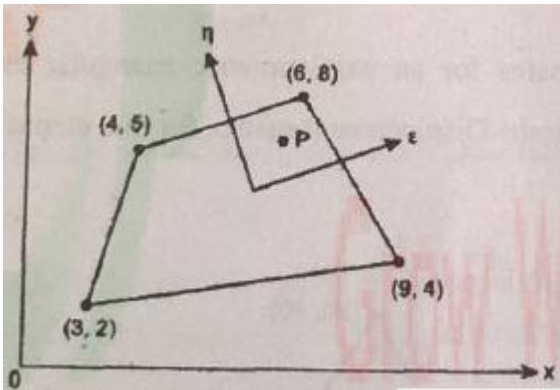
8. What meant by plane stress analysis?

Plane stress is defined to be a state of stress in which the normal stress and shear stress directed perpendicular to the plane are assumed to be zero.

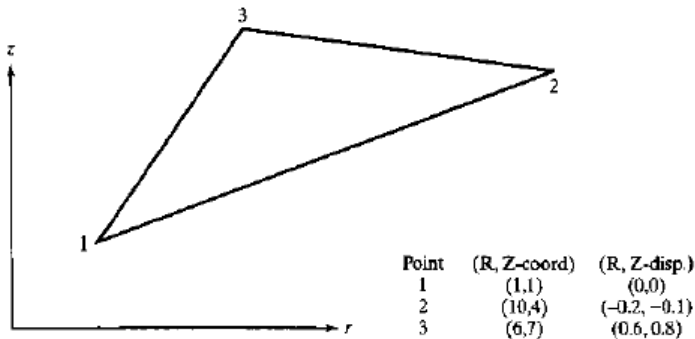
PART B

1. Derive the shape functions for a four noded rectangular element using natural coordinate system

2. Evaluate the coordinate of the point 'P' which has local coordinates $\xi=0.6$ and $\eta=0.8$



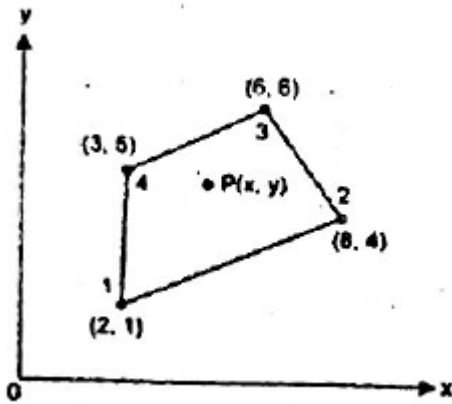
Determine the elemental stresses for the axisymmetric element



3.Evaluate using Gauss quadrature formula

$$I = \int_{-1}^1 \left[3e^x + x^2 + \frac{1}{(x+2)} \right] dx$$

4. For the isoparametric quadrilateral element the Cartesian coordinates of point 'P' are (6, 4). The loads 10kN and 12kN are acting in x and y direction on that point 'P'. Evaluate the forces.



UNIVERSITY QUESTION PAPER

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Question Paper Code : 72163

B.E./B Tech. DEGREE EXAMINATION, APRIL/MAY 2017.

Sixth/Seventh Semester

Mechanical Engineering

ME 6603 — FINITE ELEMENT ANALYSIS

(Common to Mechanical Engineering (Sandwich)/Automobile Engineering/Manufacturing Engineering/Mechanical and Automation Engineering)

(Regulations 2013)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. What do you mean by constitutive law?
2. Why polynomial type interpolation functions are mostly used in FEM?
3. Write down the expression of longitudinal vibration of bar element.
4. What are the difference between boundary value problem and initial value problem?
5. What is QST Element? www.recentquestionpaper.com
6. Write down the stress-strain relationship matrix for plane strain condition.
7. What are the assumptions used in thin plate and thick plate elements?
8. What are the ways which a three dimensional problems can be reduced to a two dimensional approach?
9. What are essential and natural boundary conditions? Give some examples.
10. Write down the stiffness matrix equation for four noded isoparametric element.

PART B — (5 × 16 = 80 marks)

11. (a) The following differential equation is available for a physical phenomenon.

$$\frac{d^2 y}{dx^2} + 50 = 0; 0 \leq x \leq 10$$

The Trial function is $y = a_1 x(10 - x)$

The boundary conditions are : $y(0) = 0$
 $y(10) = 0.$

Find the value of the parameter a_1 by the following methods.

- (i) Least square method
 (ii) Galerkin's method.

(16)

Or

- (b) Find the deflection at the centre of the simply supported beam of span length 'l' subjected to uniformly distributed load throughout its length as shown in Fig. 11(b) using (i) point collocation method (ii) sub-domain method.

(16)

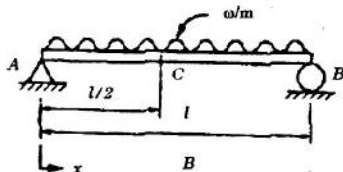


Fig. 11(b)

12. (a) Consider a bar as shown in Fig 12(a). An axial load of 200 kN is applied at point p. Take $A_1 = 2400 \text{ mm}^2$, $E_1 = 70 \times 10^9 \text{ N/mm}^2$, $A_2 = 600 \text{ mm}^2$, $E_2 = 200 \times 10^9 \text{ N/mm}^2$. Calculate the following, (i) The nodal displacement at point p, (ii) Stress in each element (iii) Reaction force.

(16)

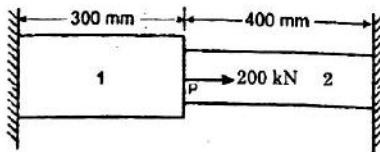


Fig. 12(a)

Or

- (b) Consider a three bar truss as shown in Fig.12(b). Take $E = 2 \times 10^5$ N/mm². Calculate the nodal displacement. Take $A_1 = 2000$ mm², $A_2 = 2500$ mm², $A_3 = 2500$ mm². (16)

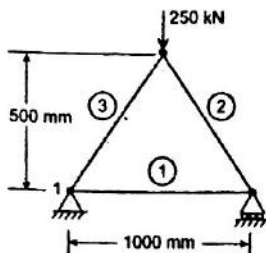


Fig. 12 (b)

13. (a) Determine the stiffness matrix for the CST Element shown in Fig 13(a). The coordinates are given in mm. Assume plane strain conditions. $E = 210$ GPa, $\nu = 0.25$ and $t = 10$ mm. (16)

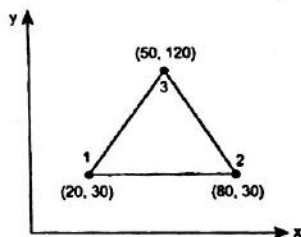


Fig 13(a)

Or

- (b) Derive the expression of shape function for heat transfer in 2D element. (16)
14. (a) Determine the stiffness matrix for the axisymmetric element shown in Fig 14(a). Take E is 2.1×10^5 N/mm², $\nu = 0.25$. The coordinates are in mm. (16)

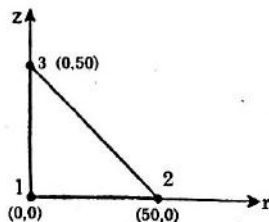


Fig 14(a)

Or

- (b) The nodal co-ordinates for an axisymmetric triangular element are given in Fig 14(b). Evaluate the strain-displacement matrix. (16)

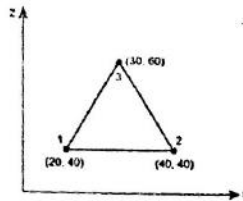


Fig 14(b)

15. (a) For the isoparametric quadrilateral element shown in Fig.15(a), the Cartesian coordinates of point 'P' are (6,4). The loads 10 kN and 12 kN are acting in x and y direction on that point P. Evaluate the nodal forces. (16)

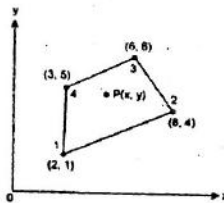


Fig 15(a)

Or

- (b) Evaluate the Jacobian matrix for the isoparametric quadrilateral element shown in Fig.15(b). (16)

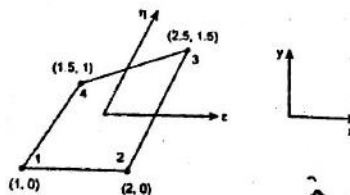


Fig. 15 (b)

Question Paper Code : 41413

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2018

Sixth/Seventh Semester

Mechanical Engineering

ME 6603 – FINITE ELEMENT ANALYSIS

(Regulations 2013)

(Common to Mechanical Engineering (Sandwich)/Automobile Engineering/
Manufacturing Engineering, Mechanical and Automation Engineering)

Time : Three Hours

Maximum : 100 Marks

Answer ALL questions.

PART – A

(10×2=20 Marks)

1. Compare the Ritz technique with the nodal approximation technique.
2. Differentiate between primary and secondary variables with suitable examples.
3. What are the properties of stiffness matrix ?
4. Write the conduction, convection and thermal load matrices for 1 D heat transfer through a fin.
5. Write down the shape functions for a 4 noded quadrilateral element.
6. Distinguish between scalar and vector variable problems in 2D.
7. Write the Strain Displacement matrix for a 3 noded triangular element.
8. Distinguish between plate and shell elements.
9. What are the advantages of natural coordinates ?
10. Derive the Jacobian of transformation for a 1D quadratic element.



11. a) A tapered bar made of steel is suspended vertically with the larger end rigidly clamped and the smaller end acted on by a pull of 10^5 N. The areas at the larger and smaller ends are 80 cm^2 and 20 cm^2 respectively. The length of the bar is 3 m. The bar weighs 0.075 N/cc . Young's modulus of the bar material is $E = 2 \times 10^7 \text{ N/cm}^2$. Obtain an approximate expression for the deformation of the rod using Ritz technique. Determine the maximum displacement at the tip of the bar.

(OR)

- b) The Governing Equation for one dimensional heat transfer through a fin of length l attached to a hot source as shown in fig. 11 b is given by

$$\frac{d}{dx} \left[-kA \frac{dT}{dx} \right] + hp(T - T_\infty) = 0$$

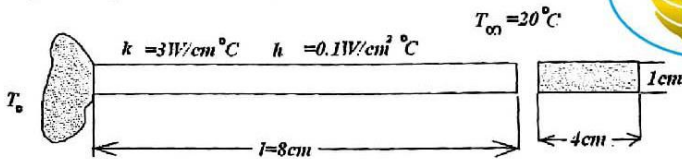


fig. 11 b

If the free end of the fin is insulated, give the boundary conditions and determine using the Collocation technique the temperature distribution in the fin. Report the temperature at the free end.

12. a) Determine the deflection in the beam, loaded as shown in Fig. 12 a, at the mid-span and at a length of 0.5 m from left support. Determine also the reactions at the fixed ends. $E = 200 \text{ GPa}$. $I_1 = 20 \times 10^{-6} \text{ m}^4$, $I_2 = 10 \times 10^{-6} \text{ m}^4$.

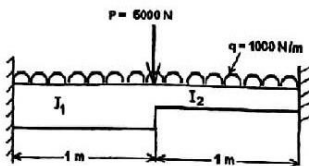


fig. 12 a

(OR)

- b) Determine the first two natural frequencies of longitudinal vibration of the stepped steel bar shown in Fig. 12 b and plot the mode shapes. All dimensions are in mm. $E = 200 \text{ GPa}$ and $\rho = 0.78 \text{ kg/cc}$. $A = 4 \text{ cm}^2$, length $l = 500 \text{ mm}$.

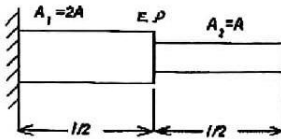


fig. 12 b



13. a) i) Determine three points on the 50°C contour line for the rectangular element shown in Fig. 13 a. The nodal values are $T_1 = 42^\circ\text{C}$, $T_2 = 54^\circ\text{C}$, $T_3 = 56^\circ\text{C}$ and $T_4 = 46^\circ\text{C}$. (8)

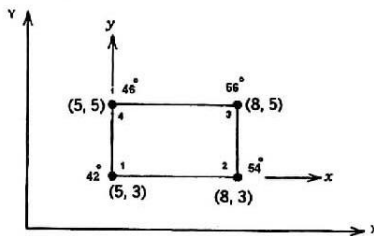


fig. 13 a

- ii) Derive the conductance matrix for a 3 noded triangular element whose nodal coordinates are known. The element is to be used for two dimensional heat transfer in a plate fin. (5)

(OR)

- b) A square shaft of cross section $1 \text{ cm} \times 1 \text{ cm}$ as shown in Fig. 13 b is to be analysed for determining the stress distribution. Considering geometric and boundary condition symmetry $1/8$ th of the cross section was modeled using four equisized triangular elements as shown. The element stiffness matrix and force vector for a triangle whose nodal coordinates are $(0, 0)$, $(0.25, 0)$ and $(0.25, 0.25)$ are given below. Carry out the assembly and determine the assembled stiffness matrix. Impose the boundary conditions and explain how the unknown stress function values at the nodes can be used to determine the shear stress.



$$\text{Stiffness matrix } [K] = \frac{1}{2} \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \quad \text{Load vector } \{f\} = \begin{Bmatrix} 29.1 \\ 29.1 \\ 29.1 \end{Bmatrix} \quad (13)$$

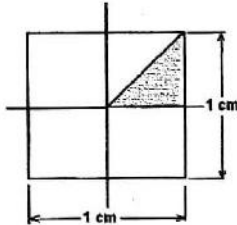
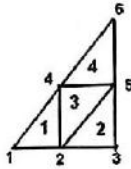


fig. 13 b



14. a) i) A thin plate of thickness 5 mm is subjected to an axial loading as shown in the Fig. 14 a. It is divided into two triangular elements by dividing it diagonally. Determine the Strain displacement matrix [B], load vector and the constitutive matrix. How will you derive the stiffness matrix? (Need not be determined). What will be the size of the assembled stiffness matrix? What are the boundary conditions? $E = 2 \times 10^7 \text{ N/cm}^2$ $\mu = 0.3$. (8)

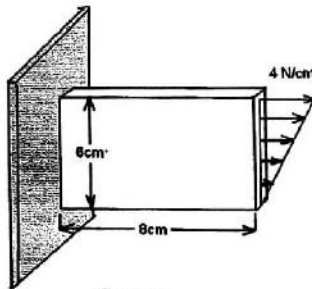


fig. 14 a

- ii) Differentiate between plane stress and plane strain analysis. (5)

(OR)

Question Paper Code : 80668

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2016.

Sixth Semester

Mechanical Engineering

ME 6603 — FINITE ELEMENT ANALYSIS

(Common to Mechanical and Automation Engineering and Manufacturing Engineering and Seventh Semester Mechanical Engineering (Sandwich) and Automobile Engineering)

(Regulations 2013)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. List the various methods of solving boundary value problems.
2. What is meant by Post Processing?
3. Why polynomials are generally used as shape function?
4. What is dynamic analysis?
5. State the assumptions in the theory of pure torsion.
6. What is an LST element?
7. What is meant by plane stress analysis?
8. Write the strain-displacement matrix for a CST element.
9. What is the purpose of isoparametric elements?
10. What are the advantages of Gauss quadrature numerical integration for isoparametric elements?

PART B — (5 × 16 = 80 marks)

11. (a) A beam AB of span ' l ' simply supported at ends and carrying a concentrated load W at the centre ' C ' as shown in Fig. 11(a). Determine the deflection at midspan by using Rayleigh-Ritz method and compare with exact solution.

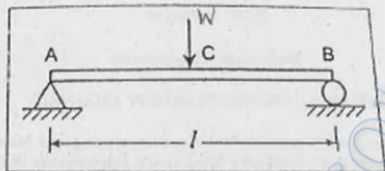


Fig. 11(a)

Or

- (b) A physical phenomenon is governed by the differential equation $(d^2w/dx^2) - 10x^2 = 5$ for $0 \leq x \leq 1$. The boundary conditions are given by $w(0) = w(1) = 0$. Assuming a trial solution $w(x) = a_0 + a_1x + a_2x^2 + a_3x^3$ determine using Galerkin method the variation of ' w ' with respect to x .
12. (a) For the bar element as shown in the Fig. 12(a). Calculate the nodal displacements and elemental stresses.

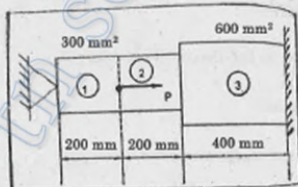


Fig. 12(a)

- (b) Determine the eigen values for the stepped bar shown in Fig. 12(b).

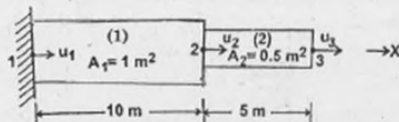


Fig. 12(b)

13. (a) The x, y coordinates of nodes i, j and k of a triangular element are given by $(0, 0)$, $(3, 0)$ and $(1.5, 4)$ mm respectively. Evaluate the shape functions N_1, N_2 and N_3 at an interior point $P(2, 2.5)$ mm of the element. Evaluate the Strain-displacement relation matrix B for the above same triangular element and explain how stiffness matrix is obtained assuming scalar variable problem.

Or

- (b) Calculate the temperature distribution in the stainless steel fin shown in Fig. 13(b). The region can be discretized into 3 elements of equal size.

$$h = 0.0025 \text{ W/cm}^2 \text{ } ^\circ\text{C} \quad T_a = 25^\circ\text{C}$$

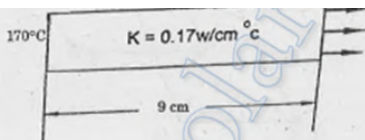


Fig. 13(b)

14. (a) For the triangular element as shown in the Fig. 14(a) determine the strain-displacement matrix $[B]$ and constitutive matrix $[D]$. Assume plane stress conditions. Take $\mu = 0.3$, $E = 30 \times 10^6 \text{ N/m}^2$ and thickness $t = 0.1 \text{ m}$. Also calculate the element stiffness matrix for the triangular element.

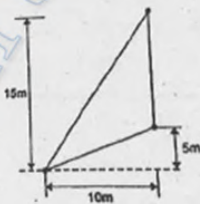


Fig. 14(a)

Or

$\mu = 0.3$ and $E =$

- (b) For the axisymmetric element shown in the Fig. 14(b), determine the stiffness matrix. Let $E = 2.1 \times 10^5 \text{ N/mm}^2$ and $\mu = 0.25$. The co ordinates are in mm.

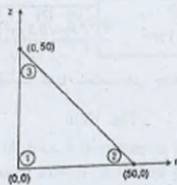


Fig. 14(b)

15. (a) Evaluate the Jacobian matrix for the linear quadrilateral element as shown the Fig. 15(a).

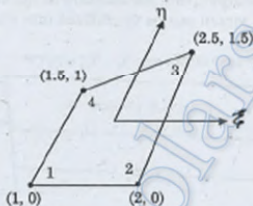


Fig. 15(a)

Or

- (b) Evaluate the integral by two point Gaussian quadrature

$$I = \int_{-1}^1 \int_{-1}^1 (2x^2 + 3xy + 4y^2) dx dy. \text{ Gauss points are } +0.57735 \text{ and } -0.57735 \text{ each of weight } 1.0000.$$

Question Paper Code : 50882

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2017

Sixth/Seventh Semester

Mechanical Engineering

ME 6603 – FINITE ELEMENT ANALYSIS

(Regulations 2013)

(Common to Mechanical Engineering (Sandwich)/Automobile Engineering/
Manufacturing Engineering/Mechanical and Automation Engineering)

Time : Three Hours

Maximum : 100 Marks

Answer ALL questions.

PART – A

(10×2=20 Marks)

1. List the various weighted residual methods.
2. Write the stiffness matrix for a one dimensional 2 noded linear element.
3. Give the Governing equation and the primary and secondary variables associated with the one dimensional beam element.
4. Write the natural frequency of bar of length 'L', Young's modulus 'E' and cross section 'A' fixed at one end and carrying lumped mass 'M' at the other end.
5. Write the governing equation for the torsion of non-circular sections and give the associated boundary conditions.
6. Why a CST element so called ?
7. What are the ways by which a 3D problem can be reduced to a 2D problem ?
8. Write down the shape functions for a 4 noded bi-linear rectangular element.
9. What are the advantages of natural coordinate system ?
10. Write the Jacobian for the one dimensional 2 noded linear element.

PART – B

(5×16=80 Marks)

11. a) Using any one of the Weighted Residual Method, find the displacement of given governing equation.

$$\frac{d}{dx} \left[x \frac{du}{dx} \right] - \frac{2}{x^2} = 0, 1 < x < 2$$

$$\text{at } x = 1, u = 2, \text{ at } x = 2, x \frac{du}{dx} = -\frac{1}{2}$$

(16)

(OR)



- b) Using Collocation method, find the solution of given governing equation

$$\frac{d^2\phi}{dx^2} + \phi + x = 0, 0 \leq x \leq 1 \text{ subject to the boundary conditions } \phi(0) = \phi(1) = 0.$$

Use $x = \frac{1}{4}$ and $\frac{3}{4}$ as the collocation points.

(16)

12. a) Determine the maximum deflection for the beam loaded as shown in Fig. 12(a), Youngs modulus 200 GPa and density $0.78 \times 10^6 \text{ kg/m}^3$. The beam is of T cross section shown in Fig. 12(b).

(16)

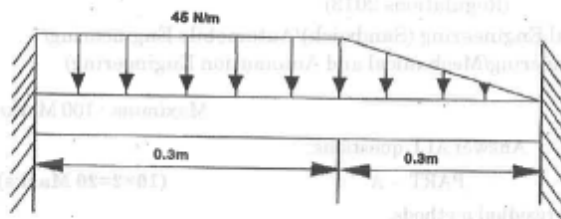


Fig. 12(a)

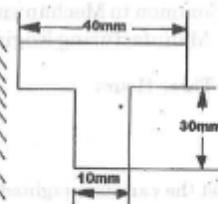


Fig. 12(b)

(OR)

- b) A metallic fin 20 mm wide and 4 mm thick is attached to a furnace whose wall temperature is 180°C . The length of the fin is 120 mm. If the thermal conductivity of the material of the fin is $350 \text{ W/m}^\circ\text{C}$ and convection coefficient is $9 \text{ W/m}^2^\circ\text{C}$, determine the temperature distribution assuming that the tip of the fin is open to the atmosphere and that the ambient temperature is 25°C .
13. a) For the square shaft of cross section $1 \text{ cm} \times 1 \text{ cm}$ as shown in Fig. 13(a), it was decided to determine the stress distribution using FEM by solving for the stress function values. Considering geometric and boundary condition symmetry, $1/8^{\text{th}}$ of the cross section was modeled using two triangular elements and a bilinear rectangular element as shown. The element matrices are given below. Carry out the assembly and solve for the unknown stress function values.

(16)

$$\text{for triangle } K = \frac{1}{2} \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \quad r = \begin{bmatrix} 29.1 \\ 29.1 \\ 29.1 \end{bmatrix}$$

$$\text{for rectangle } K = \frac{1}{6} \begin{bmatrix} 4 & -1 & -2 & -1 \\ -1 & 4 & -1 & -2 \\ -2 & -1 & 4 & -1 \\ -1 & -2 & -1 & 4 \end{bmatrix} \quad r = \begin{bmatrix} 43.6 \\ 43.6 \\ 43.6 \\ 43.6 \end{bmatrix}$$

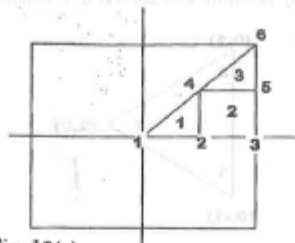


Fig. 13(a)

(OR)

- b) Determine the temperature distribution in the rectangular fin shown in Fig. 13(b). The upper half can be meshed taking into account symmetry using triangular elements.

(16)

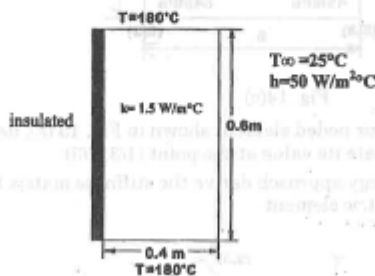


Fig. 13 (b)

14. a) i) The nodal co-ordinates for an axis-symmetric triangular element are given below.
 $r_1=10\text{ mm}$, $r_2=40\text{ mm}$, $r_3=40\text{ mm}$, $z_1=10\text{ mm}$, $z_2=10\text{ mm}$, $z_3=50\text{ mm}$.
 Evaluate strain displacement matrix (10)
 ii) Nodal values of the triangular element is shown in Fig. 14 (a). Evaluate element shape functions and calculate the value of temperature at a point whose coordinates are given (5,7). (6)

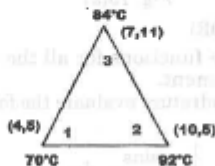


Fig. 14(a)

(OR)



- b) i) Assuming plane stress condition, evaluate stiffness matrix for the element shown in Fig. 14(b). Assume $E = 200$ GPa, Poisson's ratio 0.3. (10)

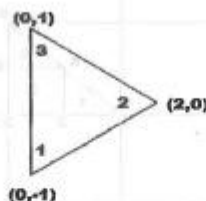


Fig. 14(b)

- ii) Determine the pressure at the location (7, 4) in a rectangular plate with the data shown in Fig. 14(c) and also draw 50 MPa contour line. (6)

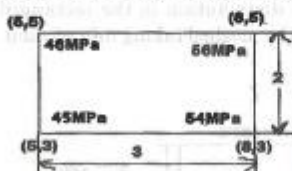


Fig. 14(c)

15. a) i) For the four noded element shown in Fig. 15 (a), determine the Jacobian and evaluate its value at the point (1/3, 1/3) and evaluate its value at the point (1/3, 1/3) (6)
- ii) Using energy approach derive the stiffness matrix for a 1D linear isoparametric element. (8)

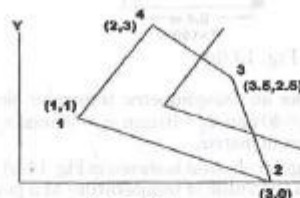


Fig. 15(a)

(OR)

- b) i) Derive the shape functions for all the corner nodes of a nine noded quadrilateral element. (8)
- ii) Using Gauss Quadrature evaluate the following integral using 1, 2 and 3 point integration. (8)

$$\int_{-1}^1 \frac{\sin s}{S(1-s^2)} ds$$